Some number theory and combinatorics questions
February 25, 2010

(1) How many 2×2 invertible matrices are there mod p?

(2) How many different full house hands can be made from a 52 card deck if the A of spades is thrown out?

(3) Find r and s such that $gcd(119, 315) = 119r + 315s$,

(4) Alice and Bob wish to set up a public key cryptosystem. Their first step is to agree on a public modulus $p = 17$ and the primitive root $a = 11$. Alice publishes her public key as the number 5 (remember it is the primitive root raised to her secret key) and Bob publishes 14 as his public key. What is the common key between Alice and Bob ($a^{\text{secret key for Alice} \cdot \text{secret key for Bob}}$). The powers of 3 mod 17 are

$$3^1 = 3, 3^2 = 9, 10, 13, 5, 15, 11, 16, 14, 8, 7, 4, 12, 2, 6, 1$$

(5) Find an integer x such that
$$202x \equiv 33 \ (mod \ 431)$$

(6) Determine by computing a Jacobi/Legendre symbol if
$$x^2 + 14x \equiv 194 \ (mod \ 389)$$
has a solution.