HOMEWORK ASSIGNMENT NO. 2

DATE: ASSIGNED OCT 24, 2011 - DUE NOVEMBER 14, 2011

Your assignment should include complete sentences and explanations and not just a few equations or numbers. A solution will not receive full credit unless you explain what your answer represents and where it came from. You may discuss the homework with other students in the class, but please write your own solutions. The first question is adapted from "What is the Name of This Book?" by Raymond Smullyan.

(1) There are four doors X,Y,Z,W leading out of the room, at least one of the doors is the exit, behind the others is 'certain death.' There are 8 people to give you a clue to which door you should choose. Each one is either a knight and always tells the truth or is a knave and always lies. They make the following statements:
  (a) X is a good door
  (b) At least one of the doors Y, Z is good.
  (c) (a) and (b) are both knights.
  (d) X and Y are both good doors.
  (e) X and Z are both good doors.
  (f) Either (d) or (e) is a knight
  (g) If (c) is a knight so is (f).
  (h) If (g) and I are both knights, so is (a).

  Let $Knight(x)$ represent the sentence "x is a knight" and and $Good(u)$ represent "u is a good door." So, for instance, the first statement that (a) made is $Good(X)$. Translate the remaining 7 statements above into a statement using logical connectives and determine how to exit the room without facing certain death.

(2) Explain in English what the following mathematical statements mean. Which of the statements are true, which are false? Why?
  (a) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, x < y$
  (b) $\exists x \in \mathbb{R}, \forall y \in \mathbb{R}, x < y$
  (c) $\exists x \in \mathbb{R}, \exists y \in \mathbb{R}, x < y$
  (d) $\forall x \in \mathbb{R}, \forall y \in \mathbb{R}, x < y$
  (e) $\forall x \in \mathbb{Z}, \exists y, z \in \mathbb{Z}, x^2 + y^2 = z^2$
  (f) $\exists x, y, z \in \mathbb{Z}, x^2 + y^2 = z^2$
  (g) $\forall x \in \mathbb{R}, \sqrt{x^2} = x$
  (h) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, x^2 + y^2 = 9$

(3) Any string of symbols in mathematics is called a word (it does not refer to being a "word in English") and the length of the word is the number of symbols in it. The set of symbols that make up the word are called the letters and the set of all possible letters is called the alphabet. Example if $w = ABCBCBCBAB$, then w is a word in the alphabet A, B, C of length 10.

  Consider the following statements about words in the letters R and S. Give an example of one word that makes the following statements true and one word that makes the statement false (if possible) and explain why your words make the statement true or false. If the sentence is always true or always false for all words in R and S, then explain why. Example: If the statement is "The word w has at least three R's." then $w = RRRRR$ makes the statement true because it has five R's and $w = SSSSS$ makes the statement false because it has no R's.

(a) The word w begins with an S and ends with an R, or begins with an R and ends with an S, or has a length less than 7.
(b) The word w begins and ends with the same letter, and w contains at least three R's, and has a length less than 7.
(c) If w is of length less than 5 and contains the substring $SRRS$, then it begins with an R.
(d) If w contains an odd number of R's and an odd number of S's, then it is of even length.
(e) If every S in w is followed immediately by an R, then there are at least two Rs in the word.
(f) If w contains $SRRS$ in consecutive positions and every R in w is followed immediately by an S, then w contains at least three Rs.