SEQUENCES AND SETS OF OBJECTS AND THE OLEIS

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The following are examples of what we want to think of as "widgets" or "doodles." These are sets of objects and they are graded by a a number n by what we referred to in earlier exercises as a 'type' or a 'size.'

Write the first 6-8 terms of the following sequences. If the description of the set seems ambiguous then make precise the set of words you are counting before proceeding. Assume that the sequences start at $n = 0$, write a formula for a_n if possible (HINT: the empty word is a word of length 0). The OLEIS sequence number can be found by going to the web site 'The On-Line Encyclopedia of Integer Sequences' and entering the first terms which you calculated.
`http://www.research.att.com/~njas/sequences/index.html`

There are two possible outcomes for this problem.
(1) It may well be that the sequence that you enter is not in the database. Your next step if you find no information in the database will be to calculate more terms (try to get as high as 20), try to arrive at a formula for a_n, and then submit the sequence to the OLEIS.
(2) It could be that your sequence is in the database. If the description in the database clearly matches your sequence then you are lucky and you are done. If the description is not the same then explain why you know that your sequence is equal to the one in the database (it is not sufficient to say that it is the same because the first x values are the same, you need an argument to show they are the same).

We will do this as a Forum exercise. Do this problem RIGHT AWAY because it may take a while to complete all steps. If you find your sequence in the database I want you to convince me that they are the same. If your sequence is not in the database you should submit it with as much data as you can create.

(1) The number of words of length n created with the letters a and b with no adjacent letters forming the pattern aba. _____
Formula? $a_n =$ _____ OLEIS sequence number _____
(2) The number of words of length n created with the letters a and b with no adjacent letters forming the pattern aba or bab. _____
Formula? $a_n =$ _____ OLEIS sequence number _____
(3) The number of words of length n created with the letters a and b with no adjacent letters forming the pattern $abba$. _____
OLEIS sequence number _____
(4) The number of words of length n created with the letters a and b with no adjacent letters forming the pattern aa or aba. _____
OLEIS sequence number _____
(5) The number of words of length n created with the letters a and b with no adjacent letters forming the pattern aba or $abba$. _____
OLEIS sequence number _____

(6) The number of words of length n created with the letters a and b with no adjacent letters forming the pattern $abab$ or $abba$. _____
OLEIS sequence number _____

(7) The number of words of length n created with the letters a, b, c with at most half of the letters are a's. _____
OLEIS sequence number _____

(8) The number of words of length n created with the letters a, b, c with no consecutive letters being equal bs or equal cs. _____
OLEIS sequence number _____

(9) The number of words of length n created with the letters a, b, c with no subword (not just adjacent but any subsequence of letters read from left to right) are of the form abc. _____
OLEIS sequence number _____

(10) The number of words of length n created with the letters a, b, c with at least as many a's as b's and at least as many b's as c's. _____
OLEIS sequence number _____

(11) The number of words of length n created with the letters a, b, c with at least as many a's as b's and at least as many b's as c's and no as adjacent. _____
OLEIS sequence number _____

(12) The number of words of length n created with the letters a, b, c with at least as many a's as b's and at least as many b's as c's and no adjacent letters forming the pattern aba. _____
OLEIS sequence number _____

(13) The number of words of length n created with the letters a, b, c with at least as many a's as b's and at least as many b's as c's and no adjacent letters forming the pattern aba or bab. _____
OLEIS sequence number _____

(14) The number of words of length n created with the letters a, b, c with at least as many a's as b's and at least as many b's as c's and no adjacent letters forming the pattern aba or abc. _____
OLEIS sequence number _____

(15) The number of words of length n created with the letters a, b, c with at least as many a's as b's and at least as many b's as c's and no adjacent letters forming the pattern aba and no subwords (any non adjacent subsequence of letters) of the form abc. _____
OLEIS sequence number _____

(16) A non-empty word is called 'Lyndon' if it is strictly smaller in dictionary order than all of the proper suffixes. (e.g. a is Lyndon because it has no proper suffixes, aa is not Lyndon because aa is not smaller than a in dictionary order, aab is Lyndon because it is strictly smaller in dictionary order than ab and b). The number of Lyndon words which do not have the pattern $abba$. _____
OLEIS sequence number _____

(17) The number of subsets of the numbers $\{1, 2, \ldots, n\}$ consisting of at most 3 elements and at most two of those are even. _____
OLEIS sequence number _____

(18) The number of subsets of the numbers $\{1, 2, \ldots, n\}$ consisting of at most 4 elements and at most two of those are even. _____
OLEIS sequence number _____

(19) The number of subsets of the numbers $\{1, 2, \ldots, n\}$ with a sum of the entries less than or equal to $n\ ln\ n$.

_____

OLEIS sequence number _____

(20) The number of subsets of the numbers $\{1, 2, \ldots, n\}$ with a sum of the entries less than or equal to $n\ ln\ n + n$. _____

OLEIS sequence number _____

(21) The number of subsets of the numbers $\{1, 2, \ldots, n\}$ with at most 5 entries and a sum of the entries less than or equal to $n\ ln\ n$. _____

OLEIS sequence number _____