WIDGETS AND DOODLES I

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Say that $a_0, a_1, a_2, \ldots$ is a sequence of non-negative integers where a_n represents the number of "widgets of type n." Assume similarly that b_n represents the number of "doodles of type n." Below are a list of algebraic expressions labeled 1 through 19 and another list of combinatorial descriptions labeled (a) through (p). Match each one of the algebraic expressions with the combinatorial description such that the number of elements in the combinatorial description is equal to the expression. Here is the tough part: three of the equations do not have a combinatorial description. Write one for each of those.

(1) $a_n + b_n$
(2) $a_{n-1} + a_n$
(3) $a_n + b_m$
(4) $\binom{n}{k} a_n$
(5) $\binom{n}{0} a_0 + \binom{n}{1} a_1 + \cdots \binom{n}{n-1} a_{n-1} + \binom{n}{n} a_n$
(6) $a_0 + a_1 + a_2 + \cdots + a_n$
(7) $a_1 + 2a_2 + 3a_3 + \cdots na_n$
(8) $a_n b_n$
(9) $(a_0 + a_1 + \cdots + a_n)(b_0 + b_1 + \cdots + b_n)$
(10) a_n^2
(11) $a_1 a_2 \cdots a_{n-1} a_n$
(12) $a_0 b_n + a_1 b_{n-1} + \cdots + a_n b_0$
(13) $a_0 b_1 + a_1 b_2 + \cdots + a_{n-1} b_n$
(14) $a_0 b_0 + a_1 b_1 + a_2 b_2 + \cdots + a_n b_n$
(15) $\binom{n}{0} a_n b_0 + \binom{n}{1} a_{n-1} b_1 + \cdots + \binom{n}{n-1} a_1 b_{n-1} + \binom{n}{n} a_0 b_n$
(16) na_n
(17) $a_n + a_{n-2} + \cdots + a_{n \ mod \ 2}$
(18) $na_0 + (n-1)a_1 + \cdots + a_{n-1}$
(19) $a_0 + a_2 + \cdots + a_{2n}$

(a) A pair consisting of a widget and a doodle such that the doodle is of type less than or equal to n and the type of the doodle is one larger than the type of the widget.
(b) Sequences of length n where the k^{th} element of the sequence is a widget of type k.
(c) A pair consisting of one widget and one doodle, both of the same type and each of type less than or equal to n.
(d) A pair whose first element is a widget of type n and whose second element is a doodle of the same type.
(e) A pair consisting of a subset of the numbers 1 through n and a widget which is the same type as the size of the subset.
(f) A pair consisting of a widget and a doodle, each one is of type less than or equal to n.
(g) The number of widgets such that the type of the widget plus n is even.
(h) A pair which is either a widget of type n and the number 1 or it is a doodle of type n and the number 2.
(i) A pair whose first element is a subset of the integers 1 through n of size k and whose second element is a widget of type n.
(j) A pair consisting of two widgets both of type n.
(k) A pair consisting of a positive integer k and a widget such that the type of the widget plus the integer is less than or equal to n.
(l) A widget of even type which is less than or equal to $2n$.
(m) A widget that is either of type n or type $n-1$.
(n) A pair consisting of a widget and a doodle such that the type of the widget plus the type of the doodle is n.
(o) A triple consisting of a widget and a doodle and a subset of the integers 1 through n such that the type of the widget and the size of the subset are the same and the type of the widget plus the type of the doodle must be n.
(p) A pair consisting of a widget whose type is less than or equal to n and an integer between 1 and the type of the widget.