CHECKOFF LIST OF PROBLEMS FROM MATH 5020

MARCH 21, 2004

Problems from Tucker's "Applied Combinatorics": Assignment do as many as necessary.

There were a couple of "Prove by counting a set in two different ways:" first question:

$$\binom{n+1}{k+1} = \binom{n}{k} + \binom{n}{k+1}$$

$$\binom{n+2}{k+2} = \binom{n}{k} + 2\binom{n}{k+1} + \binom{n}{k+2}$$

$$\binom{n+r}{k+r} = \binom{n}{k}\binom{r}{0} + \binom{n}{k+1}\binom{r}{1} + \cdots + \binom{n}{k+r}\binom{r}{r}$$

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first question:

$$\binom{n+k+1}{k} = \binom{n}{0} + \binom{n+1}{1} + \binom{n+2}{2} + \cdots + \binom{n+k}{k}$$

$$\binom{n+k+2}{k} = \binom{n}{0}(k+1) + \binom{n+1}{1}k + \binom{n+2}{2}(k-1) + \cdots + \binom{n+k}{k}$$

I don't think that i asked you to prove the general case of the second problem but I did write it down in class.

$$\binom{n+k+m}{k} = \binom{n}{0}\binom{k+m-1}{m-1} + \binom{n+1}{1}\binom{k+m-2}{m-1} + \binom{n+2}{2}\binom{k+m-3}{m-1} + \cdots + \binom{n+k}{k}\binom{m-1}{m-1}$$

Verify that if $a \equiv a' \ (mod\ n)$ and $b \equiv b' \ (mod\ n)$ then $a \pm b \equiv a' \pm b' \ (mod\ n)$, $aa' \equiv bb' \ (modn)$.

I have mentioned in my notes that I have assigned the following problems from the text "Number Theory" by George Andrews (numbers in parentheses were probably not ever assigned, but you can do them anyway if you want):
Section 1.1 # 1-18
Section 1.2 # (1), 2, (3), 4, 5
Section 2.1 # (6), 7
Section 2.2 # 2,3, 6, (8), 9, 12

The following problems I assigned to be done on the FORUM. I may or may not have mentioned them in class:
Section 1.1 # 3,4,5,6,8,9,11,12,13,14,15,16,18

Section 1.2 #2,4,5

Section 2.2 # 2,3,6,9,12

Section 5.2 # 3, 16, 19

From January 5, 2004:
How many rearrangements are there of the letters of the word 'GREATGRANDFATHER' where the word 'GREAT' appears consecutively (in that order). ____________

From January 12, 2004:
I gave an RSA problem that was to be done on the computer. This is posted on the web page.

In Lotto 6/49, what is the probability of holding a ticket with 4 of the 6 winning numbers correct and the bonus? ____________

From January 19, 2004:
Buy a lottery ticket. ____________

Verify that $d(n)$ and $\sigma(n)$ are multiplicative functions by first proving that $d(p_1^{\alpha_1} \cdots p_k^{\alpha_k}) = (\alpha_1 + 1)(\alpha_2 + 1) \cdots (\alpha_k + 1)$ (note: this one you should be able to prove by the multiplication principle) and that $\sigma(p_1^{\alpha_1} \cdots p_k^{\alpha_k}) = \prod_{i=1}^{k} \frac{p_i^{\alpha_i+1}-1}{p_i-1}$ (note: prove this by induction on k and the proof is in the book). Convince yourself that these formulas show that d and σ are multiplicative.

Go to your favorite search engine and enter "Encyclopedia of Integer Sequences" and enter into the database "1,1,3,4,11,..." which is what we calculated to be the first five number of if the sequence $a_n :=$ then number of words of 1s and 2s with at least as many 1s as 2s. Use the database to find the entry that we are looking for (you might need to calculate an extra term or two) and get the formula for this sequence. ____________

Let $a_i =$ the number of widgets of size i and $b_i =$ the number of doodles of size i (we had been talking about the concept of sequences representing the number of objects in some set and "widgets" and "doodles" are my words to represent an arbitrary set of objects).
Explain what $a_i + b_i$ represents.
Explain what $a_i b_j$ represents.
Explain what $a_n b_0 + a_{n-1} b_1 + a_{n-2} b_2 + \cdots + a_1 b_{n-1} + a_0 b_n$ represents.

From January 26, 2004:
Problem #15 in section 1.1. Please look on the web page for my "hint" on this one. ______________________

Any additional problems that I mentioned could be done:

______________________ ______________________
______________________ ______________________
______________________ ______________________

From Febrauary 9, 2004: You should do some of each of the following problems. I will be looking for evidence that you know how to answer questions from each of the following sets of problems (#9 is less important).

____1. Some connections between sequences and sets of objects: Part I
____2. Some connections between sequences and sets of objects: Part II
____3. Some connections between algebraic expressions and sequences : Part I
____4. Some Connections Between Algebraic Expressions and Sequences : Part II
____5. Some Connections Between Algebraic Expressions and Sequences : Part III
____6. Some Connections Between Algebraic Expressions and Sequences : Part IV
____7. Some Connections Between Algebraic Expressions and Combinatorial Descriptions : Part I
____8. Some Connections Between Algebraic Expressions and Combinatorial Identities
____9. Some Connections Between Combinatorial Identities and Algebraic Expressions
____10. Partition generating functions : Part I
____11. Partition generating functions : Part II

Draw the Young diagrams for all partitions of 9 with less than or equal to 3 parts and the Young diagrams for all partitions of 9 with parts of size 1,2 or 3.
Explain the following formulas

$$\prod_{i\geq 1} \frac{1}{1-q^i} = 1 + \sum_{k\geq 1} q^{k^2} \prod_{i=1}^{k} \frac{1}{(1-q^i)^2}$$

$$\prod_{i\geq 1} \frac{1}{1-zq^i} = 1 + \sum_{k\geq 1} \left(z^k q^{k^2} \prod_{i=1}^{k} \frac{1}{(1-q^i)} \prod_{i=1}^{k} \frac{1}{(1-zq^i)} \right)$$

$$\prod_{i\geq 1} \frac{1}{1-zq^i} = 1 + \sum_{k\geq 1} \left(z^k q^{k} \prod_{i=1}^{k} \frac{1}{(1-q^i)} \right)$$

$$\prod_{i\geq 1} 1+zq^i = 1 + \sum_{k\geq 1} \left(z^k q^{k(k+1)/2} \prod_{i=1}^{k} \frac{1}{(1-q^i)} \right)$$

Clearly defines each of the problems : ______________________
Possible to tell which problems have been done : ______________________
Clearly has mastered each of the concepts in the homework assignments :
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Remarks on organization: ______________________

Overall this assignment receives an: A+/A/B+/B/C+/C/D+/D/F