UNEXAM #4

ASSIGNED: MARCH 23, 2006, DUE: MARCH 30, 2006

Remember that the important aspect of this assignment is not the answer, but the solution. Please justify the following *using generating functions*. For the first problem you may require a computer to do the calculation, but your explanation doesn't need it. Note that each one of the following problems is equivalent to a problem in number theory (e.g. the first is find the number of solutions to the equation $4x_1 + 6x_2 + x_3 = n$ with $x_i \geq 0$ and $x_3 \leq x_2$).

I will be grading you on a 5 point scale again. Please provide me with a single, clear, short solution which includes all details. Each of these problems boils down to essentially one thing: organize the question in a way that you can apply the multiplication principle of generating functions. When you explain this you should tell me (1) how one should rearrange what we are looking for so that it can be represented as a set of tuples where each entry is independent of the others, (2) how to write down the generating function for each of the entries in the tuple, (3) that the generating function for the number of tuples will be the product or sum of generating functions, and (4) (if necessary) how to find the answer to the question.

(1) In an exhibition football game between the Toronto Argonaugts and Mrs. Jones' 3^{rd} grade class the final score for the game was 254 to 0. How many different combinations of field goals, touchdowns and extra points could have yielded this score? Recall that a field goal is 4 points, a touchdown is 6 points and an extra point is 1 point (which can only be scored after a touchdown).

Answer: 210

(2) What is the generating function for the number of solutions to the equation

$$x_1 + 2x_2 + 3x_3 + 4x_4 + 5x_5 = n$$

with $x_i \geq 0$ and where x_1 is odd, x_2 is even and $x_3 \neq x_4$?
Hint: The condition of $x_3 \neq x_4$ is the opposite of $x_3 = x_4$. Subtract the generating function for solutions with $x_3 = x_4$ from the generating function for all solutions.

Answer:

$$\frac{q^4 + q^5 - 2q^8}{(1-q^2)(1-q^3)(1-q^4)^2(1-q^5)(1-q^7)}$$

(3) To make a of bracelet with yellow, orange, red, green and blue beads you must first choose the number of beads with each color. Given that you have an unlimited supply of yellow, orange and red beads but only 9 green beads and 6 blue, how many ways are there of choosing n beads to go on a bracelet?

Answer:

$$\binom{n+4}{4} - \binom{n-6}{4} - \binom{n-3}{4} + \binom{n-13}{4}$$