QUESTION FOR JOURNAL ENTRY # 7

This is roughly the problem *Square Take-away* from Thinking Mathematically.

Take a rectangular piece of paper and remove from it the largest possible square. Repeat the process with the left-over rectangle. Does the process terminate? If so, how many steps does it take (what does this depend on)?

Consider the following examples of these 8×5, 40×25, $1 \times \frac{8}{5}$, 3×2, 4×6, 5×5, $1 \times n$, 9×1, 9×2, 9×3, 9×4, $\sqrt{2} \times 2\sqrt{2}$, $1 \times \sqrt{2}$, $2 \times (1 + \sqrt{5})$. How many steps does it take for each of these? What does your answer depend on?

Of course the first step is to establish a conjecture and the next step is to justify that your conjecture is correct. Your solution should explain your observations if the ratio of the sides of the rectangle is a rational number and then when it is not a rational number. Moreover you will need to compute data for lots of examples (more than those that I suggested above) to see if you can observe a pattern for the number of steps that it takes to remove the whole rectangle.