MATH 2590 - ASSIGNMENT 2

OCTOBER 5, 2010

If we divide an integer a by by a positive integer n then $a = qn + r$ where q is called the 'quotient,' and r the 'remainder' with $0 \leq r < n$. If a and b are integers, I will say that $a \equiv b$ $(mod\ n)$ if $a - b$ is 'divisible' by n (meaning that the remainder is 0 when I divide $a - b$ by n). (Example: $2 \equiv 7$ $(mod\ 5)$ because $2 - 7 = -5$ which is divisible by 5) When I have an *expression* $(mod\ n)$ this means that after I figure out what *expression* is, I should find the remainder of that expression when I divide by n. (Example: $3 \cdot 3 - 2$ $(mod\ 5)$ is 7 $(mod\ 5)$ and since $7 = 1 \cdot 5 + 2$ then this is 2 $(mod\ 5)$).

Part I - Modular arithmetic

(1) Find the remainder:
(a) $-3\ (mod\ 20)$ (b) $55\ (mod\ 4)$ (c) $238712638\ (mod\ 10)$ (d) $-45\ (mod\ 11)$

(2) Compute the following:
(a) $3 \cdot 17\ (mod\ 20)$ (b) $5 \cdot 5 + 12\ (mod\ 4)$ (c) $5\ (mod\ 4)$ (d) $5 \cdot 5\ (mod\ 4)$
(e) $12\ (mod\ 4)$ (f) $55 \cdot 31 \cdot 61 \cdot 103\ (mod\ 10)$ (g) $5^3\ (mod\ 7)$

(3) Compute powers of 2 $(mod\ 5)$
(a) $2^2\ (mod\ 5)$ (b) $2^3\ (mod\ 5)$ (c) $2^4\ (mod\ 5)$ (d) $2^5\ (mod\ 5)$

(4) Compute powers of 2 $(mod\ 7)$
(a) $2^2\ (mod\ 7)$ (b) $2^3\ (mod\ 7)$ (c) $2^4\ (mod\ 7)$ (d) $2^5\ (mod\ 7)$ (e) $2^6\ (mod\ 7)$

(5) Make a table of the powers of 2 $(mod\ 9)$ up to the point where the table repeats

(6) Make a table of the powers of 2 $(mod\ 11)$ up to the point where the table repeats

(7) Make a table of the powers of 2 $(mod\ 13)$ up to the point where the table repeats

(8) Make a table of the powers of 2 $(mod\ 15)$ up to the point where the table repeats

Part II - Shuffling cards

(1) How many shuffles of a 4 card deck brings the cards back to the original order?
(2) How many shuffles of a 6 card deck brings the cards back to the original order?
(3) How many shuffles of a 8 card deck brings the cards back to the original order?
(4) How many shuffles of a 10 card deck brings the cards back to the original order?
(5) How many shuffles of a 12 card deck brings the cards back to the original order?
(6) How many shuffles of a 14 card deck brings the cards back to the original order?

Date: due Oct 26, 2010.