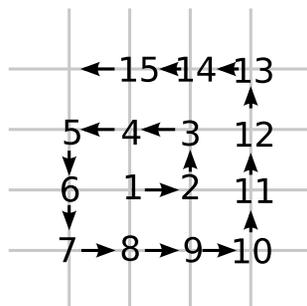# DISCUSSION FOR FIFTH TUTORIAL

DATE: NOVEMBER 21 OR 28, 2011 : DUE IN TUTORIAL DECEMBER 5, 2011 OR JANUARY 9, 2012

The following problem is (roughly) from Thinking Mathematically Second Edition p. 125 (Number Spirals).

Say that you want to number the vertices of a grid using a spiral pattern as you see in the picture below. Say that the number 1 is on the origin $(0,0)$ then 2 is placed on the vertex at $(1,0)$, 3 is placed at $(1,1)$, 4 at $(0,1)$, 5 at $(-1,1)$, 6 at $(-1,0)$, and subsequent numbers follow the pattern below.

One question I might ask is, "what are the coordinates of the number 411?" The way you might proceed to find the answer is to draw a grid sufficiently large that it contains enough squares to fill in the numbers 1 through 411 and proceed to draw a big spiral. That is an ok answer for that particular value, but it does nothing to help me find the coordinates of $17,411$ or $17,300,411$. One way to come up with a method of finding where these numbers appear is to consider some special cases:

- Where are the square numbers? even numbers? odd numbers? multiples of 3?
- What numbers are on the corners of the form (a, a)? of the form $(a, -a)$? (for $a \in \mathbb{Z}$)
- What numbers appear on particular rows or columns?

The main question you should try to answer is "given a positive integer n, what are the coordinates where that integer is placed?" This should be a formula or a procedure for finding those coordinates that should apply to all integers and in particular work on the examples above.

To extend this problem you might consider other types of spirals (e.g. ones that are more rectangular, see for example the examples below). Another possible extension would be to consider other ways of filling the plane or spirals in 3 dimensions.

7	6	5	4			9	8	7	6	5
8	1	2	3	OR		10	1	2	3	4
9	10	11	12			11	12	13	14	15