A PROOF?

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Abstract. This proof is false, but where is the error?

Question: How many non-negative integer solutions are there to the equation

$$x_1 + x_2 + \cdots + x_k = n \ ?$$

Solution: Note that every solution to this equation can be represented by a word with x_1 ones, x_2 twos, and x_3 threes, etc.

For example: $3 + 1 + 2 = 6$ can be represented by the word 111233.

Morover every word in the letters $1, 2, 3, \ldots k$ of length n represents a solution to the equation $x_1 + x_2 + \cdots + x_k = n$ by setting x_1 equal to the number of ones, x_2 equal to the number of $2s$, etc. and x_k equal to the number of ks.

Therefore we have established that there is a $1-1$ correspondence between the set of solutions to the equation $x_1 + x_2 + \cdots + x_k = n$ and the words of length n in the letters $1, 2, \ldots, k$.

For each letter in a word with the letters $1, 2, \ldots, k$ of length n is formed by picking one of k letters for each blank. Therefore, by the multiplication principle the number of words of length n is k^n. Since we have established that there is a $1-1$ correspondence between these words and the solutions, the number of solutions is also k^n.

Answer: k^n

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