COMBINATORIAL PROBLEMS FOR MATH 5020

Explain the following combinatorial questions and answers. If your explanation lists all possibilities of a representation, then it should also include an explanation (a) what that list represents and (b) of how you are convinced that you have listed all of these possibilities and don't have repeats (I need to be able to look at your solution and be able to determine that it is right without potentially listing them out myself and making the same mistake). If you use a 'division principle' you must be clear why it is possible to divide and not just say 'at this point divide number because that is the way it is.'

(1) How many ways are there of placing three different colored (black, white and red) non-attacking rooks on an 8×8 chessboard? answer: 112,896

(2) How many ways are there of placing four non-attacking rooks where two of the rooks are white and two are black (and indistinguishable) on an 8×8 chessboard? answer: 705,600

(3) How many ways are there of rolling 6 indistinguishable six-sided dice so that three of the dice are showing the same value and three are showing different values? answer: 60

(4) How many ways are there of rolling 6 indistinguishable six-sided dice so that exactly 3 different values are showing? answer: 200

(5) How many ways are there of rolling 12 indistinguishable six-sided dice so that exactly half are odd and half are even? answer: 784

(6) How many ways are there of rolling 6 different colored (and hence distinguished) six-sided dice so that the sum of the dice is 18? answer: 3,431

(7) How many ways are there of rolling 6 different colored (and hence distinguished) six-sided dice so that three of the dice are showing the same value and three are showing different values? answer: 7,200

(8) How many ways are there of rolling 6 different colored (and hence distinguished) six-sided dice so that exactly 3 different values are showing? answer: 10,800

(9) How many ways are there of rolling 12 different colored (and hence distinguished) six-sided dice so that exactly half are odd and half are even? answer: 491,051,484

(10) How many ways are there of placing 14 indistinguishable balls in 6 boxes so that the first 2 boxes have at most six of the balls. answer: 8,526

(11) How many ways are there of placing 14 indistinguishable balls in 6 boxes so that the first 2 boxes have at least half of the balls. answer: 3,102

(12) How many ways are there of making change for 35 cents using quarters, nickels, pennies (all Canadian) and U.S nickels and U.S. pennies? answer: 482

(13) How many ways are there of picking 22 marbles from 18 red ones, 5 blues and 10 pinks? answer: 56

(14) How many ways can 5 red balls, 5 blue balls, 10 green balls and 3 yellow balls be ordered so that green balls are are not adjacent? answer: 72,144,072

(15) How many ways can 5 red balls, 5 blue balls, 4 green balls and 3 yellow balls be ordered so that no more than 3 red balls are adjacent? answer: 166,846,680

(16) How many ways can 5 red balls, 5 blue balls, 4 green balls and 3 yellow balls be ordered so that exactly 2 blue balls are adjacent? answer: 47,567,520

(17) How many ways are there of placing a bishop and a rook on an 8×8 chessboard such that the bishop cannot attack the rook?

(18) How many ways are there of placing a bishop and a rook on an 8×8 chessboard such that the bishop cannot attack the rook and the rook cannot attack the bishop?

(19) How many ways are there of placing a knight and a rook on an 8×8 chessboard such that the knight cannot attack the rook?

(20) How many ways are there of placing a knight and a rook on an 8×8 chessboard such that the knight cannot attack the rook and the rook cannot attack the knight?