FINAL EXAM REVIEW - MATH 1021 SECTION A

You will be allowed one page of notes and a calculator (ones that can't do linear algebra) for this exam.

(1) Find the line which best approximates the points $(0,0)$, $(1,1)$, $(2,4)$, $(3,6)$ using the least squares approximation method. If the line is given by the equation $f(x)$, find the values of $f(0)$, $f(1)$, $f(2)$ and $f(3)$. Which point does this line come closest to?

(2) Find the quadratic which best approximates these points.

(3) Find the dimension of the null space of the matrix

$$A = \begin{bmatrix} 1 & 3 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

(4) Find a basis of the span of the columns of the matrix A.

(5) Are the following vectors linearly independent?

$$\left\{ \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ 1 \\ -2 \end{bmatrix} \right\}$$

If not, then find non-zero coefficients such that a linear combination of these vectors is 0.

(6) Let

$$B = \begin{bmatrix} 14 & -22 & -43 & 4 \\ -2 & 4 & 7 & -1 \\ 6 & -10 & -19 & 2 \\ 8 & -14 & -26 & 3 \end{bmatrix}$$

Find a basis of the eigenspace of B corresponding to the eigenvalue 1.

(7) Find the eigenvalues of the matrix

$$C = \begin{bmatrix} -4 & -10 & -32 \\ 11 & 67 & 222 \\ -4 & -20 & -66 \end{bmatrix}$$

(8) What is the dimension of each of the eigenspaces of C for each of the eigenvalues that you found in the last question.

(9) What is the equation of the line which lies in the plane $2x - y + z = 4$ and is perpendicular to the line $(2,1,-1) + t(-1,0,1)$.

(10) What is the equation of the plane which includes the the line $(2,1,-1) + t(-1,0,1)$ and and the point $(-1,1,4)$.

(11) Find a matrix P such that PQP^{-1} is diagonal where

$$Q = \begin{bmatrix} -1 & -10 & 6 \\ 0 & -11 & 6 \\ 0 & -18 & 10 \end{bmatrix}.$$

(12) Assume that $a_0 = 1$ and $a_1 = 1$ and $a_n = 2a_{n-1} + a_{n-2}$. Give a formula for a_n.

(13) Find the point on the line $(2, 1, -1) + t(-1, 2, 1)$ which is closest to the point $(4, 0, 1)$.

(14) Is the set of vectors $\{[2 - a + b, -1 + a, c]^T : a, b, c \in \mathbb{R}\}$ a subspace of $\mathbb{R}^3$?

(15) What is the dimension of the span of the vectors $\left\{ \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix} \right\}$.

(16) Find the solution to the following system of linear equations:

$$\begin{aligned} 5x_1 + x_2 - x_3 &= 12 \\ 9x_1 + x_2 - x_3 &= 22 \\ x_1 - x_2 + 5x_3 &= 12 \end{aligned}$$

(17) Does the set of vectors $\{[x, y, z]^T : 3x - y + z = 4\}$ form a subspace of $\mathbb{R}^3$?

(18) Simplify the following expression so that it is a simple product of the matrices R, $(R^T)^{-1}$, R^{-1}, R^T, A, $(A^T)^{-1}$, A^{-1} and A^T,

$$(R^{-1}A^T)^T((A^{-1}R^TA^T)^{-1}A^T)^T.$$

(19) Give the parametric equations which describe the plane $x + y - 6z = 4$.

(20) Find a basis for the subspace $\{[x, y, z, w]^T : 3x - y + z + 2w = 0 \text{ and } 2x - y + w = 0\}$.