Combinatorial Algebra meets Algebraic Combinatorics

Eeventh Annual Meeting

Dalhousie University, Halifax, Canada

FRIDAY JANUARY 24 (Chemistry 223)

2:30 – 3:20	Roth	*Decomposing inversion sets of permutations*
3:30 – 3:50	Traves	*The Heinz problem*
4 – 4:30	COFFEE	
4:30 – 4:50	Galetto	*An algorithm for determining actions of semisimple Lie groups on free resolutions*
5 – 5:20	Erey	*Multigraded Betti numbers of simplicial trees*
5:30 – 5:50	Huntemann	*The algebra of placement games*

SATURDAY JANUARY 25 (Chase 319)

9 – 9:50	Skandera	*Hecke algebra characters and quantum chromatic symmetric functions*
10 – 10:30	COFFEE	
10:30 – 10:50	Serrano	*The immaculate basis of the non-commutative symmetric functions*
11 – 11:20	Williams	*Cataland*
11:30 – 2	LUNCH	
2 – 2:50	Miller	*Binomial irreducible decomposition*
3 – 3:20	Appleby	*Constructing hives from Hermitian matrix pairs, with interpretations of path operators and crystal graphs*
3:30 – 4:30	COFFEE	
4:30 – 4:50	Basili	*On the maximum nilpotent orbit intersecting a centralizer in* $M(n;K)$
5 – 5:30	Iarrobino	*Combinatorics of two commuting nilpotent matrices*
6:30 –	DINNER	Cellar Bar & Grill: 5677 Brenton Pl, Halifax, NS B3J 1E4 (902)835-1592

SUNDAY JANUARY 26 (Chase 319)

9 – 9:50	Tymoczko	*Generalized splines*
10 – 10:30	COFFEE	
10:30 – 10:50	Morales	*Combinatorics of diagrams of permutations*
11 – 11:50	Armstrong	*Rational Catalan combinatorics*

Abstracts:

1. Mike Roth (Queen's) - *Decomposing inversion sets of permutations*

   ABSTRACT: The first part of this talk will describe a particular combinatorial problem involving permutations. The motivation for this problem comes from two seemingly different areas: (1) Each solution to this problem corresponds to a particular face of the Littlewood-Richardson cone, and (2) solutions to this problem govern cup products of the cohomology of line bundles on homogenous varieties.

   The second part of the talk will present a recursive description of all the solutions to the problem.

   This is joint work with R. Dewji, I. Dimitrov, A. McCabe, J. Wilson, and D. Wehlau.

2. William Traves (US Naval Academy) - *The Heinz problem*

   ABSTRACT: I'll show how to use intersection theory to compute the characteristic numbers of completely reducible curves of degree 3 and 4. For example, we can solve the Heinz problem: Find the number of generic arrangements of 3 lines that are tangent to three given lines and contain 3 given points (in general position). I'll use this opportunity to advertise an interesting research program dealing with the enumerative geometry of hyperplane arrangements. This is joint work with my colleague Max Wakefield and our student Tom Paul.

3. Federico Galetto (Queen's) - *An algorithm for determining actions of semisimple Lie groups on free resolutions*

   ABSTRACT: The action of a group on a graded module over a polynomial ring extends to the entire minimal free resolution of the module. One could ask whether it is possible to recover the action of the group on the resolution by calculating the resolution explicitly in a computer algebra system. I will present an algorithm that gives a positive answer to this question for semisimple Lie groups in characteristic zero. The algorithm is based on an analysis of how the weights of a torus propagate along maps of free modules over a polynomial ring.

4. Nursel Erey (Dalhousie) - *Multigraded Betti numbers of simplicial trees*

   ABSTRACT: Facet ideals of simplicial complexes generalize edge ideals of graphs if one considers a graph as a 1 dimensional simplicial complex. We generalize the known results about multigraded Betti numbers from trees to simplicial trees. We discuss some techniques that allows us to recover some results on Betti numbers of edge ideals.

5. Svenja Huntemann (Dalhousie) - *The algebra of placement games*

   ABSTRACT: Placement games are a subclass of combinatorial games. After giving the necessary background in combinatorial game theory, we will introduce a construction that assigns two simplicial complexes to a placement game using square-free monomials. We will demonstrate that a placement game played on a graph is equivalent to a placement game played on each of these simplicial complexes. We will explore the combinatorics and algebra of these game complexes via square-free monomial ideals.

6. Mark Skandera (Lehigh) - *Hecke algebra characters and quantum chromatic symmetric functions*

   ABSTRACT: We discuss generating functions for Hecke algebra characters, formulas for the evaluation of these characters at Kazhdan-Lusztig basis elements of the Hecke algebra, and related symmetric functions defined by Shareshian and Wachs. While certain posets called unit interval orders may provide the key to connecting the Hecke algebra elements and symmetric functions, we propose to describe the connection in terms of a class of directed graphs which arose implicitly in papers of Goulden-Jackson, Greene, Stanley-Stembridge, and Haiman. Using these directed graphs, we conjecture a combinatorial formula for coefficients in the power sum expansion of the quantum chromatic symmetric functions.

   This is joint work with Sam Clearman, Matthew Hyatt, and Brittany Shelton

7. Luis Serrano (UQAM) - *The immaculate basis of the non-commutative symmetric functions*

   ABSTRACT: We introduce a new basis of the non-commutative symmetric functions whose elements have Schur functions as their commutative images. Dually, we build a basis of the quasi-symmetric functions which expand positively in the fundamental quasi-symmetric functions and decompose Schur functions according to a signed combinatorial formula. These bases have many interesting properties similar to those of the Schur basis, and we will outline a few of them.

8. Nathan Williams (UQAM) - *Cataland*

   ABSTRACT: I will talk about two combinatorial miracles relating poset-theoretic objects with Coxeter-theoretic objects. The first miracle is that there are the same number of linear extensions of the root poset as reduced words of the longest element (occasionally), while the second is that there are the same number of order ideals in the root poset as certain group elements (usually). I will conjecturally place these miracles on remarkably similar footing and examine the generality at which we should expect such statements to be true.

9. Ezra Miller (Duke) - *Binomial irreducible decomposition*

   ABSTRACT: This talk presents a response to Problem 7.5 in the paper *Binomial ideals*, by Eisenbud and Sturmfels: "Does every binomial ideal have an irreducible decomposition into binomial ideals? Find a combinatorial characterization of irreducible binomial ideals." Joint work with Thomas Kahle and Chris O'Neill.

10. Glenn Appleby (Santa Clara University, California) - *Constructing hives from Hermitian matrix pairs, with interpretations of path operators and crystal graphs*

    ABSTRACT: Knutson and Tao's work on the Horn conjectures used combinatorial invariants called hives and honeycombs to relate spectra of sums of Hermitian matrices to Littlewood-Richardson coefficients and problems in representation theory, but these relationships remained implicit. Here, let M and N be two $n \times n$ Hermitian matrices. We will show how to determine a hive $\mathcal{H}(M, N)_{ijk}$ using linear algebra constructions from this matrix pair. With this construction, one may also define an explicit Littlewood-Richardson filling (enumerated by the Littlewood-Richardson coefficient $c^{\lambda}_{\mu\nu}$ associated to the matrix pair). We then relate rotations of orthonormal bases of eigenvectors of M and N to deformations of honeycombs (and hives), which we interpret in terms of the structure of crystal graphs and Littelmann's path operators. We find in many cases that the crystal structure is determined *more* simply from the perspective of rotations than that of path operators. Joint with Tamsen Whitehead.

11. Roberta Basili (Perugia) - *On the maximum nilpotent orbit intersecting a centralizer in* $M(n; K)$

    ABSTRACT: Recently several authors have published results about the problem of Ønding which pairs of partitions of n correspond to pairs of commuting $n \times n$ nilpotent matrices. We describe a maximal nilpotent subalgebra SN_B of the centralizer of a given nilpotent $n \times n$ matrix with Jordan form, we show that SN_B intersects the maximum nilpotent orbit intersecting the minimal affine subspace defined by the condition that some coordinates are 0 of which SN_B is a subvariety. Then we prove that the maximum partition which forms with a given partition a pair with the previous property can be found by a simple algorithm which was conjectured by Polona Oblak.

12. Anthony Iarrobino (Northeastern) - *Combinatorics of two commuting nilpotent matrices*

    ABSTRACT: The Jordan type of a nilpotent matrix is the partition given by the sizes of its Jordan blocks. We consider pairs of partitions P, Q where Q is the Jordan type of a generic nilpotent matrix commuting with a given matrix of type P. Several – R. Basili, P. Oblak, T. Košir, L. Khatami, D. Panyushev, have studied the map $D : P \to Q(P)$. In 2012 P. Oblak made a conjecture about the inverse image $D^{-1}(Q)$ when Q has two parts, later refined by R. Zhao. We generalize this conjecture to all "stable" Q. and show most of it for $k = 2$ parts. We conjecture also that there is a bijection between $D^{-1}(Q)$ and the set $\mathcal{DHL}(Q)$ of partitions with diagonal hook lengths Q.

    This is joint work work with Leila Khatami, Bart van Steriteghem, and Rui Zhao.

13. Julianna Tymoczko (Smith College) - *Generalized splines*

    ABSTRACT: Splines are a kind of smooth, piecewise-polynomial approximation originally developed for engineering applications but now used widely in computer graphics, data interpolation, and other fields. Billera and others pioneered an algebraic approach to splines, using sophisticated techniques from commutative and homological algebra. Independently, geometers and topologists developed a construction of equivariant cohomology rings for large classes of varieties that turns out to coincide with the ring of splines.

    In this talk, we describe how to generalize the construction of splines to a more natural algebraic and combinatorial setting, starting from a commutative ring and a graph G. We'll also show how powerful this construction can be: how the ring of splines naturally decomposes in terms of splines for certain subgraphs of G, and how to construct generating sets for the generalized splines. We'll end with a number of open questions with connections to Schubert calculus, geometric representation theory, and approximation theory.

14. Alejandro Morales (UQAM) - *Combinatorics of diagrams of permutations*

ABSTRACT: In his study of the totally nonnegative Grassmannian, Postnikov introduced several combinatorial objects linked to a Grassmannian permutation $w = w_\lambda$. We study the connection between these objects when w is no longer required to be Grassmannian. These objects include regions in the inversion hyperplane arrangement of w, rook placements on the complement of the diagram of w, "Le"-fillings of the diagram of w, and permutations below w in the strong Bruhat order. We show that for any fixed permutation w the number of regions equals the number of rook placements and the number of certain fillings related to "Le"-fillings. Then thanks to a conjecture of Postnikov, settled by Hultman-Linusson-Shareshian-Sjöstrand, we relate this number of regions/placements/fillings and one of its q-analogues to the number of permutations below w in the Bruhat order. This last relation settles part of a conjecture with Klein and Lewis. This is joint work with Joel Lewis.

15. Drew Armstrong (Miami) - *Rational Catalan combinatorics*

ABSTRACT: Enumerative combinatorics is based on the study of combinatorial sequences, i.e. indexed by the positive integers. Allow me to say that "rational combinatorics" is the study of combinatorial sequences indexed by the positive rational numbers. The role previously played by induction is now played by continued fractions and the Euclidean algorithm. As proof-of-concept I will describe a theory of "rational Catalan combinatorics". For each positive rational number x there is an integer Catalan number $\mathrm{Cat}(x)$. The Euclidean algorithm is encoded via the concept of a "derived Catalan number" $\mathrm{Cat}'(x)$. If we write $x = a/(b-a)$ where a, b are positive coprime integers, then the prototypical objects counted by $\mathrm{Cat}(x)$ are the lattice paths in an a by b rectangle staying above the diagonal. These rational Dyck paths can be converted to: simultaneous (a, b)-core partitions, rational noncrossing partitions, and a rational associahedron. Dyck paths can be decorated to obtain rational parking functions. Most generally, rational Catalan combinatorics (RCC) can be seen in the character theory of rational Cherednik algebras (RCA) of reflection groups. I will adumbrate mysteries.