SEQUENCES AND SETS OF OBJECTS AND THE OLEIS

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Write the first 6-8 terms of the following sequences. Assume that the sequences start at $n = 0$, write a formula for a_n if possible (HINT: the empty word is a word of length 0). The OLEIS sequence number can be found by going to the web site 'The On-Line Encyclopedia of Integer Sequences' and entering the first terms which you calculated.
`http://www.research.att.com/~njas/sequences/index.html`
It may well be that the sequence that you enter is not in the database. Your next step if you find no information in the database will be to calculate more terms (try to get as high as 20) and try to arrive at a formula for a_n. Again, this might not be possible. Speak to me because we might be able to solve this problem together. :

(1) The number of solutions to $x_1 + x_2 + x_3 + x_4 = n$ with $x_i \geq 0$ with x_4 odd and x_3 even. _____
Formula? $a_n =$ _____ OLEIS sequence number _____

(2) The number of solutions to $x_1 + x_2 + x_3 + x_4 = n$ with $i \geq x_i \geq 0$ with x_4 even and x_3 even. _____
Formula? $a_n =$ _____ OLEIS sequence number _____

(3) The number of words of length n created with the letters a and b such that no a is adjacent to a b. _____
Formula? $a_n =$ _____ OLEIS sequence number _____

(4) The number of words of length n created with the letters a and b such that every a is separated by at least two b's. _____
Formula? $a_n =$ _____ OLEIS sequence number _____

(5) The number of words of length n created with the letters a and b such that every a is separated by at least three b's. _____
Formula? $a_n =$ _____ OLEIS sequence number _____

(6) The number of words of length n created with the letters a, b, c with at least half of the letters are a's. _____
Formula? $a_n =$ _____ OLEIS sequence number _____

(7) The number of words of length n created with the letters a, b, c with no consecutive letters being equal. _____
Formula? $a_n =$ _____ OLEIS sequence number _____

(8) The number of words of length n created with the letters a, b, c with all c's appearing after all of the b's. _____
Formula? $a_n =$ _____ OLEIS sequence number _____

(9) The number of words of length n created with the letters a, b, c with at least as many a's as b's and at least as many b's as c's. _____
Formula? $a_n =$ _____ OLEIS sequence number _____

(10) The number of words of length n created with the letters a, b, c with every b adjacent to at least one c. _____
Formula? $a_n =$ _____ OLEIS sequence number _____

(11) The number of words of length n created with the letters a, b, c with every b adjacent to at least one c and one a. _____
Formula? $a_n =$ _____ OLEIS sequence number _____

(12) The number of words of length n created with the letters a, b, c with every c not adjacent to any as. _____
Formula? $a_n =$ _____ OLEIS sequence number _____

(13) The number of words of length n created with the letters a, b, c with every b occurring in groups of two or more. _____
Formula? $a_n =$ _____ OLEIS sequence number _____

(14) The number of words of length n created with the letters a, b, c with no bs that are adjacent. _____
Formula? $a_n =$ _____ OLEIS sequence number _____

(15) The number of words of length n created with the letters a, b, c with every a and every b adjacent to at least one c. _____
Formula? $a_n =$ _____ OLEIS sequence number _____

(16) The number of words of length n created with the letters a, b, c with every b separated from every c by at least one a. _____
Formula? $a_n =$ _____ OLEIS sequence number _____

(17) The number of words of length n created with the letters a, b, c with every b separated from every c by at least two as. _____
Formula? $a_n =$ _____ OLEIS sequence number _____

(18) The number of words of length n created with the letters a, b, c with more cs than either as or bs. _____
Formula? $a_n =$ _____ OLEIS sequence number _____

(19) The number of words of length n created with the letters a, b, c with more cs than the number of as and bs put together. _____
Formula? $a_n =$ _____ OLEIS sequence number _____

(20) The number of words of length n created with the letters a, b, c with more cs than bs and more bs than as. _____
Formula? $a_n =$ _____ OLEIS sequence number _____

(21) The number of words of length n ($n \geq 1$) in 3 letters $\{a, b, c\}$ with an even number of a's such that if $w = uv$ where u and v are non-empty then w is strictly smaller than v in dictionary order. Example 1: We have that aab is strictly smaller than the both ab and b. Example 2: $aabaabccaca$ is not the lexicographically smallest of all of its tails because a is smaller in dictionary order. Example 3: $aabaab$ is not strictly smaller than aab so it does not count. _____
Formula? $a_n =$ _____ OLEIS sequence number _____

(22) The number of words of length n in two letters $\{0, 1\}$ such that every head (every subword starting at the beginning of the word) has at least as many 0s as 1s and there are an even number of 0s. _____
Formula? $a_n =$ _____ OLEIS sequence number _____

(23) The number of words of length $2n$ in two letters $\{0, 1\}$ such that every head (every subword starting at the beginning of the word) has at least as many 0s as 1s and there are an even number of 0s and an even number of 1s. _____
Formula? $a_n =$ _____ OLEIS sequence number _____

(24) The number of words of length $4n$ in two letters $\{0, 1\}$ such that every head (every subword starting at the beginning of the word) has at least as many 0s as 1s and there are an even number of 0s and the same number of 1s as 0s. _____
Formula? $a_n =$ _____ OLEIS sequence number _____

(25) The number of words of length n in the alphabet $\{1, 2, \ldots, n\}$ such that the letter i only occurs in positions 1 through i and the sum of the letters of the word is equal to 0 mod 3. _____
Formula? $a_n =$ _____ OLEIS sequence number _____