“An Introduction to Sage” or “Why I learned to stop worrying and love py-thon”

Mike Zabrocki
York University

What is Sage?

Sage is a computer algebra system based on python which ties together many specialized open source mathematics software packages.

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ATLAS

R

Jmol

Symmetrica

Pari

NetworkX

Maxima

PALP

Cephes

Singular

GAP

Numpy

+many more

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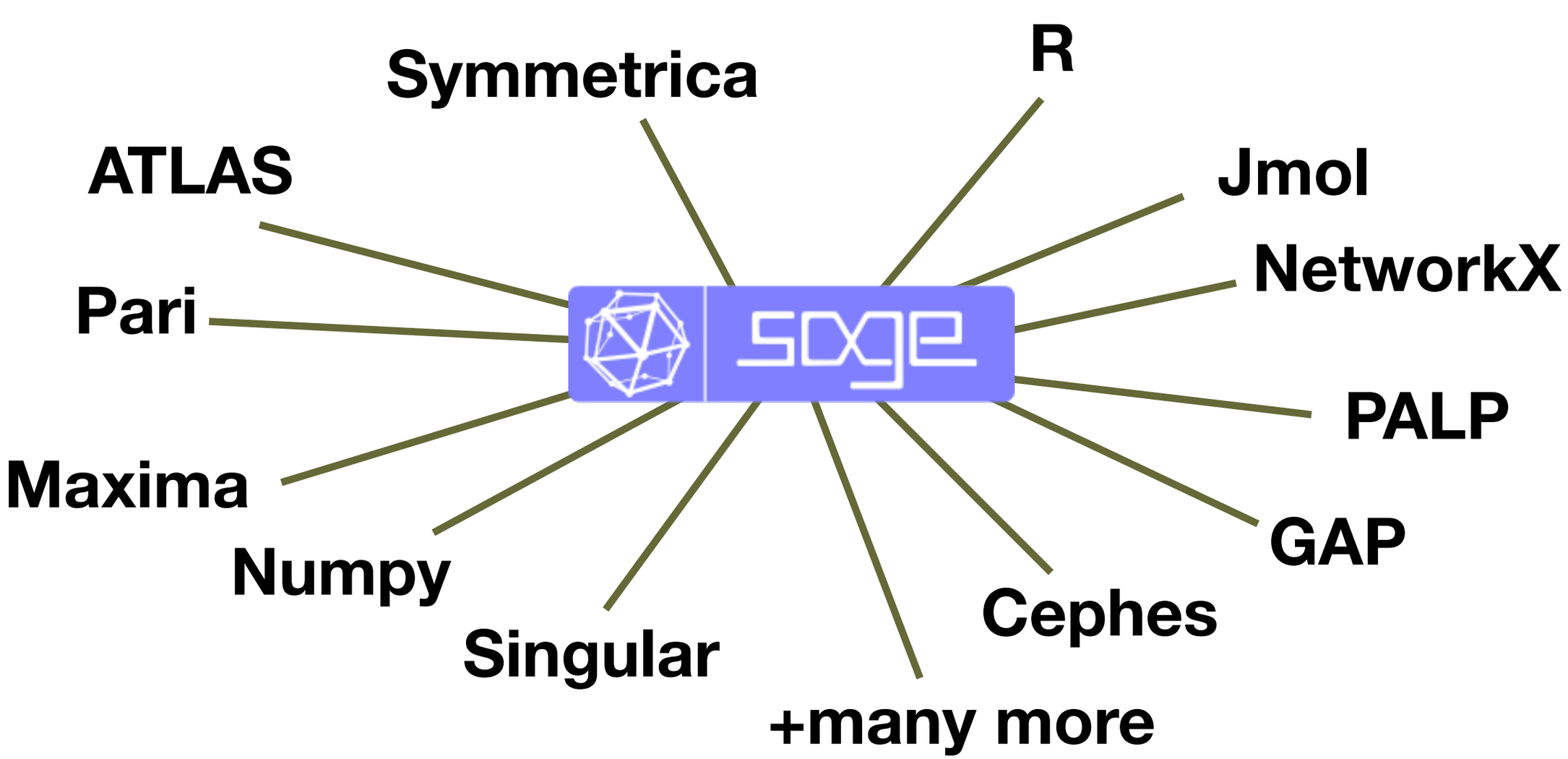

What Sage does well

- fast/vast libraries of mathematical tools
- share mathematics
- free and easy to access
- based on python

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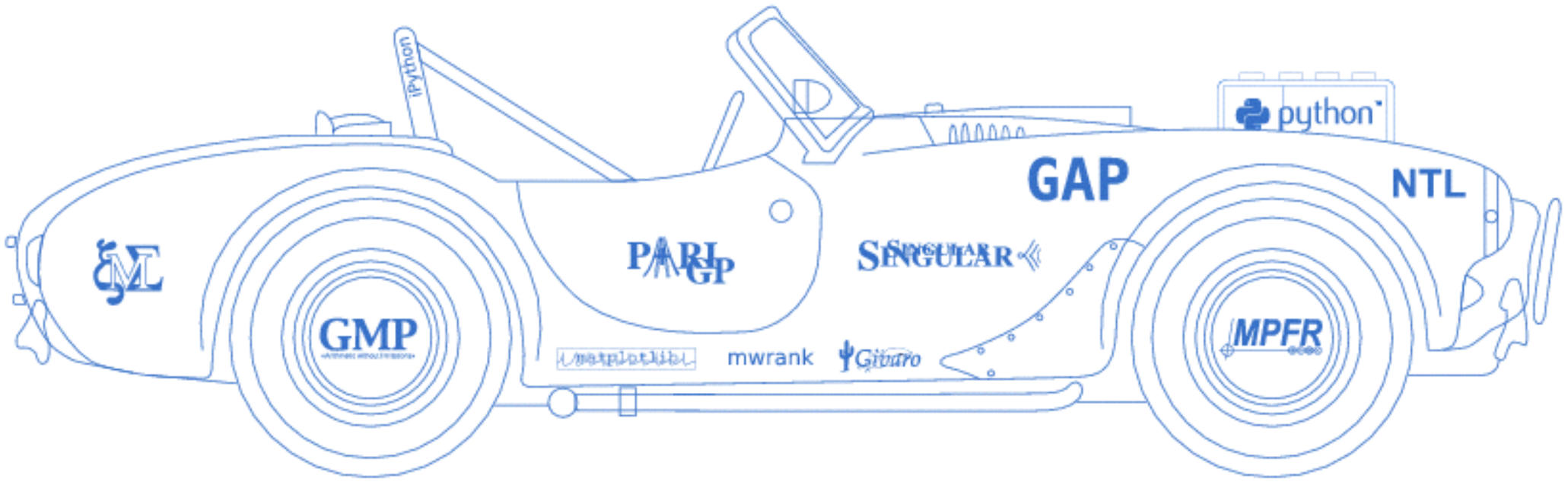
http://modular.math.washington.edu/sage

SAGE

Building »The Car«



»Every free computer algebra system I've tried has reinvented many times the wheel without being able to build the car.«

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- **share mathematics**
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share mathematics - in the classroom

‘zero knowledge’ answer key

Step 1: teach students basic commands of Sage such as taking a coefficient in a taylor series or numerical integration

Step 2: If answer is function f(x) then give as answer key the coefficient of x^100 in the taylor expansion

example:

What is the generating function for the number of solutions to the equation

$$x_1 + x_2 + 2x_3 + 3x_4 + x_5 = n$$

with $x_i \geq 0$ and where $x_1 + x_2 + 2x_3$ is even and $x_1 + 2x_3$ is less than or equal to 10, and $x_4 \neq x_5$?

Answer key: the number of solutions with $n = 100$ is 23779

share mathematics - in textbooks

excerpt from k-Schur Functions and Affine Schubert Calculus by Lam, et al.

where the element s_i represents the permutation which interchanges i and $i+1$. We will refer to the left cosets of $\tilde{S}_n/S_n$ as *affine Grassmannian elements* and they will be identified with their minimal length coset representatives, that is, the elements of $w \in \tilde{S}_n$ such that either $w = id$ or s_0 is the only elementary transposition such that $\ell(ws_0) < \ell(w)$.

Remark 1.7. The definition of affine Grassmannian elements are the special case of a more general definition. The l-Grassmannian elements are the minimal length coset representatives of $\tilde{S}_n/S_n^l$ where S_n^l is the group generated by $\{s_0, s_1, s_2, \ldots, s_{n-1}\}\backslash\{s_l\}$ and the affine Grassmannian elements are the 0-Grassmannian elements. Due to the cyclic symmetry of the affine type A Dynkin diagram, these constructions are of course all equivalent.

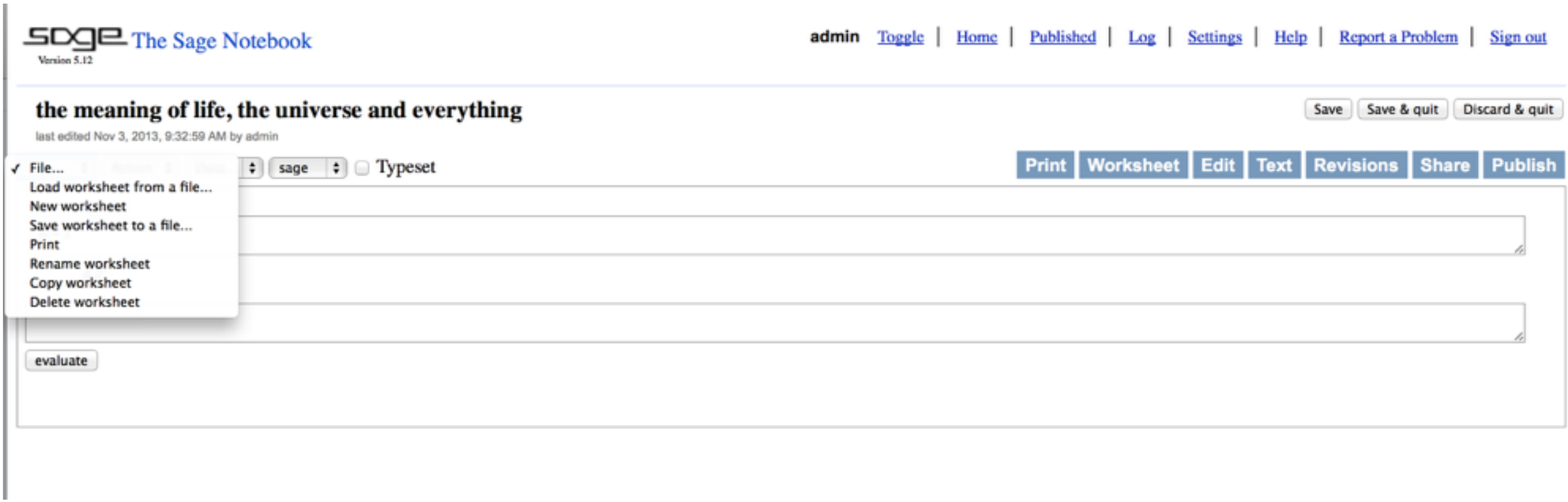
Sage Example 1.8. We can create the affine symmetric group and its generators in SAGE as

```
sage: W = WeylGroup(["A",4,1])
sage: S = W.simple_reflections()
sage: [s.reduced_word() for s in S]
[[0], [1], [2], [3], [4]]
```

For a given element, we can ask for its reduced word or create it from a word in the generators and ask whether it is Grassmannian:

```
sage: w = W.an_element(); w
[ 2  0  0  1 -2]
[ 2  0  0  0 -1]
[ 1  1  0  0 -1]
[ 1  0  1  0 -1]
[ 1  0  0  1 -1]
sage: w.reduced_word()
[0, 1, 2, 3, 4]
sage: w = W.from_reduced_word([2,1,0])
sage: w.is_affine_grassmannian()
True
```

share mathematics - through worksheets



share mathematics - through collaborations

share mathematics - by contributing to Sage

Sage Days : turning first time users into developers

What Sage does well

- fast/vast libraries of mathematical tools
- share mathematics
- **free and easy to access**
- based on python

untitled

Favorites
Handwriting
Expression
Units (SI)
Units (FPS)
Common Symbols
Matrix
Components
Greek
Arrows
Relational
Relational Round
Negated
Large Operators
Operators
Open Face
Fraktur
Script
Miscellaneous

Text | Math | Drawing | Plot | Animation | Hide

2D Math | Times New Roman | 12

Quick Help

Toggle Math/Text	F5
Evaluate	Enter
Evaluate and Display Inline	⌘=
Leave superscript/fraction	Right-arrow
Complete Symbol/Command	Escape
Navigate Placeholders	Tab
Maple Help	⌘F1
Quick Reference	⌘F2
Math Editor Shortcuts	
Interactive Assistants	
Units and Tolerances	
Assignments	a := b
Functions	f := x -> x^2
Equations	x = y
Maple Tour	

Right click on expression to perform operations.
Press ⌘? to show this list.
Show on new Documents

Maple Initialization Error

Invalid or missing license file.

invalid license return code

Click "Activate" to obtain a valid license file by registering your copy of Maple. Click "Web Store" to visit the Maplesoft web store where you can purchase an upgrade.

Quit | Web Store | Activate

Ready

Server: - Memory: - Time: - Math Mode

untitled

Favorites
Handwriting
Expression
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Activation Cancelled

Unable to determine the Host ID of this system. Please contact Maplesoft Support.

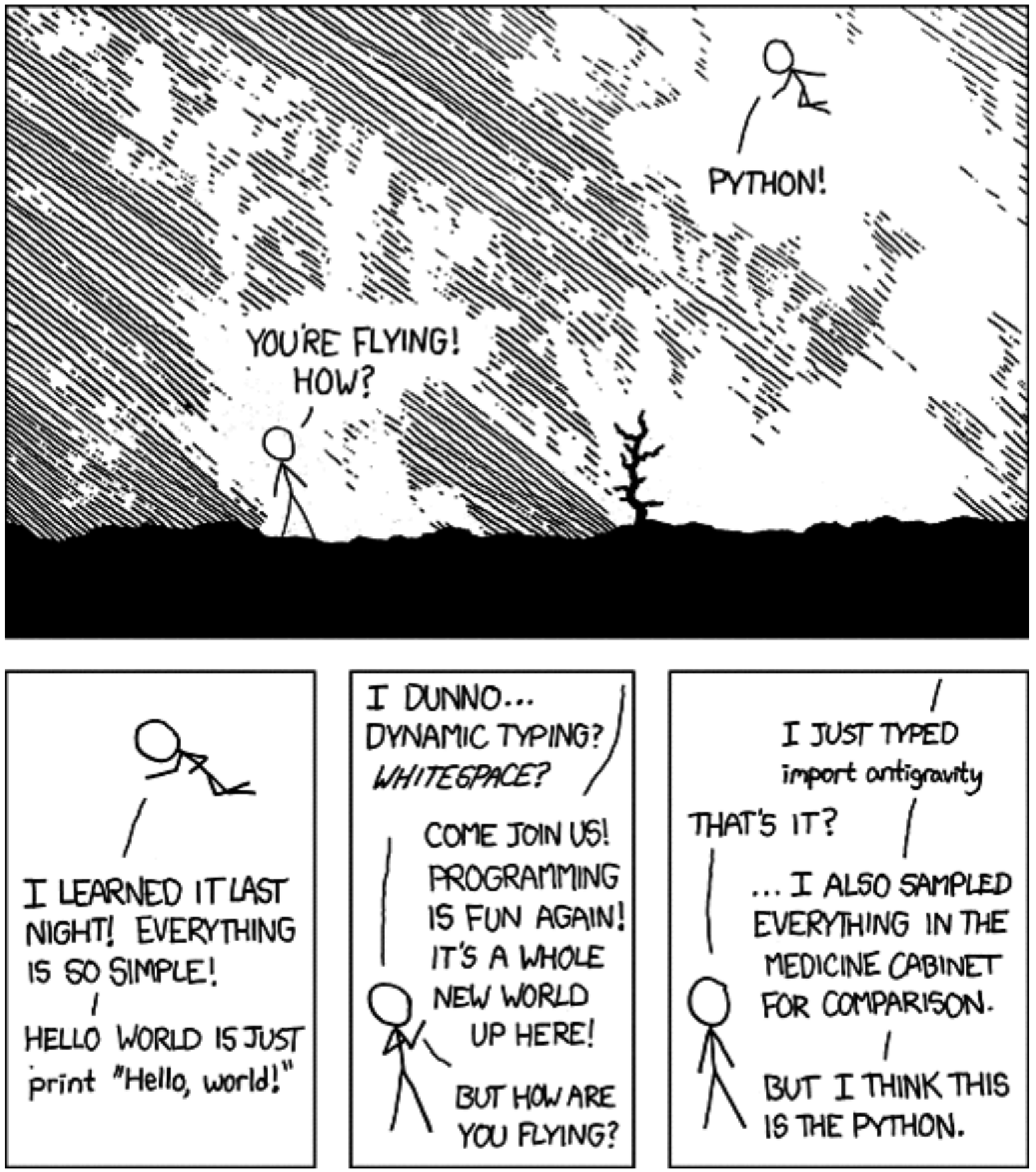
OK

Ready

Server: - Memory: - Time: - Math Mode

Why one should use sage

- fast/vast libraries of mathematical tools
- share mathematics
- free and easy to access
- **based on python**


PYTHON!
YOU'RE FLYING! HOW?
I LEARNED IT LAST NIGHT! EVERYTHING IS SO SIMPLE!
HELLO WORLD IS JUST print "Hello, world!"
I DUNNO... DYNAMIC TYPING? WHITESPACE?
COME JOIN US! PROGRAMMING IS FUN AGAIN! IT'S A WHOLE NEW WORLD UP HERE!
BUT HOW ARE YOU FLYING?
I JUST TYPED import antigravity
THAT'S IT?
... I ALSO SAMPLED EVERYTHING IN THE MEDICINE CABINET FOR COMPARISON.
BUT I THINK THIS IS THE PYTHON.

Sage is based on python (a very nice language) and has syntax close to mathematics

---

$$\sum_{i=1}^{10} i^2$$

```
sum(i^2 for i in range(1,11))
```

$$\{17x | x \in \{0,1,2,\ldots,9\} \text{ and } i \text{ is odd } \}$$

```
[17*x for x in range(10) if x%2==1]
```

Who Sage is not good for

- experts
- beginners

Who Sage is not good for

- experts doing high efficiency specialized computation
- beginners who are turned off by user friendly issues

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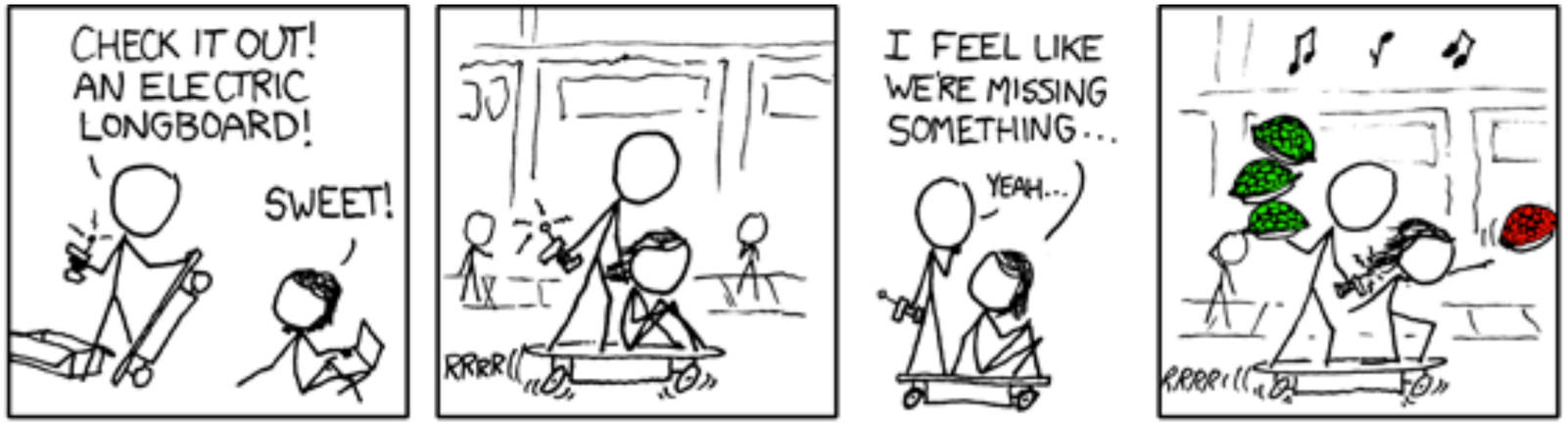
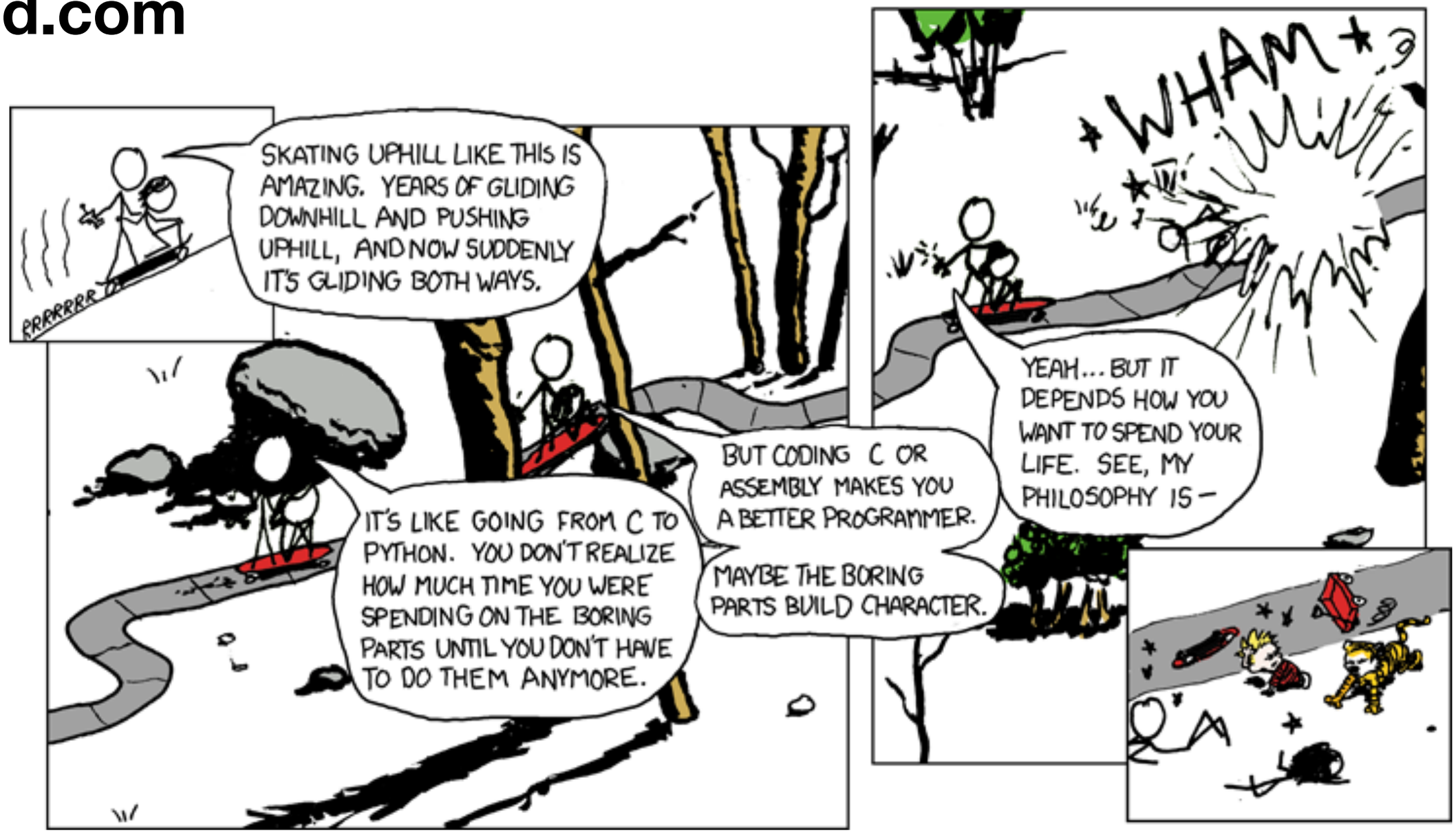
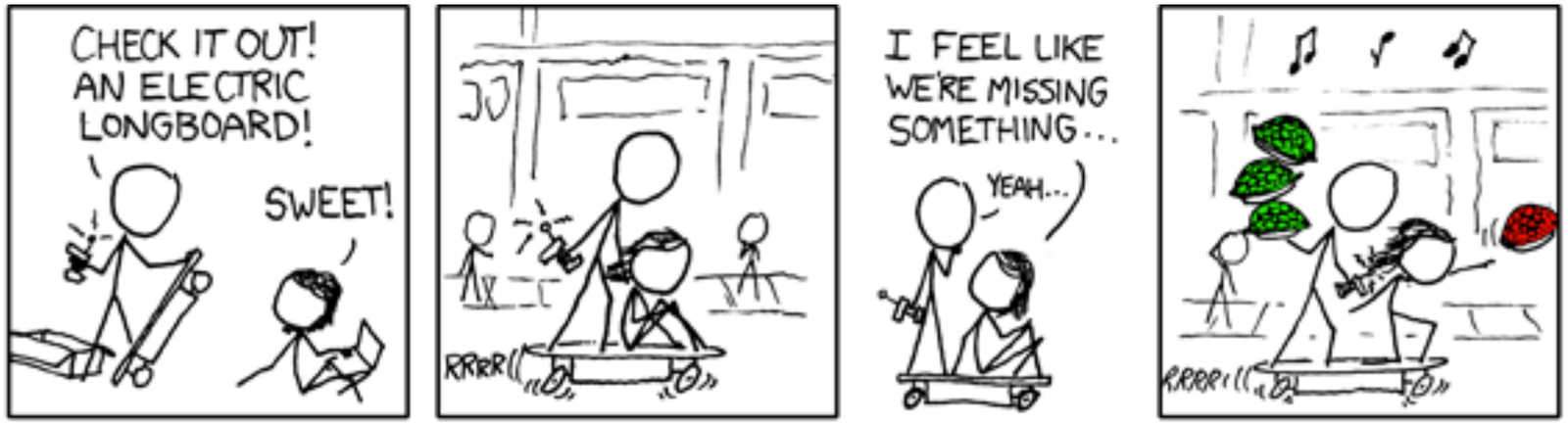

CHECK IT OUT! AN ELECTRIC LONGBOARD!
SWEET!
RRRR
I FEEL LIKE WE'RE MISSING SOMETHING...
YEAH...
RRRR

xkcd.com


SKATING UPHILL LIKE THIS IS AMAZING. YEARS OF GLIDING DOWNHILL AND PUSHING UPHILL, AND NOW SUDDENLY IT'S GLIDING BOTH WAYS.
RRARRRR
IT'S LIKE GOING FROM C TO PYTHON. YOU DON'T REALIZE HOW MUCH TIME YOU WERE SPENDING ON THE BORING PARTS UNTIL YOU DON'T HAVE TO DO THEM ANYMORE.
BUT CODING C OR ASSEMBLY MAKES YOU A BETTER PROGRAMMER.
MAYBE THE BORING PARTS BUILD CHARACTER.
WHAM
YEAH... BUT IT DEPENDS HOW YOU WANT TO SPEND YOUR LIFE. SEE, MY PHILOSOPHY IS —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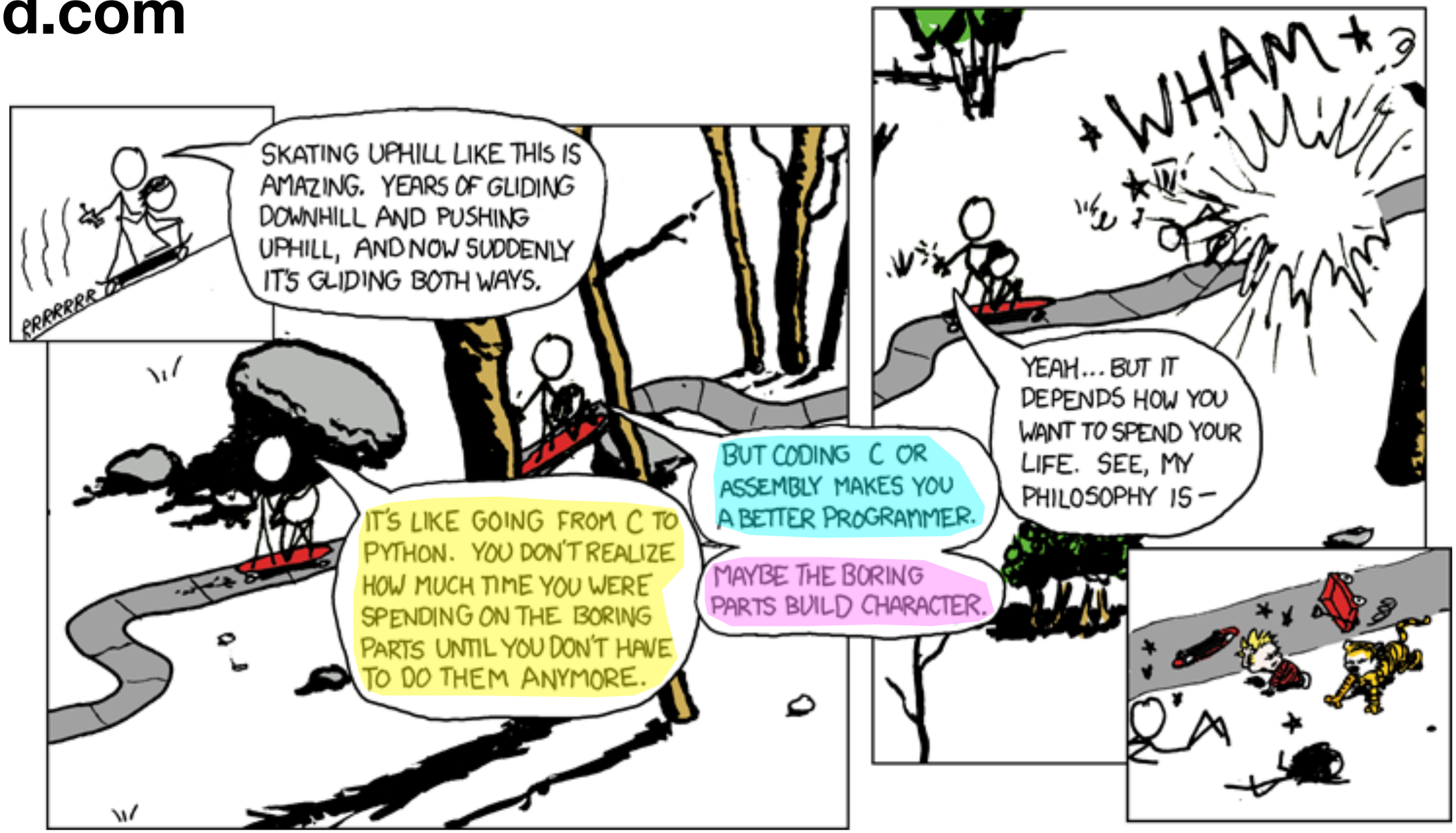
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- experts doing high efficiency specialized computation
- **beginners who are turned off by user friendly issues**

```
for i from 1 to 10:
    print i
```

```
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    print i
```

```
    Traceback (most recent call last):
      File "<stdin>", line 1, in <module>
      File "_sage_input_3.py", line 10, in <module>
        exec compile(u'open("___code___.py","w").write("# -*- coding: utf-8 -*-\\n" + _support_.preparse_workshe
      File "", line 1, in <module>

      File "/private/var/folders/_3/qq_ptsxd0bd645cgmgt_ssrm0000gn/T/tmpUxQVWB/___code___.py", line 3
        for i from _sage_const_1  to _sage_const_10 :
                  ^
    SyntaxError: invalid syntax
```

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      File "", line 1, in <module>

      File "/private/var/folders/_3/qq_ptsxd0bd645cgmgt_ssrm0000gn/T/tmpUxQVWB/___code___.py", line 3
        for i from _sage_const_1  to _sage_const_10 :
                  ^
    SyntaxError: invalid syntax
```

TMI!

```
for from 1 to 10:
    print i
```

```
'for i from ...' Bad syntax in for loop.  Help on for loops
       ^
```

```
for i in range(10):
    print factor(x^i-1)
```

```
for i in range(10):
    print factor(x^i-1)
```

evaluate

```
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
  File "_sage_input_9.py", line 10, in <module>
    exec compile(u'open("___code___.py","w").write("# -*- coding: utf-8 -*-\\n" + _support_.preparse_worksheet_
  File "", line 1, in <module>

  File "/private/var/folders/_3/qq_ptsxd0bd645cgmgt_ssrm0000gn/T/tmpAkQLpF/___code___.py", line 3, in <module>
    exec compile(u'for i in range(_sage_const_10 ):\n    print factor(x**i-_sage_const_1 )
  File "", line 2, in <module>

  File "/Applications/sage/local/lib/python2.7/site-packages/sage/rings/arith.py", line 2478, in factor
    return n.factor(**kwds)
  File "expression.pyx", line 8466, in sage.symbolic.expression.Expression.factor (sage/symbolic/expression.cpp
  File "polynomial_element.pyx", line 2892, in sage.rings.polynomial.polynomial_element.Polynomial.factor (sage
ValueError: factorization of 0 not defined
```

```
for i in range(10):
    print factor(x^i-1)
```

evaluate

```
    Traceback (most recent call last):
      File "<stdin>", line 1, in <module>
      File "_sage_input_9.py", line 10, in <module>
        exec compile(u'open("___code___.py","w").write("# -*- coding: utf-8 -*-\\n" + _support_.preparse_worksheet_
      File "", line 1, in <module>

      File "/private/var/folders/_3/qq_ptsxd0bd645cgmgt_ssrm0000gn/T/tmpAkQLpF/___code___.py", line 3, in <module>
        exec compile(u'for i in range(_sage_const_10 ):\n    print factor(x**i-_sage_const_1 )
      File "", line 2, in <module>

      File "/Applications/sage/local/lib/python2.7/site-packages/sage/rings/arith.py", line 2478, in factor
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    ValueError: factorization of 0 not defined
```

WTF?

```
for i in range(10):
    print i
```

evaluate

```
0
1
2
3
4
5
6
7
8
9
```

Demonstration

1. How to get started

2. How to get help

3. How to do something cool

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Mission: *Creating a viable free open source alternative to Magma, Maple, Mathematica and Matlab.*

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Welcome!

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The Sage Notebook

With the Sage Notebook anyone can create, collaborate on, and publish interactive worksheets. In a worksheet, one can write code using Sage, Python, and other software included in Sage.

General and Advanced Pure and Applied Mathematics

Use Sage for studying calculus, elementary to very advanced number theory, cryptography, commutative algebra, group theory, graph theory, numerical and exact linear algebra, and more.

Use an Open Source Alternative

By using Sage you help to support a viable open source alternative to Magma, Maple, Mathematica, and MATLAB. Sage includes many high-quality open source math packages.

Use Most Mathematics Software from Within Sage

Sage makes it easy for you to use most mathematics software together. Sage includes GAP, GP/PARI, Maxima, and Singular, and dozens of other open packages.

Use a Mainstream Programming Language

You work with Sage using the highly regarded scripting language Python. You can write programs that combine serious mathematics with anything else.

Acknowledgement

The Sage Notebook is based upon work supported by the National Science Foundation under grants DMS-0821725, DMS-1020378, DMS-0713225, DMS-0555776, DMS-0545904, DMS-0838212, DMS-0757627, DUE-1020378, DUE-1022574, DMS-1015114, etc. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation. See also http://sagemath.org/development-ack.html.

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File... | Action | Data... | sage | Typeset

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```
6*7
```

```
42
```

```

```

evaluate

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Help and Support

Sage standard documentation

The Sage standard documentation consists of the following documents, in both HTML and PDF versions.

View the online documentation.

- A Tour of Sage, PDF — a brief tour of some of Sage's features
  - Sage en quelques mots (Français), PDF
- Tutorial (Printed & Bound), PDF — information for beginners, recommended
  - Sage Tutorial (Deutsch), PDF
  - Sage thematische Anleitungen (Deutsch), PDF
  - Manual de Sage para principiantes (Español)
  - Introduccion: Matemáticas Elementales con Sage (Español)
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    - Sage's main tutorial

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  - Tutorial: Symbolics and Plotting (PREP)
  - Tutorial: Calculus (PREP)
  - Tutorial: Advanced-2D Plotting (PREP)
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  - Linear Programming (Mixed Integer)
  - Group Theory and Sage
  - Lie Methods and Related Combinatorics in Sage
- Number Theory
  - Number Theory and the RSA Public Key Cryptosystem
  - Introduction to the p-adics
- Combinatorics
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  - Tutorial: Symmetric Functions
  - Lie Methods and Related Combinatorics in Sage
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- Modeling Mathematics on a computer
  - How to implement new algebraic structures in Sage
  - Elements, parents, and categories in Sage: a (draft of) primer
  - Implementing a new parent: a (draft of) tutorial
- FAQ, PDF — frequently asked questions with answers
- Reference Manual, PDF — details about built-in functions
- Installation Guide, PDF — helps you install Sage (download)
- Developer's Guide, PDF — guidelines for current and prospective Sage developers
- Constructions (Printed & Bound), PDF — describes how to explain specific mathematical objects to Sage
- Numerical Sage, PDF — how to do numerical computing with Sage
- Explicit Methods in Number Theory, PDF — computing with number fields and modular forms

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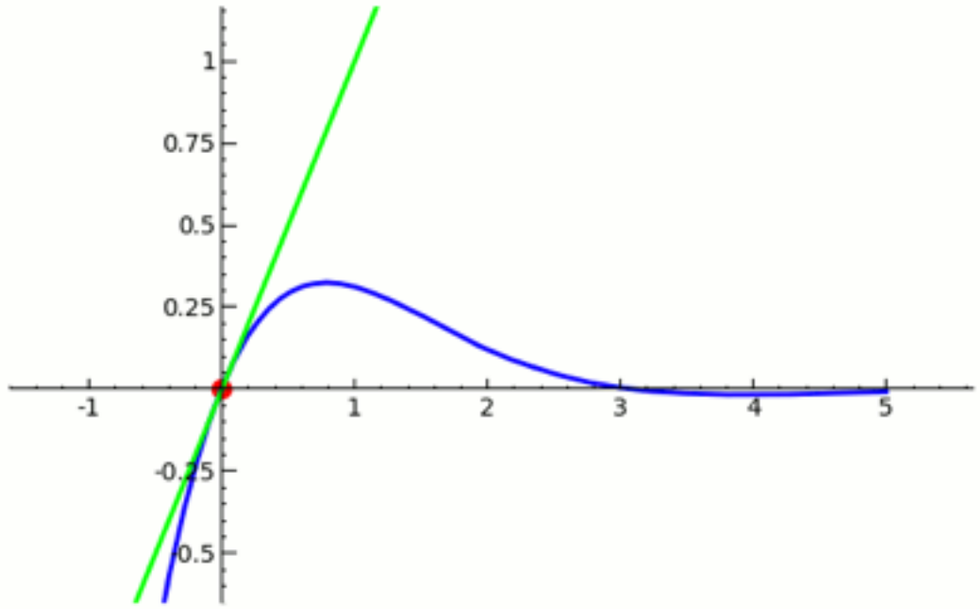
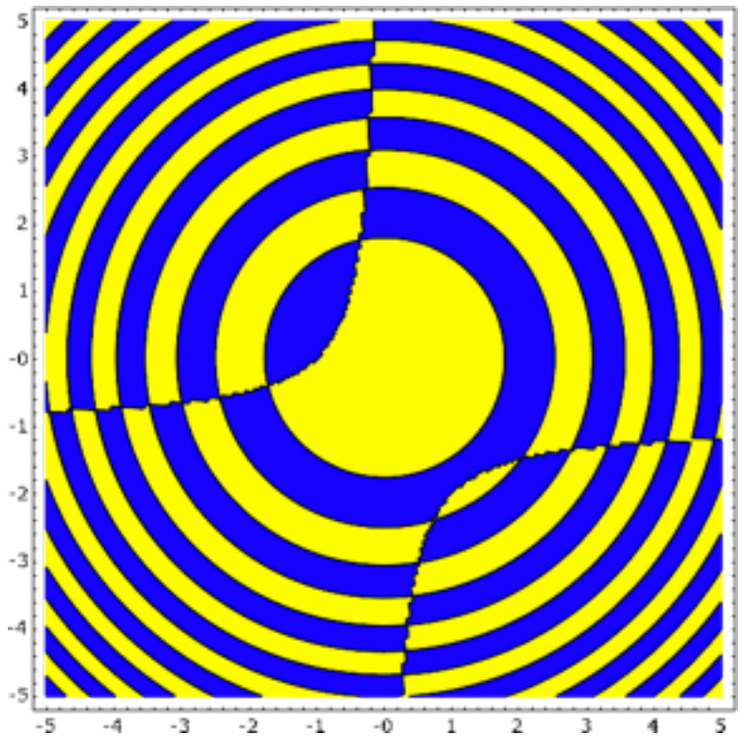
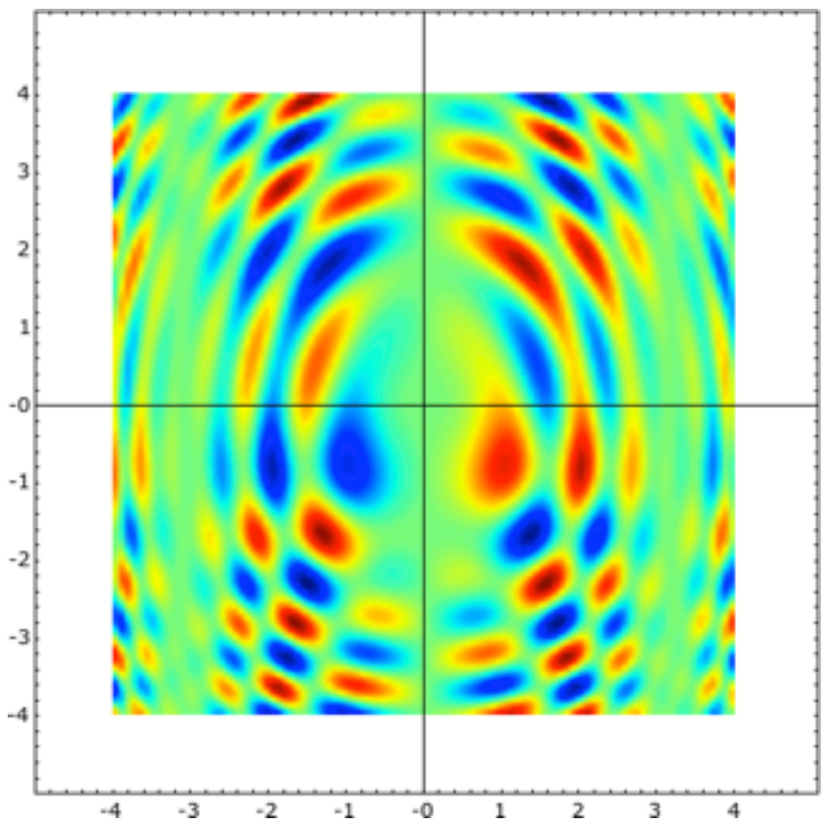
$$f(x) = e^{-x}\sin(x)$$
$$\hat{f}(x;0) = x + \mathcal{O}(x^2)$$





$$\frac{sin(x^2+y^2)}{(1+y+xy)} > 0$$



$$sin(x^2+y^2) * cos(x+y^2) * sin(y)$$