UNEXAM #2

ASSIGNED: JANUARY 26, 2012 DUE: FEBRUARY 9, 2012

Remember that the important aspect of this assignment is not the answer, but the solution. I am looking for a simple, clear, thoughtful, direct, short explanation.

(1) What is the generating function for the number of integral solutions of the equation

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = n$$

are there where each of the x_i are non-negative integers and x_1 is divisible by 6? Note: this is a sequence of numbers dependent on a parameter n. Answer:

$$\frac{1}{(1-q)^5(1-q^6)}$$

(2) Using generating the generating functions for the Fibonnaci numbers are $F(q) = \frac{1}{1-q-q^2} = \sum_{n\geq 0} F_{n+1} q^n$ explain why for $n \geq 0$:

$$F_{n+7} - F_{n+1} = 4F_{n+4}$$

Answer: Because the generating function for $q^6 \sum_{n\geq 0} F_{n+7} q^n = \frac{1}{1-q-q^2} - 1 - q - 2q^2 - 3q^3 - 5q^4 - 8q^5 = q^6 \frac{(13+8\,q)}{1-q-q^2}$ and the generating function for $q^3 \sum_{n\geq 0} F_{n+4} q^n = \frac{1}{1-q-q^2} - 1 - q - 2q^2 = q^3 \frac{(3+2\,q)}{1-q-q^2}$.