

There are several more open problems related to the research that we have been talking about over the last semester that remain unsolved.

I am providing data for several questions that we would like to know the answer to:

1) What is the coproduct on the X_A basis?

(the antipode is a map which serves as somewhat of a functional inverse in the Hopf algebra. Sometimes it has meaning in other contexts too. For a Hopf algebra which is either commutative or co-commutative (e.g. NCSym) the antipode will be an involution)

2) What is the antipode on the X_A basis?

3) What is the antipode on the P_A basis?

4) What is the antipode on the M_A basis?

5) What is the commutative image of the X_A basis?

- Looking for a pattern in the coprod X -basis

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> for i from 1 to 5 do
  print(Delta(X[{{seq(j,j=1..i)}}])=
    tenToX(coprod(X[{{seq(j,j=1..i)}}])));
od;
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$$\Delta(X_{\{(1)\}}) = X_{\{(1)\}} + XX_{\{(1)\}}$$

$$\Delta(X_{\{(1,2)\}}) = X_{\{(1,2)\}} - 2 XX_{\{(1)\}} X_{\{(1)\}} + XX_{\{(1,2)\}}$$

$$\Delta(X_{\{(1,2,3)\}}) = X_{\{(1,2,3)\}} - 3 XX_{\{(1)\}} X_{\{(1,2)\}} - 3 XX_{\{(1,2)\}} X_{\{(1)\}} + XX_{\{(1,2,3)\}}$$

$$+ 3 X_{\{(1)\}} XX_{\{(1),(2)\}} + 3 XX_{\{(1)\}} X_{\{(1),(2)\}}$$

$$\Delta(X_{\{(1,2,3,4)\}}) = X_{\{(1,2,3,4)\}} - 4 X_{\{(1)\}} XX_{\{(1),(2),(3)\}} + XX_{\{(1,2,3,4)\}}$$

$$+ 4 X_{\{(1)\}} XX_{\{(1,2),(3)\}} + 4 X_{\{(1)\}} XX_{\{(1,3),(2)\}} - 4 X_{\{(1)\}} XX_{\{(1,2,3)\}}$$

$$- 18 XX_{\{(1),(2)\}} X_{\{(1),(2)\}} + 6 XX_{\{(1,2)\}} X_{\{(1),(2)\}} + 6 X_{\{(1,2)\}} XX_{\{(1),(2)\}}$$

$$- 4 XX_{\{(1)\}} X_{\{(1),(2),(3)\}} + 4 XX_{\{(1)\}} X_{\{(1,3),(2)\}} + 4 XX_{\{(1)\}} X_{\{(1),(2,3)\}}$$

$$+ 4 XX_{\{(1)\}} X_{\{(1,2),(3)\}} - 4 XX_{\{(1)\}} X_{\{(1,2,3)\}} + 4 X_{\{(1)\}} XX_{\{(1),(2,3)\}}$$

$$- 6 XX_{\{(1,2)\}} X_{\{(1,2)\}}$$

$$\Delta(X_{\{(1,2,3,4,5)\}}) = -5 X_{\{(1)\}} XX_{\{(1,3),(4),(2)\}} - 5 X_{\{(1)\}} XX_{\{(4),(1,2),(3)\}}$$

$$+ 5 X_{\{(1)\}} XX_{\{(3,4),(1,2)\}} - 5 X_{\{(1)\}} XX_{\{(1),(3,4),(2)\}} + 5 X_{\{(1)\}} XX_{\{(2,3),(1,4)\}}$$

$$- 5 X_{\{(1)\}} XX_{\{(1),(4),(2,3)\}} + 5 X_{\{(1)\}} XX_{\{(1,3,4),(2)\}} + 5 X_{\{(1)\}} XX_{\{(4),(1,2,3)\}}$$

$$- 5 X_{\{(1)\}} XX_{\{(1),(2,4),(3)\}} - 5 X_{\{(1)\}} XX_{\{(1,2,3,4)\}} + 5 X_{\{(1)\}} XX_{\{(1,3),(2,4)\}}$$

$$- 5 X_{\{(1)\}} XX_{\{(2),(3),(1,4)\}} + 5 X_{\{(1)\}} XX_{\{(1),(2,3,4)\}} + 5 X_{\{(1)\}} XX_{\{(1,2,4),(3)\}}$$

$$- 30 XX_{\{(1),(2)\}} X_{\{(1,3),(2)\}} - 10 XX_{\{(1,2)\}} X_{\{(1),(2),(3)\}} - 5 XX_{\{(1)\}} X_{\{(2),(3),(1,4)\}}$$

$$+ 5 XX_{\{(1)\}} X_{\{(1,2,4),(3)\}} - 5 XX_{\{(1)\}} X_{\{(1),(2,4),(3)\}} - 5 XX_{\{(1)\}} X_{\{(4),(1,2),(3)\}}$$

$$- 5 XX_{\{(1)\}} X_{\{(1,3),(4),(2)\}} + 5 XX_{\{(1)\}} X_{\{(4),(1,2,3)\}} + 5 XX_{\{(1)\}} X_{\{(1,3,4),(2)\}}$$

$$- 5 XX_{\{(1)\}} X_{\{(1),(3,4),(2)\}} + 5 XX_{\{(1)\}} X_{\{(1),(2,3,4)\}} - 5 XX_{\{(1)\}} X_{\{(1),(4),(2,3)\}}$$

$$+ 5 XX_{\{(1)\}} X_{\{(1,3),(2,4)\}} + 5 XX_{\{(1)\}} X_{\{(3,4),(1,2)\}} + 5 XX_{\{(1)\}} X_{\{(2,3),(1,4)\}}$$

$$- 5 XX_{\{(1)\}} X_{\{(1,2,3,4)\}} + 70 XX_{\{(1),(2)\}} X_{\{(1),(2),(3)\}} + 5 XX_{\{(1)\}} X_{\{(1),(4),(2),(3)\}}$$

$$\begin{aligned}
& + X_{\{\{1,2,3,4,5\}\}} + XX_{\{\{1,2,3,4,5\}\}} + 5 X_{\{\{1\}\}} XX_{\{\{1\},\{4\},\{2\},\{3\}\}} + 10 X_{\{\{1,2\}\}} XX_{\{\{1\},\{2,3\}\}} \\
& + 10 X_{\{\{1,2\}\}} XX_{\{\{1,2\},\{3\}\}} - 10 X_{\{\{1,2\}\}} XX_{\{\{1,2,3\}\}} - 30 XX_{\{\{1,2\},\{3\}\}} X_{\{\{1\},\{2\}\}} \\
& + 10 XX_{\{\{1,2,3\}\}} X_{\{\{1\},\{2\}\}} + 10 XX_{\{\{1,3\},\{2\}\}} X_{\{\{1,2\}\}} - 30 XX_{\{\{1,3\},\{2\}\}} X_{\{\{1\},\{2\}\}} \\
& - 30 XX_{\{\{1\},\{2,3\}\}} X_{\{\{1\},\{2\}\}} - 10 XX_{\{\{1\},\{2\},\{3\}\}} X_{\{\{1,2\}\}} + 70 XX_{\{\{1\},\{2\},\{3\}\}} X_{\{\{1\},\{2\}\}} \\
& + 10 XX_{\{\{1\},\{2\}\}} X_{\{\{1,2,3\}\}} - 30 XX_{\{\{1\},\{2\}\}} X_{\{\{1,2\},\{3\}\}} - 30 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2,3\}\}} \\
& - 10 XX_{\{\{1,2\}\}} X_{\{\{1,2,3\}\}} + 10 XX_{\{\{1,2\}\}} X_{\{\{1,2\},\{3\}\}} + 10 XX_{\{\{1,2\}\}} X_{\{\{1\},\{2,3\}\}} \\
& + 10 XX_{\{\{1,2\}\}} X_{\{\{1,3\},\{2\}\}}
\end{aligned}$$

- n=2 coprod

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> for A in listallsp(2) do
  print(Delta(X[A]) = tenToX(coprod(X[A])));
od;

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$$\begin{aligned}
\Delta(X_{\{\{1\},\{2\}\}}) &= 2 XX_{\{\{1\}\}} X_{\{\{1\}\}} + X_{\{\{1\},\{2\}\}} + XX_{\{\{1\},\{2\}\}} \\
\Delta(X_{\{\{1,2\}\}}) &= X_{\{\{1,2\}\}} - 2 XX_{\{\{1\}\}} X_{\{\{1\}\}} + XX_{\{\{1,2\}\}}
\end{aligned}$$

- n=3 coprod

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> for A in listallsp(3) do
  print(Delta(X[A]) = tenToX(coprod(X[A])));
od;

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$$\begin{aligned}
\Delta(X_{\{\{1\},\{2\},\{3\}\}}) &= \\
& 3 X_{\{\{1\}\}} XX_{\{\{1\},\{2\}\}} + 3 XX_{\{\{1\}\}} X_{\{\{1\},\{2\}\}} + X_{\{\{1\},\{2\},\{3\}\}} + XX_{\{\{1\},\{2\},\{3\}\}} \\
\Delta(X_{\{\{1,2\},\{3\}\}}) &= X_{\{\{1,2\},\{3\}\}} + XX_{\{\{1\}\}} X_{\{\{1,2\}\}} + XX_{\{\{1,2\}\}} X_{\{\{1\}\}} + XX_{\{\{1,2\},\{3\}\}} \\
& - 2 X_{\{\{1\}\}} XX_{\{\{1\},\{2\}\}} - 2 XX_{\{\{1\}\}} X_{\{\{1\},\{2\}\}} \\
\Delta(X_{\{\{1,3\},\{2\}\}}) &= X_{\{\{1,3\},\{2\}\}} + XX_{\{\{1\}\}} X_{\{\{1,2\}\}} + XX_{\{\{1,2\}\}} X_{\{\{1\}\}} + XX_{\{\{1,3\},\{2\}\}} \\
& - 2 X_{\{\{1\}\}} XX_{\{\{1\},\{2\}\}} - 2 XX_{\{\{1\}\}} X_{\{\{1\},\{2\}\}} \\
\Delta(X_{\{\{1\},\{2,3\}\}}) &= X_{\{\{1\},\{2,3\}\}} + XX_{\{\{1\}\}} X_{\{\{1,2\}\}} + XX_{\{\{1,2\}\}} X_{\{\{1\}\}} + XX_{\{\{1\},\{2,3\}\}} \\
& - 2 X_{\{\{1\}\}} XX_{\{\{1\},\{2\}\}} - 2 XX_{\{\{1\}\}} X_{\{\{1\},\{2\}\}} \\
\Delta(X_{\{\{1,2,3\}\}}) &= X_{\{\{1,2,3\}\}} - 3 XX_{\{\{1\}\}} X_{\{\{1,2\}\}} - 3 XX_{\{\{1,2\}\}} X_{\{\{1\}\}} + XX_{\{\{1,2,3\}\}} \\
& + 3 X_{\{\{1\}\}} XX_{\{\{1\},\{2\}\}} + 3 XX_{\{\{1\}\}} X_{\{\{1\},\{2\}\}}
\end{aligned}$$

- n=4 coprod

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> for A in listallsp(4) do
  print(Delta(X[A]) = tenToX(coprod(X[A])));
od;

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$$\begin{aligned}
\Delta(X_{\{\{1\},\{4\},\{2\},\{3\}\}}) &= 6 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2\}\}} + 4 X_{\{\{1\}\}} XX_{\{\{1\},\{2\},\{3\}\}} \\
& + XX_{\{\{1\},\{4\},\{2\},\{3\}\}} + 4 XX_{\{\{1\}\}} X_{\{\{1\},\{2\},\{3\}\}} + X_{\{\{1\},\{4\},\{2\},\{3\}\}} \\
\Delta(X_{\{\{4\},\{1,2\},\{3\}\}}) &= X_{\{\{4\},\{1,2\},\{3\}\}} + XX_{\{\{1\},\{2\}\}} X_{\{\{1,2\}\}} - 4 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2\}\}} \\
& + 2 XX_{\{\{1\}\}} X_{\{\{1,2\},\{3\}\}} - 2 X_{\{\{1\}\}} XX_{\{\{1\},\{2\},\{3\}\}} + XX_{\{\{1,2\}\}} X_{\{\{1\},\{2\}\}} \\
& - 2 XX_{\{\{1\}\}} X_{\{\{1\},\{2\},\{3\}\}} + 2 X_{\{\{1\}\}} XX_{\{\{1,2\},\{3\}\}} + XX_{\{\{4\},\{1,2\},\{3\}\}} \\
\Delta(X_{\{\{1,3\},\{4\},\{2\}\}}) &= X_{\{\{1,3\},\{4\},\{2\}\}} + XX_{\{\{1\}\}} X_{\{\{1,3\},\{2\}\}} + XX_{\{\{1\},\{2\}\}} X_{\{\{1,2\}\}} \\
& - 4 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2\}\}} + XX_{\{\{1\}\}} X_{\{\{1,2\},\{3\}\}} - 2 X_{\{\{1\}\}} XX_{\{\{1\},\{2\},\{3\}\}} \\
& + XX_{\{\{1,2\}\}} X_{\{\{1\},\{2\}\}} - 2 XX_{\{\{1\}\}} X_{\{\{1\},\{2\},\{3\}\}} + X_{\{\{1\}\}} XX_{\{\{1,2\},\{3\}\}} \\
& + X_{\{\{1\}\}} XX_{\{\{1,3\},\{2\}\}} + XX_{\{\{1,3\},\{4\},\{2\}\}}
\end{aligned}$$

$$\begin{aligned}
\Delta(X_{\{\{1\},\{4\},\{2,3\}\}}) &= X_{\{\{1\},\{4\},\{2,3\}\}} + XX_{\{\{1\},\{4\},\{2,3\}\}} + XX_{\{\{1\},\{2\}\}} X_{\{\{1,2\}\}} \\
&\quad - 4 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2\}\}} + XX_{\{\{1\}\}} X_{\{\{1,2\},\{3\}\}} + X_{\{\{1\}\}} XX_{\{\{1\},\{2,3\}\}} \\
&\quad - 2 X_{\{\{1\}\}} XX_{\{\{1\},\{2\},\{3\}\}} + XX_{\{\{1\}\}} X_{\{\{1\},\{2,3\}\}} + XX_{\{\{1,2\}\}} X_{\{\{1\},\{2\}\}} \\
&\quad - 2 XX_{\{\{1\}\}} X_{\{\{1\},\{2\},\{3\}\}} + X_{\{\{1\}\}} XX_{\{\{1,2\},\{3\}\}} \\
\Delta(X_{\{\{4\},\{1,2,3\}\}}) &= X_{\{\{4\},\{1,2,3\}\}} - 3 XX_{\{\{1\},\{2\}\}} X_{\{\{1,2\}\}} + 6 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2\}\}} \\
&\quad - 3 XX_{\{\{1\}\}} X_{\{\{1,2\},\{3\}\}} + 3 X_{\{\{1\}\}} XX_{\{\{1\},\{2\},\{3\}\}} + XX_{\{\{1\}\}} X_{\{\{1,2,3\}\}} \\
&\quad - 3 XX_{\{\{1,2\}\}} X_{\{\{1\},\{2\}\}} + 3 XX_{\{\{1\}\}} X_{\{\{1\},\{2\},\{3\}\}} + X_{\{\{1\}\}} XX_{\{\{1,2,3\}\}} \\
&\quad - 3 X_{\{\{1\}\}} XX_{\{\{1,2\},\{3\}\}} + XX_{\{\{4\},\{1,2,3\}\}} \\
\Delta(X_{\{\{2\},\{3\},\{1,4\}\}}) &= 2 XX_{\{\{1\}\}} X_{\{\{1,3\},\{2\}\}} + XX_{\{\{1\},\{2\}\}} X_{\{\{1,2\}\}} - 4 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2\}\}} \\
&\quad - 2 X_{\{\{1\}\}} XX_{\{\{1\},\{2\},\{3\}\}} + X_{\{\{2\},\{3\},\{1,4\}\}} + XX_{\{\{1,2\}\}} X_{\{\{1\},\{2\}\}} \\
&\quad - 2 XX_{\{\{1\}\}} X_{\{\{1\},\{2\},\{3\}\}} + XX_{\{\{2\},\{3\},\{1,4\}\}} + 2 X_{\{\{1\}\}} XX_{\{\{1,3\},\{2\}\}} \\
\Delta(X_{\{\{2,3\},\{1,4\}\}}) &= -2 XX_{\{\{1\}\}} X_{\{\{1,3\},\{2\}\}} + X_{\{\{2,3\},\{1,4\}\}} + 4 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2\}\}} \\
&\quad - XX_{\{\{1\}\}} X_{\{\{1,2\},\{3\}\}} - X_{\{\{1\}\}} XX_{\{\{1\},\{2,3\}\}} - XX_{\{\{1\}\}} X_{\{\{1\},\{2,3\}\}} - X_{\{\{1\}\}} XX_{\{\{1,2\},\{3\}\}} \\
&\quad + 2 XX_{\{\{1,2\}\}} X_{\{\{1,2\}\}} - 2 X_{\{\{1\}\}} XX_{\{\{1,3\},\{2\}\}} + XX_{\{\{2,3\},\{1,4\}\}} \\
\Delta(X_{\{\{1,3,4\},\{2\}\}}) &= -2 XX_{\{\{1\}\}} X_{\{\{1,3\},\{2\}\}} + X_{\{\{1,3,4\},\{2\}\}} - 3 XX_{\{\{1\},\{2\}\}} X_{\{\{1,2\}\}} \\
&\quad + 6 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2\}\}} - X_{\{\{1\}\}} XX_{\{\{1\},\{2,3\}\}} + 3 X_{\{\{1\}\}} XX_{\{\{1\},\{2\},\{3\}\}} \\
&\quad - XX_{\{\{1\}\}} X_{\{\{1\},\{2,3\}\}} + XX_{\{\{1\}\}} X_{\{\{1,2,3\}\}} - 3 XX_{\{\{1,2\}\}} X_{\{\{1\},\{2\}\}} \\
&\quad + 3 XX_{\{\{1\}\}} X_{\{\{1\},\{2\},\{3\}\}} + X_{\{\{1\}\}} XX_{\{\{1,2,3\}\}} + XX_{\{\{1,3,4\},\{2\}\}} - 2 X_{\{\{1\}\}} XX_{\{\{1,3\},\{2\}\}} \\
\Delta(X_{\{\{1\},\{2,4\},\{3\}\}}) &= XX_{\{\{1\}\}} X_{\{\{1,3\},\{2\}\}} + X_{\{\{1\},\{2,4\},\{3\}\}} + XX_{\{\{1\},\{2\}\}} X_{\{\{1,2\}\}} \\
&\quad - 4 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2\}\}} + X_{\{\{1\}\}} XX_{\{\{1\},\{2,3\}\}} - 2 X_{\{\{1\}\}} XX_{\{\{1\},\{2\},\{3\}\}} \\
&\quad + XX_{\{\{1\}\}} X_{\{\{1\},\{2,3\}\}} + XX_{\{\{1,2\}\}} X_{\{\{1\},\{2\}\}} - 2 XX_{\{\{1\}\}} X_{\{\{1\},\{2\},\{3\}\}} + XX_{\{\{1\},\{2,4\},\{3\}\}} \\
&\quad + X_{\{\{1\}\}} XX_{\{\{1,3\},\{2\}\}} \\
\Delta(X_{\{\{1,3\},\{2,4\}\}}) &= -2 XX_{\{\{1\}\}} X_{\{\{1,3\},\{2\}\}} + 4 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2\}\}} - XX_{\{\{1\}\}} X_{\{\{1,2\},\{3\}\}} \\
&\quad + X_{\{\{1,3\},\{2,4\}\}} - X_{\{\{1\}\}} XX_{\{\{1\},\{2,3\}\}} - XX_{\{\{1\}\}} X_{\{\{1\},\{2,3\}\}} - X_{\{\{1\}\}} XX_{\{\{1,2\},\{3\}\}} \\
&\quad + 2 XX_{\{\{1,2\}\}} X_{\{\{1,2\}\}} - 2 X_{\{\{1\}\}} XX_{\{\{1,3\},\{2\}\}} + XX_{\{\{1,3\},\{2,4\}\}} \\
\Delta(X_{\{\{1\},\{2,3,4\}\}}) &= -3 XX_{\{\{1\},\{2\}\}} X_{\{\{1,2\}\}} + 6 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2\}\}} \\
&\quad - 3 X_{\{\{1\}\}} XX_{\{\{1\},\{2,3\}\}} + 3 X_{\{\{1\}\}} XX_{\{\{1\},\{2\},\{3\}\}} - 3 XX_{\{\{1\}\}} X_{\{\{1\},\{2,3\}\}} \\
&\quad + XX_{\{\{1\}\}} X_{\{\{1,2,3\}\}} + X_{\{\{1\},\{2,3,4\}\}} - 3 XX_{\{\{1,2\}\}} X_{\{\{1\},\{2\}\}} + 3 XX_{\{\{1\}\}} X_{\{\{1\},\{2\},\{3\}\}} \\
&\quad + X_{\{\{1\}\}} XX_{\{\{1,2,3\}\}} + XX_{\{\{1\},\{2,3,4\}\}} \\
\Delta(X_{\{\{1,2,4\},\{3\}\}}) &= -2 XX_{\{\{1\}\}} X_{\{\{1,3\},\{2\}\}} - 3 XX_{\{\{1\},\{2\}\}} X_{\{\{1,2\}\}} \\
&\quad + 6 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2\}\}} - XX_{\{\{1\}\}} X_{\{\{1,2\},\{3\}\}} + 3 X_{\{\{1\}\}} XX_{\{\{1\},\{2\},\{3\}\}} \\
&\quad + XX_{\{\{1\}\}} X_{\{\{1,2,3\}\}} - 3 XX_{\{\{1,2\}\}} X_{\{\{1\},\{2\}\}} + 3 XX_{\{\{1\}\}} X_{\{\{1\},\{2\},\{3\}\}} + X_{\{\{1\}\}} XX_{\{\{1,2,3\}\}} \\
&\quad + XX_{\{\{1,2,4\},\{3\}\}} - X_{\{\{1\}\}} XX_{\{\{1,2\},\{3\}\}} - 2 X_{\{\{1\}\}} XX_{\{\{1,3\},\{2\}\}} + X_{\{\{1,2,4\},\{3\}\}} \\
\Delta(X_{\{\{1\},\{3,4\},\{2\}\}}) &= X_{\{\{1\},\{3,4\},\{2\}\}} + XX_{\{\{1\},\{2\}\}} X_{\{\{1,2\}\}} - 4 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2\}\}} \\
&\quad + 2 X_{\{\{1\}\}} XX_{\{\{1\},\{2,3\}\}} - 2 X_{\{\{1\}\}} XX_{\{\{1\},\{2\},\{3\}\}} + 2 XX_{\{\{1\}\}} X_{\{\{1\},\{2,3\}\}} \\
&\quad + XX_{\{\{1,2\}\}} X_{\{\{1\},\{2\}\}} - 2 XX_{\{\{1\}\}} X_{\{\{1\},\{2\},\{3\}\}} + XX_{\{\{1\},\{3,4\},\{2\}\}} \\
\Delta(X_{\{\{3,4\},\{1,2\}\}}) &= X_{\{\{3,4\},\{1,2\}\}} + XX_{\{\{3,4\},\{1,2\}\}} + 4 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2\}\}} \\
&\quad - 2 XX_{\{\{1\}\}} X_{\{\{1,2\},\{3\}\}} - 2 X_{\{\{1\}\}} XX_{\{\{1\},\{2,3\}\}} - 2 XX_{\{\{1\}\}} X_{\{\{1\},\{2,3\}\}} \\
&\quad - 2 X_{\{\{1\}\}} XX_{\{\{1,2\},\{3\}\}} + 2 XX_{\{\{1,2\}\}} X_{\{\{1,2\}\}} \\
\Delta(X_{\{\{1,2,3,4\}\}}) &= 4 XX_{\{\{1\}\}} X_{\{\{1,3\},\{2\}\}} + X_{\{\{1,2,3,4\}\}} + 6 XX_{\{\{1\},\{2\}\}} X_{\{\{1,2\}\}}
\end{aligned}$$

$$\begin{aligned}
& -18 XX_{\{(1),\{2\}\}} X_{\{(1),\{2\}\}} + 4 XX_{\{(1)\}} X_{\{(1,2),\{3\}\}} + 4 X_{\{(1)\}} XX_{\{(1),\{2,3\}\}} \\
& -4 X_{\{(1)\}} XX_{\{(1),\{2,\{3\}\}} + XX_{\{(1,2,3,4)\}} + 4 XX_{\{(1)\}} X_{\{(1),\{2,3\}\}} - 4 XX_{\{(1)\}} X_{\{(1,2,3)\}} \\
& + 6 XX_{\{(1,2)\}} X_{\{(1),\{2\}\}} - 4 XX_{\{(1)\}} X_{\{(1),\{2,\{3\}\}} - 4 X_{\{(1)\}} XX_{\{(1,2,3)\}} \\
& + 4 X_{\{(1)\}} XX_{\{(1,2),\{3\}\}} - 6 XX_{\{(1,2)\}} X_{\{(1,2)\}} + 4 X_{\{(1)\}} XX_{\{(1,3),\{2\}\}}
\end{aligned}$$

n=5 coprod

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> for A in listallsp(5) do
  print(Delta(X[A]) = tenToX(coprod(X[A])));
od;

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$$\begin{aligned}
\Delta(X_{\{(5),\{1\},\{4\},\{2\},\{3\}\}}) &= 5 X_{\{(1)\}} XX_{\{(1),\{4\},\{2\},\{3\}\}} + X_{\{(5),\{1\},\{4\},\{2\},\{3\}\}} \\
&+ 10 XX_{\{(1),\{2\},\{3\}\}} X_{\{(1),\{2\}\}} + 5 XX_{\{(1)\}} X_{\{(1),\{4\},\{2\},\{3\}\}} + 10 XX_{\{(1),\{2\}\}} X_{\{(1),\{2,\{3\}\}} \\
&+ XX_{\{(5),\{1\},\{4\},\{2\},\{3\}\}} \\
\Delta(X_{\{(5),\{4\},\{1,2\},\{3\}\}}) &= -2 X_{\{(1)\}} XX_{\{(1),\{4\},\{2\},\{3\}\}} - 6 XX_{\{(1),\{2,\{3\}\}} X_{\{(1),\{2\}\}} \\
&+ 3 XX_{\{(1),\{2\}\}} X_{\{(1,2),\{3\}\}} + XX_{\{(1,2)\}} X_{\{(1),\{2,\{3\}\}} - 2 XX_{\{(1)\}} X_{\{(1),\{4\},\{2\},\{3\}\}} \\
&- 6 XX_{\{(1),\{2\}\}} X_{\{(1),\{2,\{3\}\}} + 3 XX_{\{(1)\}} X_{\{(4),\{1,2\},\{3\}\}} + 3 XX_{\{(1,2),\{3\}\}} X_{\{(1),\{2\}\}} \\
&+ 3 X_{\{(1)\}} XX_{\{(4),\{1,2\},\{3\}\}} + X_{\{(1,2)\}} XX_{\{(1),\{2,\{3\}\}} + X_{\{(5),\{4\},\{1,2\},\{3\}\}} \\
&+ XX_{\{(5),\{4\},\{1,2\},\{3\}\}} \\
\Delta(X_{\{(5),\{1,3\},\{4\},\{2\}\}}) &= -2 X_{\{(1)\}} XX_{\{(1),\{4\},\{2\},\{3\}\}} + XX_{\{(1,3),\{2\}\}} X_{\{(1),\{2\}\}} \\
&- 6 XX_{\{(1),\{2,\{3\}\}} X_{\{(1),\{2\}\}} + 2 XX_{\{(1),\{2\}\}} X_{\{(1,2),\{3\}\}} + XX_{\{(1,2)\}} X_{\{(1),\{2,\{3\}\}} \\
&+ XX_{\{(1),\{2\}\}} X_{\{(1,3),\{2\}\}} + X_{\{(5),\{1,3\},\{4\},\{2\}\}} - 2 XX_{\{(1)\}} X_{\{(1),\{4\},\{2\},\{3\}\}} \\
&- 6 XX_{\{(1),\{2\}\}} X_{\{(1),\{2,\{3\}\}} + XX_{\{(1)\}} X_{\{(4),\{1,2\},\{3\}\}} + 2 XX_{\{(1,2),\{3\}\}} X_{\{(1),\{2\}\}} \\
&+ XX_{\{(5),\{1,3\},\{4\},\{2\}\}} + X_{\{(1)\}} XX_{\{(4),\{1,2\},\{3\}\}} + X_{\{(1,2)\}} XX_{\{(1),\{2,\{3\}\}} \\
&+ 2 X_{\{(1)\}} XX_{\{(1,3),\{4\},\{2\}\}} + 2 XX_{\{(1)\}} X_{\{(1,3),\{4\},\{2\}\}} \\
\Delta(X_{\{(5),\{1\},\{4\},\{2,3\}\}}) &= -2 X_{\{(1)\}} XX_{\{(1),\{4\},\{2\},\{3\}\}} + XX_{\{(5),\{1\},\{4\},\{2,3\}\}} \\
&- 6 XX_{\{(1),\{2,\{3\}\}} X_{\{(1),\{2\}\}} + 2 XX_{\{(1),\{2\}\}} X_{\{(1,2),\{3\}\}} + XX_{\{(1,2)\}} X_{\{(1),\{2,\{3\}\}} \\
&+ 2 X_{\{(1)\}} XX_{\{(1),\{4\},\{2,3\}\}} - 2 XX_{\{(1)\}} X_{\{(1),\{4\},\{2\},\{3\}\}} + XX_{\{(1),\{2,3\}\}} X_{\{(1),\{2\}\}} \\
&- 6 XX_{\{(1),\{2\}\}} X_{\{(1),\{2,\{3\}\}} + XX_{\{(1)\}} X_{\{(4),\{1,2\},\{3\}\}} + XX_{\{(1),\{2\}\}} X_{\{(1),\{2,3\}\}} \\
&+ 2 XX_{\{(1)\}} X_{\{(1),\{4\},\{2,3\}\}} + 2 XX_{\{(1,2),\{3\}\}} X_{\{(1),\{2\}\}} + X_{\{(5),\{1\},\{4\},\{2,3\}\}} \\
&+ X_{\{(1)\}} XX_{\{(4),\{1,2\},\{3\}\}} + X_{\{(1,2)\}} XX_{\{(1),\{2,\{3\}\}} \\
\Delta(X_{\{(5),\{4\},\{1,2,3\}\}}) &= 3 X_{\{(1)\}} XX_{\{(1),\{4\},\{2\},\{3\}\}} + 9 XX_{\{(1),\{2,\{3\}\}} X_{\{(1),\{2\}\}} \\
&- 6 XX_{\{(1),\{2\}\}} X_{\{(1,2),\{3\}\}} - 3 XX_{\{(1,2)\}} X_{\{(1),\{2,\{3\}\}} + 3 XX_{\{(1)\}} X_{\{(1),\{4\},\{2\},\{3\}\}} \\
&+ 9 XX_{\{(1),\{2\}\}} X_{\{(1),\{2,\{3\}\}} + XX_{\{(5),\{4\},\{1,2,3\}\}} - 3 XX_{\{(1)\}} X_{\{(4),\{1,2\},\{3\}\}} \\
&- 6 XX_{\{(1,2),\{3\}\}} X_{\{(1),\{2\}\}} + 2 X_{\{(1)\}} XX_{\{(4),\{1,2,3\}\}} + XX_{\{(1,2,3)\}} X_{\{(1),\{2\}\}} \\
&+ 2 XX_{\{(1)\}} X_{\{(4),\{1,2,3\}\}} + XX_{\{(1),\{2\}\}} X_{\{(1,2,3\}\}} - 3 X_{\{(1)\}} XX_{\{(4),\{1,2\},\{3\}\}} \\
&- 3 X_{\{(1,2)\}} XX_{\{(1),\{2,\{3\}\}} + X_{\{(5),\{4\},\{1,2,3\}\}} \\
\Delta(X_{\{(5),\{2\},\{3\},\{1,4\}\}}) &= -2 X_{\{(1)\}} XX_{\{(1),\{4\},\{2\},\{3\}\}} - 6 XX_{\{(1),\{2,\{3\}\}} X_{\{(1),\{2\}\}} \\
&+ XX_{\{(1),\{2\}\}} X_{\{(1,2),\{3\}\}} + XX_{\{(1,2)\}} X_{\{(1),\{2,\{3\}\}} - 2 XX_{\{(1)\}} X_{\{(1),\{4\},\{2\},\{3\}\}} \\
&- 6 XX_{\{(1),\{2\}\}} X_{\{(1),\{2,\{3\}\}} + XX_{\{(1,2),\{3\}\}} X_{\{(1),\{2\}\}} + X_{\{(5),\{2\},\{3\},\{1,4\}\}} \\
&+ XX_{\{(5),\{2\},\{3\},\{1,4\}\}} + X_{\{(1,2)\}} XX_{\{(1),\{2,\{3\}\}} + 2 XX_{\{(1,3),\{2\}\}} X_{\{(1),\{2\}\}} \\
&+ X_{\{(1)\}} XX_{\{(2),\{3\},\{1,4\}\}} + 2 X_{\{(1)\}} XX_{\{(1,3),\{4\},\{2\}\}} + 2 XX_{\{(1)\}} X_{\{(1,3),\{4\},\{2\}\}} \\
&+ XX_{\{(1)\}} X_{\{(2),\{3\},\{1,4\}\}} + 2 XX_{\{(1),\{2\}\}} X_{\{(1,3),\{2\}\}}
\end{aligned}$$

$$\begin{aligned}
\Delta(X_{\{\{5\}, \{2,3\}, \{1,4\}\}}) &= 4 XX_{\{\{1\}, \{2\}, \{3\}\}} X_{\{\{1\}, \{2\}\}} - XX_{\{\{1\}, \{2\}\}} X_{\{\{1,2\}, \{3\}\}} \\
&+ 4 XX_{\{\{1\}, \{2\}\}} X_{\{\{1\}, \{2\}, \{3\}\}} - XX_{\{\{1,2\}, \{3\}\}} X_{\{\{1\}, \{2\}\}} + XX_{\{\{5\}, \{2,3\}, \{1,4\}\}} \\
&+ X_{\{\{1\}\}} XX_{\{\{2,3\}, \{1,4\}\}} - X_{\{\{1\}\}} XX_{\{\{1\}, \{4\}, \{2,3\}\}} - X_{\{\{1\}\}} XX_{\{\{4\}, \{1,2\}, \{3\}\}} \\
&- XX_{\{\{1\}\}} X_{\{\{1\}, \{4\}, \{2,3\}\}} - XX_{\{\{1\}\}} X_{\{\{4\}, \{1,2\}, \{3\}\}} + XX_{\{\{1\}\}} X_{\{\{2,3\}, \{1,4\}\}} \\
&- XX_{\{\{1\}, \{2\}\}} X_{\{\{1\}, \{2,3\}\}} + 2 XX_{\{\{1,2\}\}} X_{\{\{1,2\}, \{3\}\}} - XX_{\{\{1\}, \{2,3\}\}} X_{\{\{1\}, \{2\}\}} \\
&+ 2 X_{\{\{1,2\}\}} XX_{\{\{1,2\}, \{3\}\}} + X_{\{\{5\}, \{2,3\}, \{1,4\}\}} - 2 XX_{\{\{1,3\}, \{2\}\}} X_{\{\{1\}, \{2\}\}} \\
&- 2 X_{\{\{1\}\}} XX_{\{\{1,3\}, \{4\}, \{2\}\}} - 2 XX_{\{\{1\}\}} X_{\{\{1,3\}, \{4\}, \{2\}\}} - 2 XX_{\{\{1\}, \{2\}\}} X_{\{\{1,3\}, \{2\}\}}
\end{aligned}$$

$$\begin{aligned}
\Delta(X_{\{\{5\}, \{1,3,4\}, \{2\}\}}) &= 3 X_{\{\{1\}\}} XX_{\{\{1\}, \{4\}, \{2\}, \{3\}\}} + 9 XX_{\{\{1\}, \{2\}, \{3\}\}} X_{\{\{1\}, \{2\}\}} \\
&+ XX_{\{\{1\}\}} X_{\{\{4\}, \{1,2,3\}\}} - 3 XX_{\{\{1\}, \{2\}\}} X_{\{\{1,2\}, \{3\}\}} + XX_{\{\{1\}\}} X_{\{\{1,3,4\}, \{2\}\}} \\
&+ X_{\{\{1\}\}} XX_{\{\{4\}, \{1,2,3\}\}} + X_{\{\{1\}\}} XX_{\{\{1,3,4\}, \{2\}\}} + 3 XX_{\{\{1\}\}} X_{\{\{1\}, \{4\}, \{2\}, \{3\}\}} \\
&+ 9 XX_{\{\{1\}, \{2\}\}} X_{\{\{1\}, \{2\}, \{3\}\}} - 3 XX_{\{\{1,2\}, \{3\}\}} X_{\{\{1\}, \{2\}\}} + XX_{\{\{1\}, \{2\}\}} X_{\{\{1,2,3\}\}} \\
&- 3 XX_{\{\{1,2\}\}} X_{\{\{1\}, \{2\}, \{3\}\}} - X_{\{\{1\}\}} XX_{\{\{1\}, \{4\}, \{2,3\}\}} - XX_{\{\{1\}\}} X_{\{\{1\}, \{4\}, \{2,3\}\}} \\
&- XX_{\{\{1\}, \{2\}\}} X_{\{\{1\}, \{2,3\}\}} - XX_{\{\{1\}, \{2,3\}\}} X_{\{\{1\}, \{2\}\}} + XX_{\{\{5\}, \{1,3,4\}, \{2\}\}} \\
&- 2 XX_{\{\{1,3\}, \{2\}\}} X_{\{\{1\}, \{2\}\}} - 2 X_{\{\{1\}\}} XX_{\{\{1,3\}, \{4\}, \{2\}\}} + X_{\{\{5\}, \{1,3,4\}, \{2\}\}} \\
&- 2 XX_{\{\{1\}\}} X_{\{\{1,3\}, \{4\}, \{2\}\}} - 2 XX_{\{\{1\}, \{2\}\}} X_{\{\{1,3\}, \{2\}\}} + XX_{\{\{1,2,3\}\}} X_{\{\{1\}, \{2\}\}} \\
&- 3 X_{\{\{1,2\}\}} XX_{\{\{1\}, \{2\}, \{3\}\}}
\end{aligned}$$

$$\begin{aligned}
\Delta(X_{\{\{5\}, \{1\}, \{2,4\}, \{3\}\}}) &= -2 X_{\{\{1\}\}} XX_{\{\{1\}, \{4\}, \{2\}, \{3\}\}} + X_{\{\{1\}\}} XX_{\{\{1\}, \{2,4\}, \{3\}\}} \\
&- 6 XX_{\{\{1\}, \{2\}, \{3\}\}} X_{\{\{1\}, \{2\}\}} + XX_{\{\{1\}, \{2\}\}} X_{\{\{1,2\}, \{3\}\}} - 2 XX_{\{\{1\}\}} X_{\{\{1\}, \{4\}, \{2\}, \{3\}\}} \\
&- 6 XX_{\{\{1\}, \{2\}\}} X_{\{\{1\}, \{2\}, \{3\}\}} + XX_{\{\{1\}\}} X_{\{\{1\}, \{2,4\}, \{3\}\}} + XX_{\{\{1,2\}, \{3\}\}} X_{\{\{1\}, \{2\}\}} \\
&+ XX_{\{\{1,2\}\}} X_{\{\{1\}, \{2\}, \{3\}\}} + X_{\{\{1\}\}} XX_{\{\{1\}, \{4\}, \{2,3\}\}} + XX_{\{\{1\}\}} X_{\{\{1\}, \{4\}, \{2,3\}\}} \\
&+ XX_{\{\{1\}, \{2\}\}} X_{\{\{1\}, \{2,3\}\}} + XX_{\{\{1\}, \{2,3\}\}} X_{\{\{1\}, \{2\}\}} + XX_{\{\{1,3\}, \{2\}\}} X_{\{\{1\}, \{2\}\}} \\
&+ X_{\{\{1\}\}} XX_{\{\{1,3\}, \{4\}, \{2\}\}} + XX_{\{\{1\}\}} X_{\{\{1,3\}, \{4\}, \{2\}\}} + XX_{\{\{1\}, \{2\}\}} X_{\{\{1,3\}, \{2\}\}} \\
&+ X_{\{\{5\}, \{1\}, \{2,4\}, \{3\}\}} + X_{\{\{1,2\}\}} XX_{\{\{1\}, \{2\}, \{3\}\}} + XX_{\{\{5\}, \{1\}, \{2,4\}, \{3\}\}}
\end{aligned}$$

$$\begin{aligned}
\Delta(X_{\{\{5\}, \{1,3\}, \{2,4\}\}}) &= X_{\{\{1\}\}} XX_{\{\{1,3\}, \{2,4\}\}} + 4 XX_{\{\{1\}, \{2\}, \{3\}\}} X_{\{\{1\}, \{2\}\}} \\
&- XX_{\{\{1\}, \{2\}\}} X_{\{\{1,2\}, \{3\}\}} + XX_{\{\{1\}\}} X_{\{\{1,3\}, \{2,4\}\}} + 4 XX_{\{\{1\}, \{2\}\}} X_{\{\{1\}, \{2\}, \{3\}\}} \\
&- XX_{\{\{1,2\}, \{3\}\}} X_{\{\{1\}, \{2\}\}} + X_{\{\{5\}, \{1,3\}, \{2,4\}\}} - X_{\{\{1\}\}} XX_{\{\{1\}, \{4\}, \{2,3\}\}} \\
&- X_{\{\{1\}\}} XX_{\{\{4\}, \{1,2\}, \{3\}\}} - XX_{\{\{1\}\}} X_{\{\{1\}, \{4\}, \{2,3\}\}} - XX_{\{\{1\}\}} X_{\{\{4\}, \{1,2\}, \{3\}\}} \\
&- XX_{\{\{1\}, \{2\}\}} X_{\{\{1\}, \{2,3\}\}} + 2 XX_{\{\{1,2\}\}} X_{\{\{1,2\}, \{3\}\}} - XX_{\{\{1\}, \{2,3\}\}} X_{\{\{1\}, \{2\}\}} \\
&+ 2 X_{\{\{1,2\}\}} XX_{\{\{1,2\}, \{3\}\}} + XX_{\{\{5\}, \{1,3\}, \{2,4\}\}} - 2 XX_{\{\{1,3\}, \{2\}\}} X_{\{\{1\}, \{2\}\}} \\
&- 2 X_{\{\{1\}\}} XX_{\{\{1,3\}, \{4\}, \{2\}\}} - 2 XX_{\{\{1\}\}} X_{\{\{1,3\}, \{4\}, \{2\}\}} - 2 XX_{\{\{1\}, \{2\}\}} X_{\{\{1,3\}, \{2\}\}}
\end{aligned}$$

$$\begin{aligned}
\Delta(X_{\{\{5\}, \{1\}, \{2,3,4\}\}}) &= 3 X_{\{\{1\}\}} XX_{\{\{1\}, \{4\}, \{2\}, \{3\}\}} + X_{\{\{1\}\}} XX_{\{\{4\}, \{1,2,3\}\}} \\
&+ X_{\{\{1\}\}} XX_{\{\{1\}, \{2,3,4\}\}} + 9 XX_{\{\{1\}, \{2\}, \{3\}\}} X_{\{\{1\}, \{2\}\}} - 3 X_{\{\{1,2\}\}} XX_{\{\{1\}, \{2\}, \{3\}\}} \\
&- 3 XX_{\{\{1,2\}\}} X_{\{\{1\}, \{2\}, \{3\}\}} - 3 XX_{\{\{1\}, \{2\}\}} X_{\{\{1,2\}, \{3\}\}} + XX_{\{\{1,2,3\}\}} X_{\{\{1\}, \{2\}\}} \\
&+ XX_{\{\{1\}, \{2\}\}} X_{\{\{1,2,3\}\}} + 3 XX_{\{\{1\}\}} X_{\{\{1\}, \{4\}, \{2\}, \{3\}\}} + 9 XX_{\{\{1\}, \{2\}\}} X_{\{\{1\}, \{2\}, \{3\}\}} \\
&+ X_{\{\{5\}, \{1\}, \{2,3,4\}\}} - 3 XX_{\{\{1,2\}, \{3\}\}} X_{\{\{1\}, \{2\}\}} + XX_{\{\{5\}, \{1\}, \{2,3,4\}\}} \\
&- 3 X_{\{\{1\}\}} XX_{\{\{1\}, \{4\}, \{2,3\}\}} - 3 XX_{\{\{1\}\}} X_{\{\{1\}, \{4\}, \{2,3\}\}} - 3 XX_{\{\{1\}, \{2\}\}} X_{\{\{1\}, \{2,3\}\}} \\
&- 3 XX_{\{\{1\}, \{2,3\}\}} X_{\{\{1\}, \{2\}\}} + XX_{\{\{1\}\}} X_{\{\{1\}, \{2,3,4\}\}} + XX_{\{\{1\}\}} X_{\{\{4\}, \{1,2,3\}\}}
\end{aligned}$$

$$\begin{aligned}
\Delta(X_{\{\{5\}, \{1,2,4\}, \{3\}\}}) &= 3 X_{\{\{1\}\}} XX_{\{\{1\}, \{4\}, \{2\}, \{3\}\}} + X_{\{\{1\}\}} XX_{\{\{4\}, \{1,2,3\}\}} \\
&+ 9 XX_{\{\{1\}, \{2\}, \{3\}\}} X_{\{\{1\}, \{2\}\}} - 3 X_{\{\{1,2\}\}} XX_{\{\{1\}, \{2\}, \{3\}\}} - 3 XX_{\{\{1,2\}\}} X_{\{\{1\}, \{2\}, \{3\}\}} \\
&- 4 XX_{\{\{1\}, \{2\}\}} X_{\{\{1,2\}, \{3\}\}} + XX_{\{\{1,2,3\}\}} X_{\{\{1\}, \{2\}\}} + XX_{\{\{1\}, \{2\}\}} X_{\{\{1,2,3\}\}}
\end{aligned}$$

$$\begin{aligned}
& + 3 XX_{\{\{1\}\}} X_{\{\{1\}, \{4\}, \{2\}, \{3\}\}} + 9 XX_{\{\{1\}, \{2\}\}} X_{\{\{1\}, \{2\}, \{3\}\}} + XX_{\{\{1\}\}} X_{\{\{1, 2, 4\}, \{3\}\}} \\
& - 4 XX_{\{\{1, 2\}, \{3\}\}} X_{\{\{1\}, \{2\}\}} + X_{\{\{5\}, \{1, 2, 4\}, \{3\}\}} - X_{\{\{1\}\}} XX_{\{\{4\}, \{1, 2\}, \{3\}\}} \\
& - XX_{\{\{1\}\}} X_{\{\{4\}, \{1, 2\}, \{3\}\}} + XX_{\{\{5\}, \{1, 2, 4\}, \{3\}\}} - 2 XX_{\{\{1, 3\}, \{2\}\}} X_{\{\{1\}, \{2\}\}} \\
& - 2 X_{\{\{1\}\}} XX_{\{\{1, 3\}, \{4\}, \{2\}\}} - 2 XX_{\{\{1\}\}} X_{\{\{1, 3\}, \{4\}, \{2\}\}} - 2 XX_{\{\{1\}, \{2\}\}} X_{\{\{1, 3\}, \{2\}\}} \\
& + XX_{\{\{1\}\}} X_{\{\{4\}, \{1, 2, 3\}\}} + X_{\{\{1\}\}} XX_{\{\{1, 2, 4\}, \{3\}\}} \\
\Delta(X_{\{\{5\}, \{1\}, \{3, 4\}, \{2\}\}}) & = -2 X_{\{\{1\}\}} XX_{\{\{1\}, \{4\}, \{2\}, \{3\}\}} - 6 XX_{\{\{1\}, \{2\}, \{3\}\}} X_{\{\{1\}, \{2\}\}} \\
& + X_{\{\{1, 2\}\}} XX_{\{\{1\}, \{2\}, \{3\}\}} + XX_{\{\{1, 2\}\}} X_{\{\{1\}, \{2\}, \{3\}\}} + XX_{\{\{1\}, \{2\}\}} X_{\{\{1, 2\}, \{3\}\}} \\
& - 2 XX_{\{\{1\}\}} X_{\{\{1\}, \{4\}, \{2\}, \{3\}\}} - 6 XX_{\{\{1\}, \{2\}\}} X_{\{\{1\}, \{2\}, \{3\}\}} + XX_{\{\{1, 2\}, \{3\}\}} X_{\{\{1\}, \{2\}\}} \\
& + XX_{\{\{5\}, \{1\}, \{3, 4\}, \{2\}\}} + 2 X_{\{\{1\}\}} XX_{\{\{1\}, \{4\}, \{2, 3\}\}} + 2 XX_{\{\{1\}\}} X_{\{\{1\}, \{4\}, \{2, 3\}\}} \\
& + 2 XX_{\{\{1\}, \{2\}\}} X_{\{\{1\}, \{2, 3\}\}} + 2 XX_{\{\{1\}, \{2, 3\}\}} X_{\{\{1\}, \{2\}\}} + X_{\{\{1\}\}} XX_{\{\{1\}, \{3, 4\}, \{2\}\}} \\
& + XX_{\{\{1\}\}} X_{\{\{1\}, \{3, 4\}, \{2\}\}} + X_{\{\{5\}, \{1\}, \{3, 4\}, \{2\}\}} \\
\Delta(X_{\{\{5\}, \{3, 4\}, \{1, 2\}\}}) & = 4 XX_{\{\{1\}, \{2\}, \{3\}\}} X_{\{\{1\}, \{2\}\}} - 2 XX_{\{\{1\}, \{2\}\}} X_{\{\{1, 2\}, \{3\}\}} \\
& + 4 XX_{\{\{1\}, \{2\}\}} X_{\{\{1\}, \{2\}, \{3\}\}} + X_{\{\{5\}, \{3, 4\}, \{1, 2\}\}} - 2 XX_{\{\{1, 2\}, \{3\}\}} X_{\{\{1\}, \{2\}\}} \\
& + XX_{\{\{5\}, \{3, 4\}, \{1, 2\}\}} + X_{\{\{1\}\}} XX_{\{\{3, 4\}, \{1, 2\}\}} + XX_{\{\{1\}\}} X_{\{\{3, 4\}, \{1, 2\}\}} \\
& - 2 X_{\{\{1\}\}} XX_{\{\{1\}, \{4\}, \{2, 3\}\}} - 2 X_{\{\{1\}\}} XX_{\{\{4\}, \{1, 2\}, \{3\}\}} - 2 XX_{\{\{1\}\}} X_{\{\{1\}, \{4\}, \{2, 3\}\}} \\
& - 2 XX_{\{\{1\}\}} X_{\{\{4\}, \{1, 2\}, \{3\}\}} - 2 XX_{\{\{1\}, \{2\}\}} X_{\{\{1\}, \{2, 3\}\}} + 2 XX_{\{\{1, 2\}\}} X_{\{\{1, 2\}, \{3\}\}} \\
& - 2 XX_{\{\{1\}, \{2, 3\}\}} X_{\{\{1\}, \{2\}\}} + 2 X_{\{\{1, 2\}\}} XX_{\{\{1, 2\}, \{3\}\}} \\
\Delta(X_{\{\{5\}, \{1, 2, 3, 4\}\}}) & = -4 X_{\{\{1\}\}} XX_{\{\{1\}, \{4\}, \{2\}, \{3\}\}} + 6 XX_{\{\{1\}, \{2\}, \{3\}\}} X_{\{\{1, 2\}\}} \\
& - 4 XX_{\{\{1, 2, 3\}\}} X_{\{\{1\}, \{2\}\}} - 4 XX_{\{\{1\}\}} X_{\{\{4\}, \{1, 2, 3\}\}} + XX_{\{\{1\}\}} X_{\{\{1, 2, 3, 4\}\}} \\
& + 6 XX_{\{\{1, 2\}\}} X_{\{\{1\}, \{2\}, \{3\}\}} - 4 XX_{\{\{1\}, \{2\}\}} X_{\{\{1, 2, 3\}\}} - 22 XX_{\{\{1\}, \{2\}, \{3\}\}} X_{\{\{1\}, \{2\}\}} \\
& + 10 XX_{\{\{1\}, \{2\}\}} X_{\{\{1, 2\}, \{3\}\}} - 4 XX_{\{\{1\}\}} X_{\{\{1\}, \{4\}, \{2\}, \{3\}\}} - 22 XX_{\{\{1\}, \{2\}\}} X_{\{\{1\}, \{2\}, \{3\}\}} \\
& + 10 XX_{\{\{1, 2\}, \{3\}\}} X_{\{\{1\}, \{2\}\}} + X_{\{\{5\}, \{1, 2, 3, 4\}\}} + 4 X_{\{\{1\}\}} XX_{\{\{1\}, \{4\}, \{2, 3\}\}} \\
& + 4 X_{\{\{1\}\}} XX_{\{\{4\}, \{1, 2\}, \{3\}\}} + 4 XX_{\{\{1\}\}} X_{\{\{1\}, \{4\}, \{2, 3\}\}} + 4 XX_{\{\{1\}\}} X_{\{\{4\}, \{1, 2\}, \{3\}\}} \\
& + 4 XX_{\{\{1\}, \{2\}\}} X_{\{\{1\}, \{2, 3\}\}} - 6 XX_{\{\{1, 2\}\}} X_{\{\{1, 2\}, \{3\}\}} + 4 XX_{\{\{1\}, \{2, 3\}\}} X_{\{\{1\}, \{2\}\}} \\
& - 6 X_{\{\{1, 2\}\}} XX_{\{\{1, 2\}, \{3\}\}} - 4 X_{\{\{1\}\}} XX_{\{\{4\}, \{1, 2, 3\}\}} + XX_{\{\{5\}, \{1, 2, 3, 4\}\}} \\
& + X_{\{\{1\}\}} XX_{\{\{1, 2, 3, 4\}\}} + 4 XX_{\{\{1, 3\}, \{2\}\}} X_{\{\{1\}, \{2\}\}} + 4 X_{\{\{1\}\}} XX_{\{\{1, 3\}, \{4\}, \{2\}\}} \\
& + 4 XX_{\{\{1\}\}} X_{\{\{1, 3\}, \{4\}, \{2\}\}} + 4 XX_{\{\{1\}, \{2\}\}} X_{\{\{1, 3\}, \{2\}\}} \\
\Delta(X_{\{\{4\}, \{2\}, \{3\}, \{1, 5\}\}}) & = -2 X_{\{\{1\}\}} XX_{\{\{1\}, \{4\}, \{2\}, \{3\}\}} + XX_{\{\{1\}, \{2\}, \{3\}\}} X_{\{\{1, 2\}\}} \\
& + XX_{\{\{1, 2\}\}} X_{\{\{1\}, \{2\}, \{3\}\}} - 6 XX_{\{\{1\}, \{2\}, \{3\}\}} X_{\{\{1\}, \{2\}\}} + 3 XX_{\{\{1\}\}} X_{\{\{2\}, \{3\}, \{1, 4\}\}} \\
& - 2 XX_{\{\{1\}\}} X_{\{\{1\}, \{4\}, \{2\}, \{3\}\}} - 6 XX_{\{\{1\}, \{2\}\}} X_{\{\{1\}, \{2\}, \{3\}\}} + 3 X_{\{\{1\}\}} XX_{\{\{2\}, \{3\}, \{1, 4\}\}} \\
& + X_{\{\{4\}, \{2\}, \{3\}, \{1, 5\}\}} + 3 XX_{\{\{1, 3\}, \{2\}\}} X_{\{\{1\}, \{2\}\}} + 3 XX_{\{\{1\}, \{2\}\}} X_{\{\{1, 3\}, \{2\}\}} \\
& + XX_{\{\{4\}, \{2\}, \{3\}, \{1, 5\}\}} \\
\Delta(X_{\{\{4\}, \{2, 3\}, \{1, 5\}\}}) & = XX_{\{\{1, 2\}\}} X_{\{\{1, 3\}, \{2\}\}} + 4 XX_{\{\{1\}, \{2\}, \{3\}\}} X_{\{\{1\}, \{2\}\}} \\
& - XX_{\{\{1\}, \{2\}\}} X_{\{\{1, 2\}, \{3\}\}} - 2 XX_{\{\{1\}\}} X_{\{\{2\}, \{3\}, \{1, 4\}\}} + 4 XX_{\{\{1\}, \{2\}\}} X_{\{\{1\}, \{2\}, \{3\}\}} \\
& - 2 X_{\{\{1\}\}} XX_{\{\{2\}, \{3\}, \{1, 4\}\}} - XX_{\{\{1, 2\}, \{3\}\}} X_{\{\{1\}, \{2\}\}} + XX_{\{\{1, 3\}, \{2\}\}} X_{\{\{1, 2\}\}} \\
& + X_{\{\{1\}\}} XX_{\{\{2, 3\}, \{1, 4\}\}} - X_{\{\{1\}\}} XX_{\{\{1\}, \{4\}, \{2, 3\}\}} - X_{\{\{1\}\}} XX_{\{\{4\}, \{1, 2\}, \{3\}\}} \\
& - XX_{\{\{1\}\}} X_{\{\{1\}, \{4\}, \{2, 3\}\}} - XX_{\{\{1\}\}} X_{\{\{4\}, \{1, 2\}, \{3\}\}} + XX_{\{\{1\}\}} X_{\{\{2, 3\}, \{1, 4\}\}} \\
& - XX_{\{\{1\}, \{2\}\}} X_{\{\{1\}, \{2, 3\}\}} + XX_{\{\{1, 2\}\}} X_{\{\{1, 2\}, \{3\}\}} - XX_{\{\{1\}, \{2, 3\}\}} X_{\{\{1\}, \{2\}\}} \\
& + X_{\{\{1, 2\}\}} XX_{\{\{1, 2\}, \{3\}\}} - 2 XX_{\{\{1, 3\}, \{2\}\}} X_{\{\{1\}, \{2\}\}} + XX_{\{\{4\}, \{2, 3\}, \{1, 5\}\}} \\
& - 2 XX_{\{\{1\}, \{2\}\}} X_{\{\{1, 3\}, \{2\}\}} + X_{\{\{4\}, \{2, 3\}, \{1, 5\}\}}
\end{aligned}$$

$$\begin{aligned}
\Delta(X_{\{\{2,4\},\{3\},\{1,5\}\}}) &= X_{\{\{2,4\},\{3\},\{1,5\}\}} + 2 XX_{\{\{1,2\}\}} X_{\{\{1,3\},\{2\}\}} \\
&+ 4 XX_{\{\{1\},\{2\},\{3\}\}} X_{\{\{1\},\{2\}\}} - XX_{\{\{1\},\{2\}\}} X_{\{\{1,2\},\{3\}\}} - 2 XX_{\{\{1\}\}} X_{\{\{2\},\{3\},\{1,4\}\}} \\
&+ 4 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2\},\{3\}\}} - XX_{\{\{1\}\}} X_{\{\{1\},\{2,4\},\{3\}\}} - 2 X_{\{\{1\}\}} XX_{\{\{2\},\{3\},\{1,4\}\}} \\
&- XX_{\{\{1,2\},\{3\}\}} X_{\{\{1\},\{2\}\}} + 2 XX_{\{\{1,3\},\{2\}\}} X_{\{\{1,2\}\}} + X_{\{\{1\}\}} XX_{\{\{2,3\},\{1,4\}\}} \\
&+ XX_{\{\{1\}\}} X_{\{\{2,3\},\{1,4\}\}} - XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2,3\}\}} - XX_{\{\{1\},\{2,3\}\}} X_{\{\{1\},\{2\}\}} \\
&+ XX_{\{\{2,4\},\{3\},\{1,5\}\}} - 2 XX_{\{\{1,3\},\{2\}\}} X_{\{\{1\},\{2\}\}} - X_{\{\{1\}\}} XX_{\{\{1,3\},\{4\},\{2\}\}} \\
&- XX_{\{\{1\}\}} X_{\{\{1,3\},\{4\},\{2\}\}} - 2 XX_{\{\{1\},\{2\}\}} X_{\{\{1,3\},\{2\}\}} - X_{\{\{1\}\}} XX_{\{\{1\},\{2,4\},\{3\}\}} \\
\Delta(X_{\{\{2,3,4\},\{1,5\}\}}) &= XX_{\{\{1,2\}\}} X_{\{\{1,2,3\}\}} - 3 XX_{\{\{1,2\}\}} X_{\{\{1,3\},\{2\}\}} \\
&- 6 XX_{\{\{1\},\{2\},\{3\}\}} X_{\{\{1\},\{2\}\}} + 3 XX_{\{\{1\},\{2\}\}} X_{\{\{1,2\},\{3\}\}} + 3 XX_{\{\{1\}\}} X_{\{\{2\},\{3\},\{1,4\}\}} \\
&- XX_{\{\{1\}\}} X_{\{\{4\},\{1,2,3\}\}} - 6 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2\},\{3\}\}} - XX_{\{\{1\}\}} X_{\{\{1\},\{2,3,4\}\}} \\
&+ 3 X_{\{\{1\}\}} XX_{\{\{2\},\{3\},\{1,4\}\}} + 3 XX_{\{\{1,2\},\{3\}\}} X_{\{\{1\},\{2\}\}} + XX_{\{\{1,2,3\}\}} X_{\{\{1,2\}\}} \\
&- 3 XX_{\{\{1,3\},\{2\}\}} X_{\{\{1,2\}\}} - 3 X_{\{\{1\}\}} XX_{\{\{2,3\},\{1,4\}\}} - 3 XX_{\{\{1\}\}} X_{\{\{2,3\},\{1,4\}\}} \\
&+ 3 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2,3\}\}} + 3 XX_{\{\{1\},\{2,3\}\}} X_{\{\{1\},\{2\}\}} + XX_{\{\{2,3,4\},\{1,5\}\}} \\
&+ 3 XX_{\{\{1,3\},\{2\}\}} X_{\{\{1\},\{2\}\}} + 3 XX_{\{\{1\},\{2\}\}} X_{\{\{1,3\},\{2\}\}} - X_{\{\{1\}\}} XX_{\{\{4\},\{1,2,3\}\}} \\
&- X_{\{\{1\}\}} XX_{\{\{1\},\{2,3,4\}\}} + X_{\{\{2,3,4\},\{1,5\}\}} \\
\Delta(X_{\{\{3,4\},\{2\},\{1,5\}\}}) &= -XX_{\{\{1\}\}} X_{\{\{1\},\{3,4\},\{2\}\}} + XX_{\{\{1,2\}\}} X_{\{\{1\},\{2,3\}\}} \\
&+ XX_{\{\{1,2\}\}} X_{\{\{1,3\},\{2\}\}} + 4 XX_{\{\{1\},\{2\},\{3\}\}} X_{\{\{1\},\{2\}\}} - XX_{\{\{1\},\{2\}\}} X_{\{\{1,2\},\{3\}\}} \\
&- 2 XX_{\{\{1\}\}} X_{\{\{2\},\{3\},\{1,4\}\}} + 4 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2\},\{3\}\}} - 2 X_{\{\{1\}\}} XX_{\{\{2\},\{3\},\{1,4\}\}} \\
&+ X_{\{\{3,4\},\{2\},\{1,5\}\}} - XX_{\{\{1,2\},\{3\}\}} X_{\{\{1\},\{2\}\}} + XX_{\{\{1,3\},\{2\}\}} X_{\{\{1,2\}\}} + XX_{\{\{3,4\},\{2\},\{1,5\}\}} \\
&+ X_{\{\{1,2\}\}} XX_{\{\{1\},\{2,3\}\}} + X_{\{\{1\}\}} XX_{\{\{2,3\},\{1,4\}\}} - X_{\{\{1\}\}} XX_{\{\{1\},\{4\},\{2,3\}\}} \\
&- XX_{\{\{1\}\}} X_{\{\{1\},\{4\},\{2,3\}\}} + XX_{\{\{1\}\}} X_{\{\{2,3\},\{1,4\}\}} - XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2,3\}\}} \\
&- XX_{\{\{1\},\{2,3\}\}} X_{\{\{1\},\{2\}\}} - 2 XX_{\{\{1,3\},\{2\}\}} X_{\{\{1\},\{2\}\}} - 2 XX_{\{\{1\},\{2\}\}} X_{\{\{1,3\},\{2\}\}} \\
&- X_{\{\{1\}\}} XX_{\{\{1\},\{3,4\},\{2\}\}} \\
\Delta(X_{\{\{1,3,5\},\{4\},\{2\}\}}) &= 3 X_{\{\{1\}\}} XX_{\{\{1\},\{4\},\{2\},\{3\}\}} - 3 XX_{\{\{1\},\{2\},\{3\}\}} X_{\{\{1,2\}\}} \\
&+ XX_{\{\{1\},\{2\}\}} X_{\{\{1,2,3\}\}} - 3 XX_{\{\{1,2\}\}} X_{\{\{1\},\{2\},\{3\}\}} + 9 XX_{\{\{1\},\{2\},\{3\}\}} X_{\{\{1\},\{2\}\}} \\
&- XX_{\{\{1\},\{2\}\}} X_{\{\{1,2\},\{3\}\}} - XX_{\{\{1\}\}} X_{\{\{2\},\{3\},\{1,4\}\}} + 3 XX_{\{\{1\}\}} X_{\{\{1\},\{4\},\{2\},\{3\}\}} \\
&+ XX_{\{\{1\}\}} X_{\{\{1,2,4\},\{3\}\}} + 9 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2\},\{3\}\}} - XX_{\{\{1\}\}} X_{\{\{1\},\{2,4\},\{3\}\}} \\
&+ XX_{\{\{1\}\}} X_{\{\{1,3,4\},\{2\}\}} - X_{\{\{1\}\}} XX_{\{\{2\},\{3\},\{1,4\}\}} - XX_{\{\{1,2\},\{3\}\}} X_{\{\{1\},\{2\}\}} \\
&+ XX_{\{\{1,2,3\}\}} X_{\{\{1\},\{2\}\}} + X_{\{\{1,3,5\},\{4\},\{2\}\}} - XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2,3\}\}} \\
&- XX_{\{\{1\},\{2,3\}\}} X_{\{\{1\},\{2\}\}} - 4 XX_{\{\{1,3\},\{2\}\}} X_{\{\{1\},\{2\}\}} - X_{\{\{1\}\}} XX_{\{\{1,3\},\{4\},\{2\}\}} \\
&- XX_{\{\{1\}\}} X_{\{\{1,3\},\{4\},\{2\}\}} - 4 XX_{\{\{1\},\{2\}\}} X_{\{\{1,3\},\{2\}\}} + X_{\{\{1\}\}} XX_{\{\{1,2,4\},\{3\}\}} \\
&- X_{\{\{1\}\}} XX_{\{\{1\},\{2,4\},\{3\}\}} + X_{\{\{1\}\}} XX_{\{\{1,3,4\},\{2\}\}} + XX_{\{\{1,3,5\},\{4\},\{2\}\}} \\
\Delta(X_{\{\{1,3,5\},\{2,4\}\}}) &= -XX_{\{\{1,2\}\}} X_{\{\{1\},\{2,3\}\}} + XX_{\{\{1,2\}\}} X_{\{\{1,2,3\}\}} - XX_{\{\{1,2\}\}} X_{\{\{1,3\},\{2\}\}} \\
&- 6 XX_{\{\{1\},\{2\},\{3\}\}} X_{\{\{1\},\{2\}\}} + 2 XX_{\{\{1\},\{2\}\}} X_{\{\{1,2\},\{3\}\}} - 2 XX_{\{\{1\}\}} X_{\{\{1,3\},\{2,4\}\}} \\
&- XX_{\{\{1\}\}} X_{\{\{1,2,4\},\{3\}\}} - 6 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2\},\{3\}\}} + XX_{\{\{1\}\}} X_{\{\{1\},\{2,4\},\{3\}\}} \\
&- XX_{\{\{1\}\}} X_{\{\{1,3,4\},\{2\}\}} + 2 XX_{\{\{1,2\},\{3\}\}} X_{\{\{1\},\{2\}\}} + XX_{\{\{1,2,3\}\}} X_{\{\{1,2\}\}} \\
&- XX_{\{\{1,3\},\{2\}\}} X_{\{\{1,2\}\}} - 2 X_{\{\{1\}\}} XX_{\{\{1,3\},\{2,4\}\}} - X_{\{\{1,2\}\}} XX_{\{\{1\},\{2,3\}\}} \\
&- X_{\{\{1\}\}} XX_{\{\{2,3\},\{1,4\}\}} + X_{\{\{1\}\}} XX_{\{\{1\},\{4\},\{2,3\}\}} + XX_{\{\{1\}\}} X_{\{\{1\},\{4\},\{2,3\}\}} \\
&- XX_{\{\{1\}\}} X_{\{\{2,3\},\{1,4\}\}} + 2 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2,3\}\}} - XX_{\{\{1,2\}\}} X_{\{\{1,2\},\{3\}\}} \\
&+ 2 XX_{\{\{1\},\{2,3\}\}} X_{\{\{1\},\{2\}\}} - X_{\{\{1,2\}\}} XX_{\{\{1,2\},\{3\}\}} + 5 XX_{\{\{1,3\},\{2\}\}} X_{\{\{1\},\{2\}\}}
\end{aligned}$$

$$\begin{aligned}
& + X_{\{\{1\}\}} XX_{\{\{1,3\},\{4\},\{2\}\}} + XX_{\{\{1\}\}} X_{\{\{1,3\},\{4\},\{2\}\}} + 5 XX_{\{\{1\},\{2\}\}} X_{\{\{1,3\},\{2\}\}} \\
& + X_{\{\{1,3,5\},\{2,4\}\}} - X_{\{\{1\}\}} XX_{\{\{1,2,4\},\{3\}\}} + X_{\{\{1\}\}} XX_{\{\{1\},\{2,4\},\{3\}\}} - X_{\{\{1\}\}} XX_{\{\{1,3,4\},\{2\}\}} \\
& + XX_{\{\{1,3,5\},\{2,4\}\}} \\
\Delta(X_{\{\{4,5\},\{1\},\{2\},\{3\}\}}) & = 3 XX_{\{\{1\}\}} X_{\{\{1\},\{3,4\},\{2\}\}} - 2 X_{\{\{1\}\}} XX_{\{\{1\},\{4\},\{2\},\{3\}\}} \\
& - 6 XX_{\{\{1\},\{2\},\{3\}\}} X_{\{\{1\},\{2\}\}} + XX_{\{\{4,5\},\{1\},\{2\},\{3\}\}} - 2 XX_{\{\{1\}\}} X_{\{\{1\},\{4\},\{2\},\{3\}\}} \\
& - 6 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2\},\{3\}\}} + XX_{\{\{1\},\{2\},\{3\}\}} X_{\{\{1,2\}\}} + XX_{\{\{1,2\}\}} X_{\{\{1\},\{2\},\{3\}\}} \\
& + 3 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2,3\}\}} + 3 XX_{\{\{1\},\{2,3\}\}} X_{\{\{1\},\{2\}\}} + 3 X_{\{\{1\}\}} XX_{\{\{1\},\{3,4\},\{2\}\}} \\
& + X_{\{\{4,5\},\{1\},\{2\},\{3\}\}} \\
\Delta(X_{\{\{4,5\},\{1,2\},\{3\}\}}) & = -2 XX_{\{\{1\}\}} X_{\{\{1\},\{3,4\},\{2\}\}} + XX_{\{\{1,2\}\}} X_{\{\{1\},\{2,3\}\}} \\
& + 4 XX_{\{\{1\},\{2\},\{3\}\}} X_{\{\{1\},\{2\}\}} - 2 XX_{\{\{1\},\{2\}\}} X_{\{\{1,2\},\{3\}\}} + 4 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2\},\{3\}\}} \\
& - 2 XX_{\{\{1,2\},\{3\}\}} X_{\{\{1\},\{2\}\}} + XX_{\{\{1\}\}} X_{\{\{3,4\},\{1,2\}\}} + X_{\{\{1\}\}} XX_{\{\{3,4\},\{1,2\}\}} \\
& + X_{\{\{4,5\},\{1,2\},\{3\}\}} + X_{\{\{1,2\}\}} XX_{\{\{1\},\{2,3\}\}} - 2 X_{\{\{1\}\}} XX_{\{\{4\},\{1,2\},\{3\}\}} \\
& - 2 XX_{\{\{1\}\}} X_{\{\{4\},\{1,2\},\{3\}\}} - 2 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2,3\}\}} + XX_{\{\{1,2\}\}} X_{\{\{1,2\},\{3\}\}} \\
& - 2 XX_{\{\{1\},\{2,3\}\}} X_{\{\{1\},\{2\}\}} + X_{\{\{1,2\}\}} XX_{\{\{1,2\},\{3\}\}} - 2 X_{\{\{1\}\}} XX_{\{\{1\},\{3,4\},\{2\}\}} \\
& + XX_{\{\{4,5\},\{1,2\},\{3\}\}} \\
\Delta(X_{\{\{4,5\},\{1,3\},\{2\}\}}) & = -2 XX_{\{\{1\}\}} X_{\{\{1\},\{3,4\},\{2\}\}} + XX_{\{\{1,2\}\}} X_{\{\{1\},\{2,3\}\}} \\
& + XX_{\{\{1,2\}\}} X_{\{\{1,3\},\{2\}\}} + 4 XX_{\{\{1\},\{2\},\{3\}\}} X_{\{\{1\},\{2\}\}} - 2 XX_{\{\{1\},\{2\}\}} X_{\{\{1,2\},\{3\}\}} \\
& + 4 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2\},\{3\}\}} - 2 XX_{\{\{1,2\},\{3\}\}} X_{\{\{1\},\{2\}\}} + XX_{\{\{1\}\}} X_{\{\{3,4\},\{1,2\}\}} \\
& + X_{\{\{1\}\}} XX_{\{\{3,4\},\{1,2\}\}} + XX_{\{\{1,3\},\{2\}\}} X_{\{\{1,2\}\}} + X_{\{\{1,2\}\}} XX_{\{\{1\},\{2,3\}\}} \\
& - 2 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2,3\}\}} - 2 XX_{\{\{1\},\{2,3\}\}} X_{\{\{1\},\{2\}\}} + X_{\{\{4,5\},\{1,3\},\{2\}\}} \\
& - 2 X_{\{\{1\}\}} XX_{\{\{1,3\},\{4\},\{2\}\}} - 2 XX_{\{\{1\}\}} X_{\{\{1,3\},\{4\},\{2\}\}} - 2 X_{\{\{1\}\}} XX_{\{\{1\},\{3,4\},\{2\}\}} \\
& + XX_{\{\{4,5\},\{1,3\},\{2\}\}} \\
\Delta(X_{\{\{4,5\},\{1\},\{2,3\}\}}) & = -2 XX_{\{\{1\}\}} X_{\{\{1\},\{3,4\},\{2\}\}} + 2 XX_{\{\{1,2\}\}} X_{\{\{1\},\{2,3\}\}} \\
& + 4 XX_{\{\{1\},\{2\},\{3\}\}} X_{\{\{1\},\{2\}\}} - 2 XX_{\{\{1\},\{2\}\}} X_{\{\{1,2\},\{3\}\}} + 4 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2\},\{3\}\}} \\
& - 2 XX_{\{\{1,2\},\{3\}\}} X_{\{\{1\},\{2\}\}} + XX_{\{\{1\}\}} X_{\{\{3,4\},\{1,2\}\}} + X_{\{\{1\}\}} XX_{\{\{3,4\},\{1,2\}\}} \\
& + 2 X_{\{\{1,2\}\}} XX_{\{\{1\},\{2,3\}\}} - 2 X_{\{\{1\}\}} XX_{\{\{1\},\{4\},\{2,3\}\}} - 2 XX_{\{\{1\}\}} X_{\{\{1\},\{4\},\{2,3\}\}} \\
& - 2 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2,3\}\}} - 2 XX_{\{\{1\},\{2,3\}\}} X_{\{\{1\},\{2\}\}} + XX_{\{\{4,5\},\{1\},\{2,3\}\}} \\
& - 2 X_{\{\{1\}\}} XX_{\{\{1\},\{3,4\},\{2\}\}} + X_{\{\{4,5\},\{1\},\{2,3\}\}} \\
\Delta(X_{\{\{4,5\},\{1,2,3\}\}}) & = 3 XX_{\{\{1\}\}} X_{\{\{1\},\{3,4\},\{2\}\}} - 3 XX_{\{\{1,2\}\}} X_{\{\{1\},\{2,3\}\}} \\
& + XX_{\{\{1,2\}\}} X_{\{\{1,2,3\}\}} + X_{\{\{4,5\},\{1,2,3\}\}} - 6 XX_{\{\{1\},\{2\},\{3\}\}} X_{\{\{1\},\{2\}\}} \\
& + 6 XX_{\{\{1\},\{2\}\}} X_{\{\{1,2\},\{3\}\}} - 2 XX_{\{\{1\}\}} X_{\{\{4\},\{1,2,3\}\}} - 6 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2\},\{3\}\}} \\
& + 6 XX_{\{\{1,2\},\{3\}\}} X_{\{\{1\},\{2\}\}} - 3 XX_{\{\{1\}\}} X_{\{\{3,4\},\{1,2\}\}} - 3 X_{\{\{1\}\}} XX_{\{\{3,4\},\{1,2\}\}} \\
& + XX_{\{\{1,2,3\}\}} X_{\{\{1,2\}\}} - 3 X_{\{\{1,2\}\}} XX_{\{\{1\},\{2,3\}\}} + 3 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2,3\}\}} \\
& + 3 XX_{\{\{1\},\{2,3\}\}} X_{\{\{1\},\{2\}\}} - 2 X_{\{\{1\}\}} XX_{\{\{4\},\{1,2,3\}\}} + 3 X_{\{\{1\}\}} XX_{\{\{1\},\{3,4\},\{2\}\}} \\
& + XX_{\{\{4,5\},\{1,2,3\}\}} \\
\Delta(X_{\{\{2,3,4,5\},\{1\}\}}) & = 4 XX_{\{\{1\}\}} X_{\{\{1\},\{3,4\},\{2\}\}} - 4 X_{\{\{1\}\}} XX_{\{\{1\},\{4\},\{2\},\{3\}\}} \\
& - 6 XX_{\{\{1,2\}\}} X_{\{\{1\},\{2,3\}\}} + X_{\{\{1\}\}} XX_{\{\{1,2,3,4\}\}} + 4 X_{\{\{1\}\}} XX_{\{\{1\},\{2,4\},\{3\}\}} \\
& - 4 XX_{\{\{1,2,3\}\}} X_{\{\{1\},\{2\}\}} + 6 XX_{\{\{1,2\}\}} X_{\{\{1\},\{2\},\{3\}\}} - 22 XX_{\{\{1\},\{2\},\{3\}\}} X_{\{\{1\},\{2\}\}} \\
& + 6 X_{\{\{1,2\}\}} XX_{\{\{1\},\{2\},\{3\}\}} + 4 XX_{\{\{1\},\{2\}\}} X_{\{\{1,2\},\{3\}\}} - 4 XX_{\{\{1\},\{2\}\}} X_{\{\{1,2,3\}\}} \\
& - 4 XX_{\{\{1\}\}} X_{\{\{1\},\{4\},\{2\},\{3\}\}} - 22 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2\},\{3\}\}} - 4 XX_{\{\{1\}\}} X_{\{\{1\},\{2,3,4\}\}}
\end{aligned}$$

$$\begin{aligned}
& + 4 XX_{\{\{1,2\},\{3\}\}} X_{\{\{1\},\{2\}\}} - 6 X_{\{\{1,2\}\}} XX_{\{\{1\},\{2,3\}\}} + XX_{\{\{2,3,4,5\},\{1\}\}} \\
& + 4 X_{\{\{1\}\}} XX_{\{\{1\},\{4\},\{2,3\}\}} + 4 XX_{\{\{1\}\}} X_{\{\{1\},\{4\},\{2,3\}\}} + 10 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2,3\}\}} \\
& + 10 XX_{\{\{1\},\{2,3\}\}} X_{\{\{1\},\{2\}\}} + 4 XX_{\{\{1,3\},\{2\}\}} X_{\{\{1\},\{2\}\}} + 4 XX_{\{\{1\},\{2\}\}} X_{\{\{1,3\},\{2\}\}} \\
& - 4 X_{\{\{1\}\}} XX_{\{\{1\},\{2,3,4\}\}} + 4 X_{\{\{1\}\}} XX_{\{\{1\},\{3,4\},\{2\}\}} + X_{\{\{2,3,4,5\},\{1\}\}} \\
& + 4 XX_{\{\{1\}\}} X_{\{\{1\},\{2,4\},\{3\}\}} + XX_{\{\{1\}\}} X_{\{\{1,2,3,4\}\}} \\
\Delta(X_{\{\{1,2,4,5\},\{3\}\}}) = & 2 XX_{\{\{1\}\}} X_{\{\{1\},\{3,4\},\{2\}\}} - 4 X_{\{\{1\}\}} XX_{\{\{1\},\{4\},\{2\},\{3\}\}} \\
& - XX_{\{\{1,2\}\}} X_{\{\{1\},\{2,3\}\}} + X_{\{\{1\}\}} XX_{\{\{1,2,3,4\}\}} - 4 XX_{\{\{1,2\}\}} X_{\{\{1,3\},\{2\}\}} \\
& + 2 X_{\{\{1\}\}} XX_{\{\{1\},\{2,4\},\{3\}\}} - 4 XX_{\{\{1,2,3\}\}} X_{\{\{1\},\{2\}\}} + 6 XX_{\{\{1,2\}\}} X_{\{\{1\},\{2\},\{3\}\}} \\
& - 22 XX_{\{\{1\},\{2\},\{3\}\}} X_{\{\{1\},\{2\}\}} + 6 X_{\{\{1,2\}\}} XX_{\{\{1\},\{2\},\{3\}\}} + 5 XX_{\{\{1\},\{2\}\}} X_{\{\{1,2\},\{3\}\}} \\
& - 2 XX_{\{\{1\}\}} X_{\{\{1,3,4\},\{2\}\}} - 4 XX_{\{\{1\},\{2\}\}} X_{\{\{1,2,3\}\}} + 4 XX_{\{\{1\}\}} X_{\{\{2\},\{3\},\{1,4\}\}} \\
& - 2 XX_{\{\{1\}\}} X_{\{\{1,2,4\},\{3\}\}} - 4 XX_{\{\{1\}\}} X_{\{\{1\},\{4\},\{2\},\{3\}\}} + XX_{\{\{1,2,4,5\},\{3\}\}} \\
& - 22 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2\},\{3\}\}} + 4 X_{\{\{1\}\}} XX_{\{\{2\},\{3\},\{1,4\}\}} + X_{\{\{1,2,4,5\},\{3\}\}} \\
& + 5 XX_{\{\{1,2\},\{3\}\}} X_{\{\{1\},\{2\}\}} - 4 XX_{\{\{1,3\},\{2\}\}} X_{\{\{1,2\}\}} - X_{\{\{1,2\}\}} XX_{\{\{1\},\{2,3\}\}} \\
& - 2 X_{\{\{1\}\}} XX_{\{\{1,2,4\},\{3\}\}} - 2 X_{\{\{1\}\}} XX_{\{\{1,3,4\},\{2\}\}} + 2 X_{\{\{1\}\}} XX_{\{\{4\},\{1,2\},\{3\}\}} \\
& + 2 XX_{\{\{1\}\}} X_{\{\{4\},\{1,2\},\{3\}\}} + 5 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2,3\}\}} - XX_{\{\{1,2\}\}} X_{\{\{1,2\},\{3\}\}} \\
& + 5 XX_{\{\{1\},\{2,3\}\}} X_{\{\{1\},\{2\}\}} - X_{\{\{1,2\}\}} XX_{\{\{1,2\},\{3\}\}} + 8 XX_{\{\{1,3\},\{2\}\}} X_{\{\{1\},\{2\}\}} \\
& + 2 X_{\{\{1\}\}} XX_{\{\{1,3\},\{4\},\{2\}\}} + 2 XX_{\{\{1\}\}} X_{\{\{1,3\},\{4\},\{2\}\}} + 8 XX_{\{\{1\},\{2\}\}} X_{\{\{1,3\},\{2\}\}} \\
& + 2 X_{\{\{1\}\}} XX_{\{\{1\},\{3,4\},\{2\}\}} + 2 XX_{\{\{1\}\}} X_{\{\{1\},\{2,4\},\{3\}\}} + XX_{\{\{1\}\}} X_{\{\{1,2,3,4\}\}} \\
\Delta(X_{\{\{2,4,5\},\{1\},\{3\}\}}) = & -XX_{\{\{1\}\}} X_{\{\{1\},\{3,4\},\{2\}\}} + 3 X_{\{\{1\}\}} XX_{\{\{1\},\{4\},\{2\},\{3\}\}} \\
& - 2 X_{\{\{1\}\}} XX_{\{\{1\},\{2,4\},\{3\}\}} + XX_{\{\{1,2,3\}\}} X_{\{\{1\},\{2\}\}} - 3 XX_{\{\{1,2\}\}} X_{\{\{1\},\{2\},\{3\}\}} \\
& + 9 XX_{\{\{1\},\{2\},\{3\}\}} X_{\{\{1\},\{2\}\}} - 3 X_{\{\{1,2\}\}} XX_{\{\{1\},\{2\},\{3\}\}} + XX_{\{\{1\}\}} X_{\{\{1,3,4\},\{2\}\}} \\
& + XX_{\{\{1\},\{2\}\}} X_{\{\{1,2,3\}\}} + 3 XX_{\{\{1\}\}} X_{\{\{1\},\{4\},\{2\},\{3\}\}} + 9 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2\},\{3\}\}} \\
& + XX_{\{\{1\}\}} X_{\{\{1\},\{2,3,4\}\}} + X_{\{\{2,4,5\},\{1\},\{3\}\}} + X_{\{\{1\}\}} XX_{\{\{1,3,4\},\{2\}\}} \\
& - 4 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2,3\}\}} - 4 XX_{\{\{1\},\{2,3\}\}} X_{\{\{1\},\{2\}\}} - 2 XX_{\{\{1,3\},\{2\}\}} X_{\{\{1\},\{2\}\}} \\
& - 2 XX_{\{\{1\},\{2\}\}} X_{\{\{1,3\},\{2\}\}} + X_{\{\{1\}\}} XX_{\{\{1\},\{2,3,4\}\}} - X_{\{\{1\}\}} XX_{\{\{1\},\{3,4\},\{2\}\}} \\
& + XX_{\{\{2,4,5\},\{1\},\{3\}\}} - 2 XX_{\{\{1\}\}} X_{\{\{1\},\{2,4\},\{3\}\}} \\
\Delta(X_{\{\{2,4,5\},\{1,3\}\}}) = & XX_{\{\{1,2\}\}} X_{\{\{1,2,3\}\}} - XX_{\{\{1,2\}\}} X_{\{\{1,3\},\{2\}\}} - X_{\{\{1\}\}} XX_{\{\{3,4\},\{1,2\}\}} \\
& - 2 X_{\{\{1\}\}} XX_{\{\{1,3\},\{2,4\}\}} - 6 XX_{\{\{1\},\{2\},\{3\}\}} X_{\{\{1\},\{2\}\}} + 2 XX_{\{\{1\},\{2\}\}} X_{\{\{1,2\},\{3\}\}} \\
& - XX_{\{\{1\}\}} X_{\{\{1,3,4\},\{2\}\}} - 6 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2\},\{3\}\}} - XX_{\{\{1\}\}} X_{\{\{1\},\{2,3,4\}\}} \\
& - XX_{\{\{1\}\}} X_{\{\{3,4\},\{1,2\}\}} - 2 XX_{\{\{1\}\}} X_{\{\{1,3\},\{2,4\}\}} + 2 XX_{\{\{1,2\},\{3\}\}} X_{\{\{1\},\{2\}\}} \\
& + XX_{\{\{1,2,3\}\}} X_{\{\{1,2\}\}} - XX_{\{\{1,3\},\{2\}\}} X_{\{\{1,2\}\}} - X_{\{\{1\}\}} XX_{\{\{1,3,4\},\{2\}\}} \\
& + X_{\{\{1\}\}} XX_{\{\{4\},\{1,2\},\{3\}\}} + XX_{\{\{1\}\}} X_{\{\{4\},\{1,2\},\{3\}\}} + 4 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2,3\}\}} \\
& - 2 XX_{\{\{1,2\}\}} X_{\{\{1,2\},\{3\}\}} + 4 XX_{\{\{1\},\{2,3\}\}} X_{\{\{1\},\{2\}\}} - 2 X_{\{\{1,2\}\}} XX_{\{\{1,2\},\{3\}\}} \\
& + X_{\{\{2,4,5\},\{1,3\}\}} + 3 XX_{\{\{1,3\},\{2\}\}} X_{\{\{1\},\{2\}\}} + 2 X_{\{\{1\}\}} XX_{\{\{1,3\},\{4\},\{2\}\}} \\
& + 2 XX_{\{\{1\}\}} X_{\{\{1,3\},\{4\},\{2\}\}} + 3 XX_{\{\{1\},\{2\}\}} X_{\{\{1,3\},\{2\}\}} - X_{\{\{1\}\}} XX_{\{\{1\},\{2,3,4\}\}} \\
& + XX_{\{\{2,4,5\},\{1,3\}\}} \\
\Delta(X_{\{\{1,2,3,4,5\}\}}) = & -5 XX_{\{\{1\}\}} X_{\{\{1\},\{3,4\},\{2\}\}} + 5 X_{\{\{1\}\}} XX_{\{\{1\},\{4\},\{2\},\{3\}\}} \\
& + 10 XX_{\{\{1,2\}\}} X_{\{\{1\},\{2,3\}\}} - 10 XX_{\{\{1,2\}\}} X_{\{\{1,2,3\}\}} + 10 XX_{\{\{1,2\}\}} X_{\{\{1,3\},\{2\}\}} \\
& + 5 X_{\{\{1\}\}} XX_{\{\{3,4\},\{1,2\}\}} + 5 X_{\{\{1\}\}} XX_{\{\{1,3\},\{2,4\}\}} + 70 XX_{\{\{1\},\{2\},\{3\}\}} X_{\{\{1\},\{2\}\}} \\
& - 30 XX_{\{\{1\},\{2\}\}} X_{\{\{1,2\},\{3\}\}} + 5 XX_{\{\{1\}\}} X_{\{\{1,3,4\},\{2\}\}} + 5 X_{\{\{1\}\}} XX_{\{\{1,2,4\},\{3\}\}}
\end{aligned}$$

$$\begin{aligned}
& -5 X_{\{\{1\}\}} XX_{\{\{1\}, \{2,4\}, \{3\}\}} - 5 X_{\{\{1\}\}} XX_{\{\{1,2,3,4\}\}} + 10 XX_{\{\{1,2,3\}\}} X_{\{\{1\}, \{2\}\}} \\
& - 5 XX_{\{\{1\}\}} X_{\{\{2\}, \{3\}, \{1,4\}\}} - 10 X_{\{\{1,2\}\}} XX_{\{\{1\}, \{2\}, \{3\}\}} + 5 XX_{\{\{1\}\}} X_{\{\{1\}, \{4\}, \{2\}, \{3\}\}} \\
& + 5 XX_{\{\{1\}\}} X_{\{\{4\}, \{1,2,3\}\}} + 70 XX_{\{\{1\}, \{2\}\}} X_{\{\{1\}, \{2\}, \{3\}\}} + 5 XX_{\{\{1\}\}} X_{\{\{1\}, \{2,3,4\}\}} \\
& - 5 X_{\{\{1\}\}} XX_{\{\{2\}, \{3\}, \{1,4\}\}} - 10 XX_{\{\{1,2\}\}} X_{\{\{1\}, \{2\}, \{3\}\}} + 5 XX_{\{\{1\}\}} X_{\{\{3,4\}, \{1,2\}\}} \\
& + 5 XX_{\{\{1\}\}} X_{\{\{1,3\}, \{2,4\}\}} + 10 XX_{\{\{1\}, \{2\}\}} X_{\{\{1,2,3\}\}} + XX_{\{\{1,2,3,4,5\}\}} \\
& - 30 XX_{\{\{1,2\}, \{3\}\}} X_{\{\{1\}, \{2\}\}} + 5 XX_{\{\{1\}\}} X_{\{\{1,2,4\}, \{3\}\}} - 5 XX_{\{\{1\}\}} X_{\{\{1,2,3,4\}\}} \\
& - 10 XX_{\{\{1,2,3\}\}} X_{\{\{1,2\}\}} + 10 XX_{\{\{1,3\}, \{2\}\}} X_{\{\{1,2\}\}} - 5 XX_{\{\{1\}\}} X_{\{\{1\}, \{2,4\}, \{3\}\}} \\
& + 10 X_{\{\{1,2\}\}} XX_{\{\{1\}, \{2,3\}\}} + 5 X_{\{\{1\}\}} XX_{\{\{1,3,4\}, \{2\}\}} + 5 X_{\{\{1\}\}} XX_{\{\{2,3\}, \{1,4\}\}} \\
& - 5 X_{\{\{1\}\}} XX_{\{\{1\}, \{4\}, \{2,3\}\}} - 5 X_{\{\{1\}\}} XX_{\{\{4\}, \{1,2\}, \{3\}\}} - 5 XX_{\{\{1\}\}} X_{\{\{1\}, \{4\}, \{2,3\}\}} \\
& - 5 XX_{\{\{1\}\}} X_{\{\{4\}, \{1,2\}, \{3\}\}} + 5 XX_{\{\{1\}\}} X_{\{\{2,3\}, \{1,4\}\}} - 30 XX_{\{\{1\}, \{2\}\}} X_{\{\{1\}, \{2,3\}\}} \\
& + 10 XX_{\{\{1,2\}\}} X_{\{\{1,2\}, \{3\}\}} - 30 XX_{\{\{1\}, \{2,3\}\}} X_{\{\{1\}, \{2\}\}} + 10 X_{\{\{1,2\}\}} XX_{\{\{1,2\}, \{3\}\}} \\
& - 30 XX_{\{\{1,3\}, \{2\}\}} X_{\{\{1\}, \{2\}\}} - 5 X_{\{\{1\}\}} XX_{\{\{1,3\}, \{4\}, \{2\}\}} - 5 XX_{\{\{1\}\}} X_{\{\{1,3\}, \{4\}, \{2\}\}} \\
& - 30 XX_{\{\{1\}, \{2\}\}} X_{\{\{1,3\}, \{2\}\}} + 5 X_{\{\{1\}\}} XX_{\{\{4\}, \{1,2,3\}\}} + 5 X_{\{\{1\}\}} XX_{\{\{1\}, \{2,3,4\}\}} \\
& - 5 X_{\{\{1\}\}} XX_{\{\{1\}, \{3,4\}, \{2\}\}} + X_{\{\{1,2,3,4,5\}\}} \\
\Delta(X_{\{\{1,3,4,5\}, \{2\}\}}) &= 4 XX_{\{\{1\}\}} X_{\{\{1\}, \{3,4\}, \{2\}\}} - 4 X_{\{\{1\}\}} XX_{\{\{1\}, \{4\}, \{2\}, \{3\}\}} \\
& - 3 XX_{\{\{1,2\}\}} X_{\{\{1\}, \{2,3\}\}} - 3 XX_{\{\{1,2\}\}} X_{\{\{1,3\}, \{2\}\}} - 22 XX_{\{\{1\}, \{2\}, \{3\}\}} X_{\{\{1\}, \{2\}\}} \\
& + 4 XX_{\{\{1\}, \{2\}\}} X_{\{\{1,2\}, \{3\}\}} - 3 XX_{\{\{1\}\}} X_{\{\{1,3,4\}, \{2\}\}} + X_{\{\{1\}\}} XX_{\{\{1\}, \{2,4\}, \{3\}\}} \\
& + X_{\{\{1\}\}} XX_{\{\{1,2,3,4\}\}} - 4 XX_{\{\{1,2,3\}\}} X_{\{\{1\}, \{2\}\}} + 3 XX_{\{\{1\}\}} X_{\{\{2\}, \{3\}, \{1,4\}\}} \\
& + 6 X_{\{\{1,2\}\}} XX_{\{\{1\}, \{2\}, \{3\}\}} - 4 XX_{\{\{1\}\}} X_{\{\{1\}, \{4\}, \{2\}, \{3\}\}} - 22 XX_{\{\{1\}, \{2\}\}} X_{\{\{1\}, \{2\}, \{3\}\}} \\
& - XX_{\{\{1\}\}} X_{\{\{1\}, \{2,3,4\}\}} + 3 X_{\{\{1\}\}} XX_{\{\{2\}, \{3\}, \{1,4\}\}} + 6 XX_{\{\{1,2\}\}} X_{\{\{1\}, \{2\}, \{3\}\}} \\
& - 4 XX_{\{\{1\}, \{2\}\}} X_{\{\{1,2,3\}\}} + 4 XX_{\{\{1,2\}, \{3\}\}} X_{\{\{1\}, \{2\}\}} + XX_{\{\{1\}\}} X_{\{\{1,2,3,4\}\}} \\
& - 3 XX_{\{\{1,3\}, \{2\}\}} X_{\{\{1,2\}\}} + XX_{\{\{1\}\}} X_{\{\{1\}, \{2,4\}, \{3\}\}} - 3 X_{\{\{1,2\}\}} XX_{\{\{1\}, \{2,3\}\}} \\
& - 3 X_{\{\{1\}\}} XX_{\{\{1,3,4\}, \{2\}\}} + X_{\{\{1\}\}} XX_{\{\{1\}, \{4\}, \{2,3\}\}} + XX_{\{\{1\}\}} X_{\{\{1\}, \{4\}, \{2,3\}\}} \\
& + 7 XX_{\{\{1\}, \{2\}\}} X_{\{\{1\}, \{2,3\}\}} + 7 XX_{\{\{1\}, \{2,3\}\}} X_{\{\{1\}, \{2\}\}} + XX_{\{\{1,3,4,5\}, \{2\}\}} \\
& + 7 XX_{\{\{1,3\}, \{2\}\}} X_{\{\{1\}, \{2\}\}} + 3 X_{\{\{1\}\}} XX_{\{\{1,3\}, \{4\}, \{2\}\}} + 3 XX_{\{\{1\}\}} X_{\{\{1,3\}, \{4\}, \{2\}\}} \\
& + 7 XX_{\{\{1\}, \{2\}\}} X_{\{\{1,3\}, \{2\}\}} - X_{\{\{1\}\}} XX_{\{\{1\}, \{2,3,4\}\}} + 4 X_{\{\{1\}\}} XX_{\{\{1\}, \{3,4\}, \{2\}\}} \\
& + X_{\{\{1,3,4,5\}, \{2\}\}} \\
\Delta(X_{\{\{3,4,5\}, \{1\}, \{2\}\}}) &= -3 XX_{\{\{1\}\}} X_{\{\{1\}, \{3,4\}, \{2\}\}} + 3 X_{\{\{1\}\}} XX_{\{\{1\}, \{4\}, \{2\}, \{3\}\}} \\
& + 9 XX_{\{\{1\}, \{2\}, \{3\}\}} X_{\{\{1\}, \{2\}\}} + XX_{\{\{1,2,3\}\}} X_{\{\{1\}, \{2\}\}} - 3 X_{\{\{1,2\}\}} XX_{\{\{1\}, \{2\}, \{3\}\}} \\
& + 3 XX_{\{\{1\}\}} X_{\{\{1\}, \{4\}, \{2\}, \{3\}\}} + 9 XX_{\{\{1\}, \{2\}\}} X_{\{\{1\}, \{2\}, \{3\}\}} + 2 XX_{\{\{1\}\}} X_{\{\{1\}, \{2,3,4\}\}} \\
& - 3 XX_{\{\{1,2\}\}} X_{\{\{1\}, \{2\}, \{3\}\}} + XX_{\{\{1\}, \{2\}\}} X_{\{\{1,2,3\}\}} - 6 XX_{\{\{1\}, \{2\}\}} X_{\{\{1\}, \{2,3\}\}} \\
& - 6 XX_{\{\{1\}, \{2,3\}\}} X_{\{\{1\}, \{2\}\}} + 2 X_{\{\{1\}\}} XX_{\{\{1\}, \{2,3,4\}\}} - 3 X_{\{\{1\}\}} XX_{\{\{1\}, \{3,4\}, \{2\}\}} \\
& + X_{\{\{3,4,5\}, \{1\}, \{2\}\}} + XX_{\{\{3,4,5\}, \{1\}, \{2\}\}} \\
\Delta(X_{\{\{3,4,5\}, \{1,2\}\}}) &= XX_{\{\{3,4,5\}, \{1,2\}\}} + XX_{\{\{1,2\}\}} X_{\{\{1,2,3\}\}} - 6 XX_{\{\{1\}, \{2\}, \{3\}\}} X_{\{\{1\}, \{2\}\}} \\
& - 3 X_{\{\{1\}\}} XX_{\{\{3,4\}, \{1,2\}\}} + 3 XX_{\{\{1\}, \{2\}\}} X_{\{\{1,2\}, \{3\}\}} - 6 XX_{\{\{1\}, \{2\}\}} X_{\{\{1\}, \{2\}, \{3\}\}} \\
& - 2 XX_{\{\{1\}\}} X_{\{\{1\}, \{2,3,4\}\}} + 3 XX_{\{\{1,2\}, \{3\}\}} X_{\{\{1\}, \{2\}\}} - 3 XX_{\{\{1\}\}} X_{\{\{3,4\}, \{1,2\}\}} \\
& + XX_{\{\{1,2,3\}\}} X_{\{\{1,2\}\}} + 3 X_{\{\{1\}\}} XX_{\{\{4\}, \{1,2\}, \{3\}\}} + 3 XX_{\{\{1\}\}} X_{\{\{4\}, \{1,2\}, \{3\}\}} \\
& + 6 XX_{\{\{1\}, \{2\}\}} X_{\{\{1\}, \{2,3\}\}} - 3 XX_{\{\{1,2\}\}} X_{\{\{1,2\}, \{3\}\}} + 6 XX_{\{\{1\}, \{2,3\}\}} X_{\{\{1\}, \{2\}\}} \\
& - 3 X_{\{\{1,2\}\}} XX_{\{\{1,2\}, \{3\}\}} - 2 X_{\{\{1\}\}} XX_{\{\{1\}, \{2,3,4\}\}} + X_{\{\{3,4,5\}, \{1,2\}\}} \\
\Delta(X_{\{\{1\}, \{2,5\}, \{4\}, \{3\}\}}) &= -2 X_{\{\{1\}\}} XX_{\{\{1\}, \{4\}, \{2\}, \{3\}\}} - 6 XX_{\{\{1\}, \{2\}, \{3\}\}} X_{\{\{1\}, \{2\}\}}
\end{aligned}$$

$$\begin{aligned}
& + XX_{\{\{1\}\}} X_{\{\{2\},\{3\},\{1,4\}\}} + X_{\{\{1,2\}\}} XX_{\{\{1\},\{2\},\{3\}\}} - 2 XX_{\{\{1\}\}} X_{\{\{1\},\{4\},\{2\},\{3\}\}} \\
& - 6 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2\},\{3\}\}} + X_{\{\{1\}\}} XX_{\{\{2\},\{3\},\{1,4\}\}} + XX_{\{\{1,2\}\}} X_{\{\{1\},\{2\},\{3\}\}} \\
& + XX_{\{\{1\},\{2,5\},\{4\},\{3\}\}} + XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2,3\}\}} + XX_{\{\{1\},\{2,3\}\}} X_{\{\{1\},\{2\}\}} \\
& + 2 XX_{\{\{1,3\},\{2\}\}} X_{\{\{1\},\{2\}\}} + 2 XX_{\{\{1\},\{2\}\}} X_{\{\{1,3\},\{2\}\}} + 2 X_{\{\{1\}\}} XX_{\{\{1\},\{2,4\},\{3\}\}} \\
& + 2 XX_{\{\{1\}\}} X_{\{\{1\},\{2,4\},\{3\}\}} + X_{\{\{1\},\{2,5\},\{4\},\{3\}\}} \\
\Delta(X_{\{\{2,5\},\{1,3\},\{4\}\}}) & = XX_{\{\{1,2\}\}} X_{\{\{1,3\},\{2\}\}} + 4 XX_{\{\{1\},\{2\},\{3\}\}} X_{\{\{1\},\{2\}\}} \\
& - XX_{\{\{1\},\{2\}\}} X_{\{\{1,2\},\{3\}\}} - XX_{\{\{1\}\}} X_{\{\{2\},\{3\},\{1,4\}\}} + XX_{\{\{2,5\},\{1,3\},\{4\}\}} \\
& + 4 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2\},\{3\}\}} - X_{\{\{1\}\}} XX_{\{\{2\},\{3\},\{1,4\}\}} + XX_{\{\{1\}\}} X_{\{\{1,3\},\{2,4\}\}} \\
& + X_{\{\{2,5\},\{1,3\},\{4\}\}} - XX_{\{\{1,2\},\{3\}\}} X_{\{\{1\},\{2\}\}} + XX_{\{\{1,3\},\{2\}\}} X_{\{\{1,2\}\}} \\
& + X_{\{\{1\}\}} XX_{\{\{1,3\},\{2,4\}\}} - X_{\{\{1\}\}} XX_{\{\{4\},\{1,2\},\{3\}\}} - XX_{\{\{1\}\}} X_{\{\{4\},\{1,2\},\{3\}\}} \\
& - XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2,3\}\}} + XX_{\{\{1,2\}\}} X_{\{\{1,2\},\{3\}\}} - XX_{\{\{1\},\{2,3\}\}} X_{\{\{1\},\{2\}\}} \\
& + X_{\{\{1,2\}\}} XX_{\{\{1,2\},\{3\}\}} - 2 XX_{\{\{1,3\},\{2\}\}} X_{\{\{1\},\{2\}\}} - X_{\{\{1\}\}} XX_{\{\{1,3\},\{4\},\{2\}\}} \\
& - XX_{\{\{1\}\}} X_{\{\{1,3\},\{4\},\{2\}\}} - 2 XX_{\{\{1\},\{2\}\}} X_{\{\{1,3\},\{2\}\}} - X_{\{\{1\}\}} XX_{\{\{1\},\{2,4\},\{3\}\}} \\
& - XX_{\{\{1\}\}} X_{\{\{1\},\{2,4\},\{3\}\}} \\
\Delta(X_{\{\{2,5\},\{3\},\{1,4\}\}}) & = 2 XX_{\{\{1,2\}\}} X_{\{\{1,3\},\{2\}\}} + 4 XX_{\{\{1\},\{2\},\{3\}\}} X_{\{\{1\},\{2\}\}} \\
& - XX_{\{\{1\},\{2\}\}} X_{\{\{1,2\},\{3\}\}} - 2 XX_{\{\{1\}\}} X_{\{\{2\},\{3\},\{1,4\}\}} + 4 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2\},\{3\}\}} \\
& - 2 X_{\{\{1\}\}} XX_{\{\{2\},\{3\},\{1,4\}\}} + XX_{\{\{1\}\}} X_{\{\{1,3\},\{2,4\}\}} + X_{\{\{2,5\},\{3\},\{1,4\}\}} \\
& - XX_{\{\{1,2\},\{3\}\}} X_{\{\{1\},\{2\}\}} + 2 XX_{\{\{1,3\},\{2\}\}} X_{\{\{1,2\}\}} + X_{\{\{1\}\}} XX_{\{\{1,3\},\{2,4\}\}} \\
& - XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2,3\}\}} - XX_{\{\{1\},\{2,3\}\}} X_{\{\{1\},\{2\}\}} - 2 XX_{\{\{1,3\},\{2\}\}} X_{\{\{1\},\{2\}\}} \\
& - X_{\{\{1\}\}} XX_{\{\{1,3\},\{4\},\{2\}\}} - XX_{\{\{1\}\}} X_{\{\{1,3\},\{4\},\{2\}\}} - 2 XX_{\{\{1\},\{2\}\}} X_{\{\{1,3\},\{2\}\}} \\
& - X_{\{\{1\}\}} XX_{\{\{1\},\{2,4\},\{3\}\}} - XX_{\{\{1\}\}} X_{\{\{1\},\{2,4\},\{3\}\}} + XX_{\{\{2,5\},\{3\},\{1,4\}\}} \\
\Delta(X_{\{\{2,5\},\{1,3,4\}\}}) & = -XX_{\{\{1,2\}\}} X_{\{\{1\},\{2,3\}\}} + XX_{\{\{1,2\}\}} X_{\{\{1,2,3\}\}} + XX_{\{\{2,5\},\{1,3,4\}\}} \\
& - 2 XX_{\{\{1,2\}\}} X_{\{\{1,3\},\{2\}\}} - 6 XX_{\{\{1\},\{2\},\{3\}\}} X_{\{\{1\},\{2\}\}} - X_{\{\{1\}\}} XX_{\{\{1,3,4\},\{2\}\}} \\
& + 3 XX_{\{\{1\},\{2\}\}} X_{\{\{1,2\},\{3\}\}} + XX_{\{\{1\}\}} X_{\{\{2\},\{3\},\{1,4\}\}} - XX_{\{\{1\}\}} X_{\{\{4\},\{1,2,3\}\}} \\
& - 6 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2\},\{3\}\}} + X_{\{\{1\}\}} XX_{\{\{2\},\{3\},\{1,4\}\}} - 2 XX_{\{\{1\}\}} X_{\{\{1,3\},\{2,4\}\}} \\
& + 3 XX_{\{\{1,2\},\{3\}\}} X_{\{\{1\},\{2\}\}} + XX_{\{\{1,2,3\}\}} X_{\{\{1,2\}\}} - XX_{\{\{1\}\}} X_{\{\{1,3,4\},\{2\}\}} \\
& - 2 XX_{\{\{1,3\},\{2\}\}} X_{\{\{1,2\}\}} + X_{\{\{2,5\},\{1,3,4\}\}} - X_{\{\{1,2\}\}} XX_{\{\{1\},\{2,3\}\}} \\
& - 2 X_{\{\{1\}\}} XX_{\{\{1,3\},\{2,4\}\}} - X_{\{\{1\}\}} XX_{\{\{2,3\},\{1,4\}\}} - XX_{\{\{1\}\}} X_{\{\{2,3\},\{1,4\}\}} \\
& + 2 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2,3\}\}} + 2 XX_{\{\{1\},\{2,3\}\}} X_{\{\{1\},\{2\}\}} + 4 XX_{\{\{1,3\},\{2\}\}} X_{\{\{1\},\{2\}\}} \\
& + 4 XX_{\{\{1\},\{2\}\}} X_{\{\{1,3\},\{2\}\}} + 2 X_{\{\{1\}\}} XX_{\{\{1\},\{2,4\},\{3\}\}} - X_{\{\{1\}\}} XX_{\{\{4\},\{1,2,3\}\}} \\
& + 2 XX_{\{\{1\}\}} X_{\{\{1\},\{2,4\},\{3\}\}} \\
\Delta(X_{\{\{1\},\{2,5\},\{3,4\}\}}) & = -XX_{\{\{1\}\}} X_{\{\{1\},\{3,4\},\{2\}\}} + 2 XX_{\{\{1,2\}\}} X_{\{\{1\},\{2,3\}\}} \\
& + 4 XX_{\{\{1\},\{2\},\{3\}\}} X_{\{\{1\},\{2\}\}} - XX_{\{\{1\},\{2\}\}} X_{\{\{1,2\},\{3\}\}} + 4 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2\},\{3\}\}} \\
& - XX_{\{\{1,2\},\{3\}\}} X_{\{\{1\},\{2\}\}} + X_{\{\{1\},\{2,5\},\{3,4\}\}} + 2 X_{\{\{1,2\}\}} XX_{\{\{1\},\{2,3\}\}} \\
& + X_{\{\{1\}\}} XX_{\{\{2,3\},\{1,4\}\}} - X_{\{\{1\}\}} XX_{\{\{1\},\{4\},\{2,3\}\}} - XX_{\{\{1\}\}} X_{\{\{1\},\{4\},\{2,3\}\}} \\
& + XX_{\{\{1\}\}} X_{\{\{2,3\},\{1,4\}\}} - XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2,3\}\}} - XX_{\{\{1\},\{2,3\}\}} X_{\{\{1\},\{2\}\}} \\
& - 2 XX_{\{\{1,3\},\{2\}\}} X_{\{\{1\},\{2\}\}} - 2 XX_{\{\{1\},\{2\}\}} X_{\{\{1,3\},\{2\}\}} - 2 X_{\{\{1\}\}} XX_{\{\{1\},\{2,4\},\{3\}\}} \\
& - X_{\{\{1\}\}} XX_{\{\{1\},\{3,4\},\{2\}\}} - 2 XX_{\{\{1\}\}} X_{\{\{1\},\{2,4\},\{3\}\}} + XX_{\{\{1\},\{2,5\},\{3,4\}\}} \\
\Delta(X_{\{\{2,3,5\},\{1\},\{4\}\}}) & = 3 X_{\{\{1\}\}} XX_{\{\{1\},\{4\},\{2\},\{3\}\}} + 9 XX_{\{\{1\},\{2\},\{3\}\}} X_{\{\{1\},\{2\}\}} \\
& - XX_{\{\{1\},\{2\}\}} X_{\{\{1,2\},\{3\}\}} + X_{\{\{1\}\}} XX_{\{\{1,2,4\},\{3\}\}} + XX_{\{\{1,2,3\}\}} X_{\{\{1\},\{2\}\}}
\end{aligned}$$

$$\begin{aligned}
& -3 X_{\{\{1,2\}\}} XX_{\{\{1\},\{2\},\{3\}\}} + 3 XX_{\{\{1\}\}} X_{\{\{1\},\{4\},\{2\},\{3\}\}} + 9 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2\},\{3\}\}} \\
& + XX_{\{\{1\}\}} X_{\{\{1\},\{2,3,4\}\}} + XX_{\{\{1\}\}} X_{\{\{1,2,4\},\{3\}\}} - 3 XX_{\{\{1,2\}\}} X_{\{\{1\},\{2\},\{3\}\}} \\
& + XX_{\{\{1\},\{2\}\}} X_{\{\{1,2,3\}\}} - XX_{\{\{1,2\},\{3\}\}} X_{\{\{1\},\{2\}\}} + X_{\{\{2,3,5\},\{1\},\{4\}\}} \\
& - X_{\{\{1\}\}} XX_{\{\{1\},\{4\},\{2,3\}\}} - XX_{\{\{1\}\}} X_{\{\{1\},\{4\},\{2,3\}\}} - 3 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2,3\}\}} \\
& - 3 XX_{\{\{1\},\{2,3\}\}} X_{\{\{1\},\{2\}\}} - 2 XX_{\{\{1,3\},\{2\}\}} X_{\{\{1\},\{2\}\}} - 2 XX_{\{\{1\},\{2\}\}} X_{\{\{1,3\},\{2\}\}} \\
& - 2 X_{\{\{1\}\}} XX_{\{\{1\},\{2,4\},\{3\}\}} + X_{\{\{1\}\}} XX_{\{\{1\},\{2,3,4\}\}} - 2 XX_{\{\{1\}\}} X_{\{\{1\},\{2,4\},\{3\}\}} \\
& + XX_{\{\{2,3,5\},\{1\},\{4\}\}} \\
\Delta(X_{\{\{2,3,5\},\{1,4\}\}}) &= XX_{\{\{1,2\}\}} X_{\{\{1,2,3\}\}} - 2 XX_{\{\{1,2\}\}} X_{\{\{1,3\},\{2\}\}} \\
& - 6 XX_{\{\{1\},\{2\},\{3\}\}} X_{\{\{1\},\{2\}\}} + 2 XX_{\{\{1\},\{2\}\}} X_{\{\{1,2\},\{3\}\}} - X_{\{\{1\}\}} XX_{\{\{1,2,4\},\{3\}\}} \\
& + XX_{\{\{1\}\}} X_{\{\{2\},\{3\},\{1,4\}\}} - 2 X_{\{\{1\}\}} XX_{\{\{1,3\},\{2,4\}\}} - 6 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2\},\{3\}\}} \\
& - XX_{\{\{1\}\}} X_{\{\{1\},\{2,3,4\}\}} - XX_{\{\{1\}\}} X_{\{\{1,2,4\},\{3\}\}} + X_{\{\{1\}\}} XX_{\{\{2\},\{3\},\{1,4\}\}} \\
& + 2 XX_{\{\{1,2\},\{3\}\}} X_{\{\{1\},\{2\}\}} + XX_{\{\{1,2,3\}\}} X_{\{\{1,2\}\}} + XX_{\{\{2,3,5\},\{1,4\}\}} \\
& - 2 XX_{\{\{1,3\},\{2\}\}} X_{\{\{1,2\}\}} - 2 XX_{\{\{1\}\}} X_{\{\{1,3\},\{2,4\}\}} + X_{\{\{2,3,5\},\{1,4\}\}} \\
& - X_{\{\{1\}\}} XX_{\{\{2,3\},\{1,4\}\}} - XX_{\{\{1\}\}} X_{\{\{2,3\},\{1,4\}\}} + 3 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2,3\}\}} \\
& - XX_{\{\{1,2\}\}} X_{\{\{1,2\},\{3\}\}} + 3 XX_{\{\{1\},\{2,3\}\}} X_{\{\{1\},\{2\}\}} - X_{\{\{1,2\}\}} XX_{\{\{1,2\},\{3\}\}} \\
& + 4 XX_{\{\{1,3\},\{2\}\}} X_{\{\{1\},\{2\}\}} + 2 X_{\{\{1\}\}} XX_{\{\{1,3\},\{4\},\{2\}\}} + 2 XX_{\{\{1\}\}} X_{\{\{1,3\},\{4\},\{2\}\}} \\
& + 4 XX_{\{\{1\},\{2\}\}} X_{\{\{1,3\},\{2\}\}} - X_{\{\{1\}\}} XX_{\{\{1\},\{2,3,4\}\}} \\
\Delta(X_{\{\{1,2,5\},\{4\},\{3\}\}}) &= 3 X_{\{\{1\}\}} XX_{\{\{1\},\{4\},\{2\},\{3\}\}} + 9 XX_{\{\{1\},\{2\},\{3\}\}} X_{\{\{1\},\{2\}\}} \\
& - 2 XX_{\{\{1\},\{2\}\}} X_{\{\{1,2\},\{3\}\}} + 2 X_{\{\{1\}\}} XX_{\{\{1,2,4\},\{3\}\}} + XX_{\{\{1,2,3\}\}} X_{\{\{1\},\{2\}\}} \\
& - 2 XX_{\{\{1\}\}} X_{\{\{2\},\{3\},\{1,4\}\}} - 3 X_{\{\{1,2\}\}} XX_{\{\{1\},\{2\},\{3\}\}} + 3 XX_{\{\{1\}\}} X_{\{\{1\},\{4\},\{2\},\{3\}\}} \\
& + 9 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2\},\{3\}\}} + 2 XX_{\{\{1\}\}} X_{\{\{1,2,4\},\{3\}\}} - 2 X_{\{\{1\}\}} XX_{\{\{2\},\{3\},\{1,4\}\}} \\
& - 3 XX_{\{\{1,2\}\}} X_{\{\{1\},\{2\},\{3\}\}} + XX_{\{\{1\},\{2\}\}} X_{\{\{1,2,3\}\}} - 2 XX_{\{\{1,2\},\{3\}\}} X_{\{\{1\},\{2\}\}} \\
& + XX_{\{\{1,2,5\},\{4\},\{3\}\}} - X_{\{\{1\}\}} XX_{\{\{4\},\{1,2\},\{3\}\}} - XX_{\{\{1\}\}} X_{\{\{4\},\{1,2\},\{3\}\}} + X_{\{\{1,2,5\},\{4\},\{3\}\}} \\
& - 4 XX_{\{\{1,3\},\{2\}\}} X_{\{\{1\},\{2\}\}} - 4 XX_{\{\{1\},\{2\}\}} X_{\{\{1,3\},\{2\}\}} \\
\Delta(X_{\{\{1,2,5\},\{3,4\}\}}) &= XX_{\{\{1\}\}} X_{\{\{1\},\{3,4\},\{2\}\}} - 2 XX_{\{\{1,2\}\}} X_{\{\{1\},\{2,3\}\}} + XX_{\{\{1,2\}\}} X_{\{\{1,2,3\}\}} \\
& - 6 XX_{\{\{1\},\{2\},\{3\}\}} X_{\{\{1\},\{2\}\}} + 3 XX_{\{\{1\},\{2\}\}} X_{\{\{1,2\},\{3\}\}} - 2 X_{\{\{1\}\}} XX_{\{\{1,2,4\},\{3\}\}} \\
& - 6 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2\},\{3\}\}} - 2 XX_{\{\{1\}\}} X_{\{\{1,2,4\},\{3\}\}} + 3 XX_{\{\{1,2\},\{3\}\}} X_{\{\{1\},\{2\}\}} \\
& + XX_{\{\{1,2,3\}\}} X_{\{\{1,2\}\}} - 2 X_{\{\{1,2\}\}} XX_{\{\{1\},\{2,3\}\}} - 2 X_{\{\{1\}\}} XX_{\{\{2,3\},\{1,4\}\}} \\
& + 2 X_{\{\{1\}\}} XX_{\{\{1\},\{4\},\{2,3\}\}} + 2 XX_{\{\{1\}\}} X_{\{\{1\},\{4\},\{2,3\}\}} - 2 XX_{\{\{1\}\}} X_{\{\{2,3\},\{1,4\}\}} \\
& + 2 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2,3\}\}} - XX_{\{\{1,2\}\}} X_{\{\{1,2\},\{3\}\}} + 2 XX_{\{\{1\},\{2,3\}\}} X_{\{\{1\},\{2\}\}} \\
& - X_{\{\{1,2\}\}} XX_{\{\{1,2\},\{3\}\}} + X_{\{\{1,2,5\},\{3,4\}\}} + 4 XX_{\{\{1,3\},\{2\}\}} X_{\{\{1\},\{2\}\}} \\
& + 4 XX_{\{\{1\},\{2\}\}} X_{\{\{1,3\},\{2\}\}} + X_{\{\{1\}\}} XX_{\{\{1\},\{3,4\},\{2\}\}} + XX_{\{\{1,2,5\},\{3,4\}\}} \\
& - XX_{\{\{1\}\}} X_{\{\{3,4\},\{1,2\}\}} - X_{\{\{1\}\}} XX_{\{\{3,4\},\{1,2\}\}} \\
\Delta(X_{\{\{1\},\{3,5\},\{4\},\{2\}\}}) &= XX_{\{\{1\}\}} X_{\{\{1\},\{3,4\},\{2\}\}} - 2 X_{\{\{1\}\}} XX_{\{\{1\},\{4\},\{2\},\{3\}\}} \\
& + 2 X_{\{\{1\}\}} XX_{\{\{1\},\{2,4\},\{3\}\}} - 6 XX_{\{\{1\},\{2\},\{3\}\}} X_{\{\{1\},\{2\}\}} + X_{\{\{1,2\}\}} XX_{\{\{1\},\{2\},\{3\}\}} \\
& - 2 XX_{\{\{1\}\}} X_{\{\{1\},\{4\},\{2\},\{3\}\}} - 6 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2\},\{3\}\}} + XX_{\{\{1,2\}\}} X_{\{\{1\},\{2\},\{3\}\}} \\
& + 2 XX_{\{\{1\}\}} X_{\{\{1\},\{2,4\},\{3\}\}} + 2 XX_{\{\{1\},\{2\}\}} X_{\{\{1\},\{2,3\}\}} + 2 XX_{\{\{1\},\{2,3\}\}} X_{\{\{1\},\{2\}\}} \\
& + X_{\{\{1\},\{3,5\},\{4\},\{2\}\}} + XX_{\{\{1,3\},\{2\}\}} X_{\{\{1\},\{2\}\}} + XX_{\{\{1\},\{2\}\}} X_{\{\{1,3\},\{2\}\}} \\
& + X_{\{\{1\}\}} XX_{\{\{1\},\{3,4\},\{2\}\}} + XX_{\{\{1\},\{3,5\},\{4\},\{2\}\}} \\
\Delta(X_{\{\{3,5\},\{4\},\{1,2\}\}}) &= XX_{\{\{1,2\}\}} X_{\{\{1,3\},\{2\}\}} - 2 X_{\{\{1\}\}} XX_{\{\{1\},\{2,4\},\{3\}\}}
\end{aligned}$$

$$\begin{aligned}
& + 4 XX_{\{\{1\}, \{2\}, \{3\}\}} X_{\{\{1\}, \{2\}\}} - 2 XX_{\{\{1\}, \{2\}\}} X_{\{\{1,2\}, \{3\}\}} + 4 XX_{\{\{1\}, \{2\}\}} X_{\{\{1\}, \{2\}, \{3\}\}} \\
& - 2 XX_{\{\{1,2\}, \{3\}\}} X_{\{\{1\}, \{2\}\}} + XX_{\{\{1,3\}, \{2\}\}} X_{\{\{1,2\}\}} - 2 XX_{\{\{1\}\}} X_{\{\{1\}, \{2,4\}, \{3\}\}} \\
& - 2 X_{\{\{1\}\}} XX_{\{\{4\}, \{1,2\}, \{3\}\}} - 2 XX_{\{\{1\}\}} X_{\{\{4\}, \{1,2\}, \{3\}\}} - 2 XX_{\{\{1\}, \{2\}\}} X_{\{\{1\}, \{2,3\}\}} \\
& + XX_{\{\{1,2\}\}} X_{\{\{1,2\}, \{3\}\}} - 2 XX_{\{\{1\}, \{2,3\}\}} X_{\{\{1\}, \{2\}\}} + X_{\{\{1,2\}\}} XX_{\{\{1,2\}, \{3\}\}} \\
& + X_{\{\{3,5\}, \{4\}, \{1,2\}\}} + XX_{\{\{1\}\}} X_{\{\{3,4\}, \{1,2\}\}} + X_{\{\{1\}\}} XX_{\{\{3,4\}, \{1,2\}\}} + XX_{\{\{3,5\}, \{4\}, \{1,2\}\}} \\
\Delta(X_{\{\{3,5\}, \{2\}, \{1,4\}\}}) = & -XX_{\{\{1\}\}} X_{\{\{1\}, \{3,4\}, \{2\}\}} + XX_{\{\{1,2\}\}} X_{\{\{1\}, \{2,3\}\}} \\
& + XX_{\{\{1,2\}\}} X_{\{\{1,3\}, \{2\}\}} + X_{\{\{1\}\}} XX_{\{\{1,3\}, \{2,4\}\}} - X_{\{\{1\}\}} XX_{\{\{1\}, \{2,4\}, \{3\}\}} \\
& + 4 XX_{\{\{1\}, \{2\}, \{3\}\}} X_{\{\{1\}, \{2\}\}} - XX_{\{\{1\}, \{2\}\}} X_{\{\{1,2\}, \{3\}\}} - XX_{\{\{1\}\}} X_{\{\{2\}, \{3\}, \{1,4\}\}} \\
& + 4 XX_{\{\{1\}, \{2\}\}} X_{\{\{1\}, \{2\}, \{3\}\}} - X_{\{\{1\}\}} XX_{\{\{2\}, \{3\}, \{1,4\}\}} - XX_{\{\{1,2\}, \{3\}\}} X_{\{\{1\}, \{2\}\}} \\
& + XX_{\{\{1,3\}, \{2\}\}} X_{\{\{1,2\}\}} + X_{\{\{1,2\}\}} XX_{\{\{1\}, \{2,3\}\}} + XX_{\{\{1\}\}} X_{\{\{1,3\}, \{2,4\}\}} \\
& - XX_{\{\{1\}\}} X_{\{\{1\}, \{2,4\}, \{3\}\}} - XX_{\{\{1\}, \{2\}\}} X_{\{\{1\}, \{2,3\}\}} - XX_{\{\{1\}, \{2,3\}\}} X_{\{\{1\}, \{2\}\}} \\
& + X_{\{\{3,5\}, \{2\}, \{1,4\}\}} - 2 XX_{\{\{1,3\}, \{2\}\}} X_{\{\{1\}, \{2\}\}} - X_{\{\{1\}\}} XX_{\{\{1,3\}, \{4\}, \{2\}\}} \\
& - XX_{\{\{1\}\}} X_{\{\{1,3\}, \{4\}, \{2\}\}} - 2 XX_{\{\{1\}, \{2\}\}} X_{\{\{1,3\}, \{2\}\}} + XX_{\{\{3,5\}, \{2\}, \{1,4\}\}} \\
& - X_{\{\{1\}\}} XX_{\{\{1\}, \{3,4\}, \{2\}\}} \\
\Delta(X_{\{\{1\}, \{3,5\}, \{2,4\}\}}) = & -XX_{\{\{1\}\}} X_{\{\{1\}, \{3,4\}, \{2\}\}} + 2 XX_{\{\{1,2\}\}} X_{\{\{1\}, \{2,3\}\}} \\
& + X_{\{\{1\}\}} XX_{\{\{1,3\}, \{2,4\}\}} - 2 X_{\{\{1\}\}} XX_{\{\{1\}, \{2,4\}, \{3\}\}} + 4 XX_{\{\{1\}, \{2\}, \{3\}\}} X_{\{\{1\}, \{2\}\}} \\
& - XX_{\{\{1\}, \{2\}\}} X_{\{\{1,2\}, \{3\}\}} + 4 XX_{\{\{1\}, \{2\}\}} X_{\{\{1\}, \{2\}, \{3\}\}} - XX_{\{\{1,2\}, \{3\}\}} X_{\{\{1\}, \{2\}\}} \\
& + 2 X_{\{\{1,2\}\}} XX_{\{\{1\}, \{2,3\}\}} + XX_{\{\{1\}\}} X_{\{\{1,3\}, \{2,4\}\}} - 2 XX_{\{\{1\}\}} X_{\{\{1\}, \{2,4\}, \{3\}\}} \\
& - X_{\{\{1\}\}} XX_{\{\{1\}, \{4\}, \{2,3\}\}} - XX_{\{\{1\}\}} X_{\{\{1\}, \{4\}, \{2,3\}\}} - XX_{\{\{1\}, \{2\}\}} X_{\{\{1\}, \{2,3\}\}} \\
& - XX_{\{\{1\}, \{2,3\}\}} X_{\{\{1\}, \{2\}\}} + XX_{\{\{1\}, \{3,5\}, \{2,4\}\}} - 2 XX_{\{\{1,3\}, \{2\}\}} X_{\{\{1\}, \{2\}\}} \\
& - 2 XX_{\{\{1\}, \{2\}\}} X_{\{\{1,3\}, \{2\}\}} + X_{\{\{1\}, \{3,5\}, \{2,4\}\}} - X_{\{\{1\}\}} XX_{\{\{1\}, \{3,4\}, \{2\}\}} \\
\Delta(X_{\{\{3,5\}, \{1,2,4\}\}}) = & XX_{\{\{1\}\}} X_{\{\{1\}, \{3,4\}, \{2\}\}} - 2 XX_{\{\{1,2\}\}} X_{\{\{1\}, \{2,3\}\}} + XX_{\{\{1,2\}\}} X_{\{\{1,2,3\}\}} \\
& - XX_{\{\{1,2\}\}} X_{\{\{1,3\}, \{2\}\}} - 2 X_{\{\{1\}\}} XX_{\{\{1,3\}, \{2,4\}\}} + 2 X_{\{\{1\}\}} XX_{\{\{1\}, \{2,4\}, \{3\}\}} \\
& - 6 XX_{\{\{1\}, \{2\}, \{3\}\}} X_{\{\{1\}, \{2\}\}} + 4 XX_{\{\{1\}, \{2\}\}} X_{\{\{1,2\}, \{3\}\}} - X_{\{\{1\}\}} XX_{\{\{1,2,4\}, \{3\}\}} \\
& - XX_{\{\{1\}\}} X_{\{\{4\}, \{1,2,3\}\}} - 6 XX_{\{\{1\}, \{2\}\}} X_{\{\{1\}, \{2\}, \{3\}\}} - XX_{\{\{1\}\}} X_{\{\{1,2,4\}, \{3\}\}} \\
& + 4 XX_{\{\{1,2\}, \{3\}\}} X_{\{\{1\}, \{2\}\}} + XX_{\{\{1,2,3\}\}} X_{\{\{1,2\}\}} - XX_{\{\{1,3\}, \{2\}\}} X_{\{\{1,2\}\}} \\
& - 2 X_{\{\{1,2\}\}} XX_{\{\{1\}, \{2,3\}\}} - 2 XX_{\{\{1\}\}} X_{\{\{1,3\}, \{2,4\}\}} + 2 XX_{\{\{1\}\}} X_{\{\{1\}, \{2,4\}, \{3\}\}} \\
& + 2 XX_{\{\{1\}, \{2\}\}} X_{\{\{1\}, \{2,3\}\}} + 2 XX_{\{\{1\}, \{2,3\}\}} X_{\{\{1\}, \{2\}\}} + 3 XX_{\{\{1,3\}, \{2\}\}} X_{\{\{1\}, \{2\}\}} \\
& + 3 XX_{\{\{1\}, \{2\}\}} X_{\{\{1,3\}, \{2\}\}} + X_{\{\{3,5\}, \{1,2,4\}\}} + XX_{\{\{3,5\}, \{1,2,4\}\}} - X_{\{\{1\}\}} XX_{\{\{4\}, \{1,2,3\}\}} \\
& + X_{\{\{1\}\}} XX_{\{\{1\}, \{3,4\}, \{2\}\}} - XX_{\{\{1\}\}} X_{\{\{3,4\}, \{1,2\}\}} - X_{\{\{1\}\}} XX_{\{\{3,4\}, \{1,2\}\}} \\
\Delta(X_{\{\{1,4,5\}, \{2\}, \{3\}\}}) = & -XX_{\{\{1\}\}} X_{\{\{1\}, \{3,4\}, \{2\}\}} + 3 X_{\{\{1\}\}} XX_{\{\{1\}, \{4\}, \{2\}, \{3\}\}} \\
& + 9 XX_{\{\{1\}, \{2\}, \{3\}\}} X_{\{\{1\}, \{2\}\}} + XX_{\{\{1,2,3\}\}} X_{\{\{1\}, \{2\}\}} - 2 XX_{\{\{1\}\}} X_{\{\{2\}, \{3\}, \{1,4\}\}} \\
& - 3 X_{\{\{1,2\}\}} XX_{\{\{1\}, \{2\}, \{3\}\}} + 3 XX_{\{\{1\}\}} X_{\{\{1\}, \{4\}, \{2\}, \{3\}\}} + 9 XX_{\{\{1\}, \{2\}\}} X_{\{\{1\}, \{2\}, \{3\}\}} \\
& - 2 X_{\{\{1\}\}} XX_{\{\{2\}, \{3\}, \{1,4\}\}} + 2 XX_{\{\{1\}\}} X_{\{\{1,3,4\}, \{2\}\}} - 3 XX_{\{\{1,2\}\}} X_{\{\{1\}, \{2\}, \{3\}\}} \\
& + XX_{\{\{1\}, \{2\}\}} X_{\{\{1,2,3\}\}} - 2 XX_{\{\{1\}, \{2\}\}} X_{\{\{1\}, \{2,3\}\}} - 2 XX_{\{\{1\}, \{2,3\}\}} X_{\{\{1\}, \{2\}\}} \\
& - 4 XX_{\{\{1,3\}, \{2\}\}} X_{\{\{1\}, \{2\}\}} - 4 XX_{\{\{1\}, \{2\}\}} X_{\{\{1,3\}, \{2\}\}} - X_{\{\{1\}\}} XX_{\{\{1\}, \{3,4\}, \{2\}\}} \\
& + X_{\{\{1,4,5\}, \{2\}, \{3\}\}} + 2 X_{\{\{1\}\}} XX_{\{\{1,3,4\}, \{2\}\}} + XX_{\{\{1,4,5\}, \{2\}, \{3\}\}} \\
\Delta(X_{\{\{1,4,5\}, \{2,3\}\}}) = & -XX_{\{\{1,2\}\}} X_{\{\{1\}, \{2,3\}\}} + XX_{\{\{1,2\}\}} X_{\{\{1,2,3\}\}} \\
& - 6 XX_{\{\{1\}, \{2\}, \{3\}\}} X_{\{\{1\}, \{2\}\}} + XX_{\{\{1,4,5\}, \{2,3\}\}} + 2 XX_{\{\{1\}, \{2\}\}} X_{\{\{1,2\}, \{3\}\}} \\
& - 6 XX_{\{\{1\}, \{2\}\}} X_{\{\{1\}, \{2\}, \{3\}\}} - 2 XX_{\{\{1\}\}} X_{\{\{1,3,4\}, \{2\}\}} + 2 XX_{\{\{1,2\}, \{3\}\}} X_{\{\{1\}, \{2\}\}}
\end{aligned}$$

```

+ XX_{(1,2,3)} X_{(1,2)} - X_{(1,2)} XX_{(1),(2,3)} - 2 X_{(1)} XX_{(2,3),(1,4)}
+ 2 X_{(1)} XX_{(1),(4),(2,3)} + X_{(1)} XX_{(4),(1,2),(3)} + 2 XX_{(1)} X_{(1),(4),(2,3)}
+ XX_{(1)} X_{(4),(1,2),(3)} - 2 XX_{(1)} X_{(2,3),(1,4)} + 3 XX_{(1),(2)} X_{(1),(2,3)}
- 2 XX_{(1,2)} X_{(1,2),(3)} + 3 XX_{(1),(2,3)} X_{(1),(2)} - 2 X_{(1,2)} XX_{(1,2),(3)}
+ 4 XX_{(1,3),(2)} X_{(1),(2)} + 4 XX_{(1),(2)} X_{(1,3),(2)} - 2 X_{(1)} XX_{(1,3,4),(2)}
+ X_{(1,4,5),(2,3)} - XX_{(1)} X_{(3,4),(1,2)} - X_{(1)} XX_{(3,4),(1,2)}
Δ(X_{(1,2,3,5),(4)}) = -4 X_{(1)} XX_{(1),(4),(2),(3)} - 3 XX_{(1,2)} X_{(1,3),(2)}
+ 3 X_{(1)} XX_{(1),(2,4),(3)} - 22 XX_{(1),(2),(3)} X_{(1),(2)} + 7 XX_{(1),(2)} X_{(1,2),(3)}
- 3 X_{(1)} XX_{(1,2,4),(3)} - 4 XX_{(1,2,3)} X_{(1),(2)} + 3 XX_{(1)} X_{(2),(3),(1,4)}
+ 6 X_{(1,2)} XX_{(1),(2),(3)} - 4 XX_{(1)} X_{(1),(4),(2),(3)} - XX_{(1)} X_{(4),(1,2,3)}
- 22 XX_{(1),(2)} X_{(1),(2),(3)} - 3 XX_{(1)} X_{(1,2,4),(3)} + 3 X_{(1)} XX_{(2),(3),(1,4)}
+ 6 XX_{(1,2)} X_{(1),(2),(3)} - 4 XX_{(1),(2)} X_{(1,2,3)} + 7 XX_{(1,2),(3)} X_{(1),(2)}
- 3 XX_{(1,3),(2)} X_{(1,2)} + XX_{(1,2,3,5),(4)} + X_{(1)} XX_{(1,2,3,4)}
+ 3 XX_{(1)} X_{(1),(2,4),(3)} + X_{(1)} XX_{(1),(4),(2,3)} + 4 X_{(1)} XX_{(4),(1,2),(3)}
+ XX_{(1)} X_{(1),(4),(2,3)} + 4 XX_{(1)} X_{(4),(1,2),(3)} + 4 XX_{(1),(2)} X_{(1),(2,3)}
- 3 XX_{(1,2)} X_{(1,2),(3)} + 4 XX_{(1),(2,3)} X_{(1),(2)} - 3 X_{(1,2)} XX_{(1,2),(3)}
+ 7 XX_{(1,3),(2)} X_{(1),(2)} + X_{(1)} XX_{(1,3),(4),(2)} + XX_{(1)} X_{(1,3),(4),(2)}
+ 7 XX_{(1),(2)} X_{(1,3),(2)} + XX_{(1)} X_{(1,2,3,4)} - X_{(1)} XX_{(4),(1,2,3)} + X_{(1,2,3,5),(4)}

```

— looking for a pattern in the antipode X-basis

```

> for i from 1 to 5 do
  print(S(X[{{seq(j,j=1..i)}}])=
    ToX(antipode(X[{{seq(j,j=1..i)}}])));
od;

S(X_{(1)}) = -X_{(1)}
S(X_{(1,2)}) = -X_{(1,2)} - 2 X_{(1),(2)}
S(X_{(1,2,3)}) = -3 X_{(1,2),(3)} - X_{(1,2,3)} - 3 X_{(1),(2,3)} - 6 X_{(1),(2),(3)}
S(X_{(1,2,3,4)}) = 4 X_{(1,3),(4),(2)} - 16 X_{(1),(4),(2,3)} - 14 X_{(4),(1,2),(3)}
- 14 X_{(1),(3,4),(2)} - 6 X_{(3,4),(1,2)} - 4 X_{(4),(1,2,3)} + 4 X_{(1),(2,4),(3)} - X_{(1,2,3,4)}
- 4 X_{(1),(2,3,4)} - 26 X_{(1),(4),(2),(3)}
S(X_{(1,2,3,4,5)}) = -150 X_{(5),(1),(4),(2),(3)} - 10 X_{(4,5),(1,2,3)} + 45 X_{(5),(1,3),(4),(2)}
- 30 X_{(5),(1),(2,3,4)} - 45 X_{(5),(3,4),(1,2)} - 5 X_{(5),(2),(3),(1,4)} + 5 X_{(1),(2,5),(3,4)}
- 5 X_{(5),(1,2,3,4)} + 5 X_{(5),(1,2,4),(3)} + 5 X_{(5),(1,3),(2,4)} + 5 X_{(2,3,5),(1),(4)}
- 120 X_{(5),(1),(4),(2,3)} - 40 X_{(4,5),(1,2),(3)} + 5 X_{(2,4,5),(1),(3)} + 5 X_{(5),(2,3),(1,4)}
- 120 X_{(5),(1),(3,4),(2)} + 10 X_{(3,5),(4),(1,2)} + 45 X_{(1),(3,5),(4),(2)}
+ 10 X_{(4,5),(1,3),(2)} + 5 X_{(5),(1,3,4),(2)} - 85 X_{(5),(4),(1,2),(3)} + 5 X_{(1),(3,5),(2,4)}
+ 70 X_{(5),(1),(2,4),(3)} - X_{(1,2,3,4,5)} - 85 X_{(4,5),(1),(2),(3)} - 45 X_{(4,5),(1),(2,3)}
- 25 X_{(3,4,5),(1),(2)} - 10 X_{(3,4,5),(1,2)} - 5 X_{(1),(2,5),(4),(3)} - 25 X_{(5),(4),(1,2,3)}
- 5 X_{(2,3,4,5),(1)}

```

- n=2 antipode

```
> for A in listallsp(2) do
  print(S(X[A]) = ToX(antipode(X[A])));
od;
```

$$S(X_{\{(1),\{2\}\}}) = X_{\{(1),\{2\}\}}$$
$$S(X_{\{\{1,2\}\}}) = -X_{\{\{1,2\}\}} - 2 X_{\{(1),\{2\}\}}$$

- n=3 antipode

```
> for A in listallsp(3) do
  print(S(X[A]) = ToX(antipode(X[A])));
od;
```

$$S(X_{\{(1),\{2\},\{3\}\}}) = -X_{\{(1),\{2\},\{3\}\}}$$
$$S(X_{\{\{1,2\},\{3\}\}}) = X_{\{\{1,2\},\{3\}\}} + 2 X_{\{(1),\{2\},\{3\}\}}$$
$$S(X_{\{\{1,3\},\{2\}\}}) = X_{\{\{1,2\},\{3\}\}} - X_{\{\{1,3\},\{2\}\}} + X_{\{(1),\{2,3\}\}} + 2 X_{\{(1),\{2\},\{3\}\}}$$
$$S(X_{\{\{1\},\{2,3\}\}}) = X_{\{\{1,2\},\{3\}\}} + 2 X_{\{(1),\{2\},\{3\}\}}$$
$$S(X_{\{\{1,2,3\}\}}) = -3 X_{\{\{1,2\},\{3\}\}} - X_{\{\{1,2,3\}\}} - 3 X_{\{(1),\{2,3\}\}} - 6 X_{\{(1),\{2\},\{3\}\}}$$

- n=4 antipode

```
> for A in listallsp(4) do
  print(S(X[A]) = ToX(antipode(X[A])));
od;
```

$$S(X_{\{(1),\{4\},\{2\},\{3\}\}}) = X_{\{(1),\{4\},\{2\},\{3\}\}}$$
$$S(X_{\{\{4\},\{1,2\},\{3\}\}}) = -X_{\{(1),\{3,4\},\{2\}\}} - 2 X_{\{(1),\{4\},\{2\},\{3\}\}}$$
$$S(X_{\{\{1,3\},\{4\},\{2\}\}}) =$$
$$-X_{\{(1),\{4\},\{2,3\}\}} - X_{\{(1),\{3,4\},\{2\}\}} + X_{\{(1),\{2,4\},\{3\}\}} - 2 X_{\{(1),\{4\},\{2\},\{3\}\}}$$
$$S(X_{\{\{1\},\{4\},\{2,3\}\}}) = -X_{\{(1),\{4\},\{2,3\}\}} - 2 X_{\{(1),\{4\},\{2\},\{3\}\}}$$
$$S(X_{\{\{4\},\{1,2,3\}\}}) =$$
$$3 X_{\{(1),\{4\},\{2,3\}\}} + 3 X_{\{(1),\{3,4\},\{2\}\}} + X_{\{(1),\{2,3,4\}\}} + 6 X_{\{(1),\{4\},\{2\},\{3\}\}}$$
$$S(X_{\{\{2\},\{3\},\{1,4\}\}}) = 2 X_{\{\{1,3\},\{4\},\{2\}\}} - 2 X_{\{(1),\{4\},\{2,3\}\}} - X_{\{\{4\},\{1,2\},\{3\}\}}$$
$$- X_{\{(1),\{3,4\},\{2\}\}} + 2 X_{\{(1),\{2,4\},\{3\}\}} - X_{\{\{2\},\{3\},\{1,4\}\}} - 2 X_{\{(1),\{4\},\{2\},\{3\}\}}$$
$$S(X_{\{\{2,3\},\{1,4\}\}}) = -2 X_{\{\{1,3\},\{4\},\{2\}\}} - X_{\{\{2,3\},\{1,4\}\}} + 2 X_{\{(1),\{4\},\{2,3\}\}} + 3 X_{\{\{4\},\{1,2\},\{3\}\}}$$
$$+ 3 X_{\{(1),\{3,4\},\{2\}\}} + 2 X_{\{\{3,4\},\{1,2\}\}} - 2 X_{\{(1),\{2,4\},\{3\}\}} + 4 X_{\{(1),\{4\},\{2\},\{3\}\}}$$
$$S(X_{\{\{1,3,4\},\{2\}\}}) = -2 X_{\{\{1,3\},\{4\},\{2\}\}} + 5 X_{\{(1),\{4\},\{2,3\}\}} + 3 X_{\{\{4\},\{1,2\},\{3\}\}} - X_{\{\{1,3,4\},\{2\}\}}$$
$$+ 2 X_{\{(1),\{3,4\},\{2\}\}} + X_{\{\{4\},\{1,2,3\}\}} - 2 X_{\{(1),\{2,4\},\{3\}\}} + X_{\{(1),\{2,3,4\}\}} + 6 X_{\{(1),\{4\},\{2\},\{3\}\}}$$
$$S(X_{\{\{1\},\{2,4\},\{3\}\}}) =$$
$$X_{\{\{1,3\},\{4\},\{2\}\}} - X_{\{(1),\{4\},\{2,3\}\}} - X_{\{\{4\},\{1,2\},\{3\}\}} - 2 X_{\{(1),\{4\},\{2\},\{3\}\}}$$
$$S(X_{\{\{1,3\},\{2,4\}\}}) = -2 X_{\{\{1,3\},\{4\},\{2\}\}} + 2 X_{\{(1),\{4\},\{2,3\}\}} + 3 X_{\{\{4\},\{1,2\},\{3\}\}}$$
$$+ 3 X_{\{(1),\{3,4\},\{2\}\}} + 2 X_{\{\{3,4\},\{1,2\}\}} - 2 X_{\{(1),\{2,4\},\{3\}\}} - X_{\{\{1,3\},\{2,4\}\}}$$
$$+ 4 X_{\{(1),\{4\},\{2\},\{3\}\}}$$
$$S(X_{\{\{1\},\{2,3,4\}\}}) =$$
$$3 X_{\{(1),\{4\},\{2,3\}\}} + 3 X_{\{\{4\},\{1,2\},\{3\}\}} + X_{\{\{4\},\{1,2,3\}\}} + 6 X_{\{(1),\{4\},\{2\},\{3\}\}}$$
$$S(X_{\{\{1,2,4\},\{3\}\}}) = -2 X_{\{\{1,3\},\{4\},\{2\}\}} + 5 X_{\{(1),\{4\},\{2,3\}\}} + 2 X_{\{\{4\},\{1,2\},\{3\}\}}$$

$$+ 3 X_{\{\{1\}, \{3,4\}, \{2\}\}} + X_{\{\{4\}, \{1,2,3\}\}} - 2 X_{\{\{1\}, \{2,4\}, \{3\}\}} + X_{\{\{1\}, \{2,3,4\}\}} + 6 X_{\{\{1\}, \{4\}, \{2\}, \{3\}\}} - X_{\{\{1,2,4\}, \{3\}\}}$$

$$S(X_{\{\{1\}, \{3,4\}, \{2\}\}}) = -X_{\{\{4\}, \{1,2\}, \{3\}\}} - 2 X_{\{\{1\}, \{4\}, \{2\}, \{3\}\}}$$

$$S(X_{\{\{3,4\}, \{1,2\}\}}) =$$

$$2 X_{\{\{4\}, \{1,2\}, \{3\}\}} + 2 X_{\{\{1\}, \{3,4\}, \{2\}\}} + X_{\{\{3,4\}, \{1,2\}\}} + 4 X_{\{\{1\}, \{4\}, \{2\}, \{3\}\}}$$

$$S(X_{\{\{1,2,3,4\}\}}) = 4 X_{\{\{1,3\}, \{4\}, \{2\}\}} - 16 X_{\{\{1\}, \{4\}, \{2,3\}\}} - 14 X_{\{\{4\}, \{1,2\}, \{3\}\}}$$

$$- 14 X_{\{\{1\}, \{3,4\}, \{2\}\}} - 6 X_{\{\{3,4\}, \{1,2\}\}} - 4 X_{\{\{4\}, \{1,2,3\}\}} + 4 X_{\{\{1\}, \{2,4\}, \{3\}\}} - X_{\{\{1,2,3,4\}\}}$$

$$- 4 X_{\{\{1\}, \{2,3,4\}\}} - 26 X_{\{\{1\}, \{4\}, \{2\}, \{3\}\}}$$

- n=5 antipode

```
> for A in listallsp(5) do
  print(S(X[A]) = ToX(antipode(X[A])));
od;
```

$$S(X_{\{\{5\}, \{1\}, \{4\}, \{2\}, \{3\}\}}) = -X_{\{\{5\}, \{1\}, \{4\}, \{2\}, \{3\}\}}$$

$$S(X_{\{\{5\}, \{4\}, \{1,2\}, \{3\}\}}) = 2 X_{\{\{5\}, \{1\}, \{4\}, \{2\}, \{3\}\}} + X_{\{\{4,5\}, \{1\}, \{2\}, \{3\}\}}$$

$$S(X_{\{\{5\}, \{1,3\}, \{4\}, \{2\}\}}) =$$

$$2 X_{\{\{5\}, \{1\}, \{4\}, \{2\}, \{3\}\}} + X_{\{\{5\}, \{1\}, \{3,4\}, \{2\}\}} - X_{\{\{1\}, \{3,5\}, \{4\}, \{2\}\}} + X_{\{\{4,5\}, \{1\}, \{2\}, \{3\}\}}$$

$$S(X_{\{\{5\}, \{1\}, \{4\}, \{2,3\}\}}) = 2 X_{\{\{5\}, \{1\}, \{4\}, \{2\}, \{3\}\}} + X_{\{\{5\}, \{1\}, \{3,4\}, \{2\}\}}$$

$$S(X_{\{\{5\}, \{4\}, \{1,2,3\}\}}) =$$

$$-6 X_{\{\{5\}, \{1\}, \{4\}, \{2\}, \{3\}\}} - 3 X_{\{\{5\}, \{1\}, \{3,4\}, \{2\}\}} - 3 X_{\{\{4,5\}, \{1\}, \{2\}, \{3\}\}} - X_{\{\{3,4,5\}, \{1\}, \{2\}\}}$$

$$S(X_{\{\{5\}, \{2\}, \{3\}, \{1,4\}\}}) = 2 X_{\{\{5\}, \{1\}, \{4\}, \{2\}, \{3\}\}} + X_{\{\{5\}, \{1\}, \{4\}, \{2,3\}\}} + 2 X_{\{\{5\}, \{1\}, \{3,4\}, \{2\}\}}$$

$$- 2 X_{\{\{1\}, \{3,5\}, \{4\}, \{2\}\}} - 2 X_{\{\{5\}, \{1\}, \{2,4\}, \{3\}\}} + X_{\{\{4,5\}, \{1\}, \{2\}, \{3\}\}} + X_{\{\{1\}, \{2,5\}, \{4\}, \{3\}\}}$$

$$S(X_{\{\{5\}, \{2,3\}, \{1,4\}\}}) = -4 X_{\{\{5\}, \{1\}, \{4\}, \{2\}, \{3\}\}} + X_{\{\{1\}, \{2,5\}, \{3,4\}\}} - 3 X_{\{\{5\}, \{1\}, \{4\}, \{2,3\}\}}$$

$$- 2 X_{\{\{5\}, \{1\}, \{3,4\}, \{2\}\}} + 2 X_{\{\{1\}, \{3,5\}, \{4\}, \{2\}\}} + 2 X_{\{\{5\}, \{1\}, \{2,4\}, \{3\}\}} - 3 X_{\{\{4,5\}, \{1\}, \{2\}, \{3\}\}}$$

$$- 2 X_{\{\{4,5\}, \{1\}, \{2,3\}\}}$$

$$S(X_{\{\{5\}, \{1,3,4\}, \{2\}\}}) = -6 X_{\{\{5\}, \{1\}, \{4\}, \{2\}, \{3\}\}} - X_{\{\{5\}, \{1\}, \{2,3,4\}\}} - 3 X_{\{\{5\}, \{1\}, \{4\}, \{2,3\}\}}$$

$$+ X_{\{\{2,4,5\}, \{1\}, \{3\}\}} - 5 X_{\{\{5\}, \{1\}, \{3,4\}, \{2\}\}} + 2 X_{\{\{1\}, \{3,5\}, \{4\}, \{2\}\}} + 2 X_{\{\{5\}, \{1\}, \{2,4\}, \{3\}\}}$$

$$- 2 X_{\{\{4,5\}, \{1\}, \{2\}, \{3\}\}} - X_{\{\{3,4,5\}, \{1\}, \{2\}\}}$$

$$S(X_{\{\{5\}, \{1\}, \{2,4\}, \{3\}\}}) =$$

$$2 X_{\{\{5\}, \{1\}, \{4\}, \{2\}, \{3\}\}} + X_{\{\{5\}, \{1\}, \{4\}, \{2,3\}\}} + X_{\{\{5\}, \{1\}, \{3,4\}, \{2\}\}} - X_{\{\{5\}, \{1\}, \{2,4\}, \{3\}\}}$$

$$S(X_{\{\{5\}, \{1,3\}, \{2,4\}\}}) = -4 X_{\{\{5\}, \{1\}, \{4\}, \{2\}, \{3\}\}} - 3 X_{\{\{5\}, \{1\}, \{4\}, \{2,3\}\}}$$

$$- 2 X_{\{\{5\}, \{1\}, \{3,4\}, \{2\}\}} + 2 X_{\{\{1\}, \{3,5\}, \{4\}, \{2\}\}} + X_{\{\{1\}, \{3,5\}, \{2,4\}\}} + 2 X_{\{\{5\}, \{1\}, \{2,4\}, \{3\}\}}$$

$$- 3 X_{\{\{4,5\}, \{1\}, \{2\}, \{3\}\}} - 2 X_{\{\{4,5\}, \{1\}, \{2,3\}\}}$$

$$S(X_{\{\{5\}, \{1\}, \{2,3,4\}\}}) =$$

$$-6 X_{\{\{5\}, \{1\}, \{4\}, \{2\}, \{3\}\}} - X_{\{\{5\}, \{1\}, \{2,3,4\}\}} - 3 X_{\{\{5\}, \{1\}, \{4\}, \{2,3\}\}} - 3 X_{\{\{5\}, \{1\}, \{3,4\}, \{2\}\}}$$

$$S(X_{\{\{5\}, \{1,2,4\}, \{3\}\}}) = -6 X_{\{\{5\}, \{1\}, \{4\}, \{2\}, \{3\}\}} - X_{\{\{5\}, \{1\}, \{2,3,4\}\}} + X_{\{\{2,3,5\}, \{1\}, \{4\}\}}$$

$$- 2 X_{\{\{5\}, \{1\}, \{4\}, \{2,3\}\}} - 5 X_{\{\{5\}, \{1\}, \{3,4\}, \{2\}\}} + 2 X_{\{\{1\}, \{3,5\}, \{4\}, \{2\}\}} + 2 X_{\{\{5\}, \{1\}, \{2,4\}, \{3\}\}}$$

$$- 3 X_{\{\{4,5\}, \{1\}, \{2\}, \{3\}\}} - X_{\{\{3,4,5\}, \{1\}, \{2\}\}}$$

$$S(X_{\{\{5\}, \{1\}, \{3,4\}, \{2\}\}}) = 2 X_{\{\{5\}, \{1\}, \{4\}, \{2\}, \{3\}\}} + X_{\{\{5\}, \{1\}, \{4\}, \{2,3\}\}}$$

$$S(X_{\{\{5\}, \{3,4\}, \{1,2\}\}}) =$$

$$-4 X_{\{\{5\}, \{1\}, \{4\}, \{2\}, \{3\}\}} - 2 X_{\{\{5\}, \{1\}, \{4\}, \{2,3\}\}} - 2 X_{\{\{4,5\}, \{1\}, \{2\}, \{3\}\}} - X_{\{\{4,5\}, \{1\}, \{2,3\}\}}$$

$$\begin{aligned} S(X_{\{\{5\}, \{1, 2, 3, 4\}\}}) &= 26 X_{\{\{5\}, \{1\}, \{4\}, \{2\}, \{3\}\}} + 4 X_{\{\{5\}, \{1\}, \{2, 3, 4\}\}} + 14 X_{\{\{5\}, \{1\}, \{4\}, \{2, 3\}\}} \\ &+ 16 X_{\{\{5\}, \{1\}, \{3, 4\}, \{2\}\}} - 4 X_{\{\{1\}, \{3, 5\}, \{4\}, \{2\}\}} - 4 X_{\{\{5\}, \{1\}, \{2, 4\}, \{3\}\}} \\ &+ 14 X_{\{\{4, 5\}, \{1\}, \{2\}, \{3\}\}} + 6 X_{\{\{4, 5\}, \{1\}, \{2, 3\}\}} + 4 X_{\{\{3, 4, 5\}, \{1\}, \{2\}\}} + X_{\{\{2, 3, 4, 5\}, \{1\}\}} \end{aligned}$$

$$\begin{aligned} S(X_{\{\{4\}, \{2\}, \{3\}, \{1, 5\}\}}) &= 2 X_{\{\{5\}, \{1\}, \{4\}, \{2\}, \{3\}\}} - 3 X_{\{\{5\}, \{1, 3\}, \{4\}, \{2\}\}} \\ &+ 3 X_{\{\{5\}, \{2\}, \{3\}, \{1, 4\}\}} + 3 X_{\{\{5\}, \{1\}, \{4\}, \{2, 3\}\}} - X_{\{\{4\}, \{2\}, \{3\}, \{1, 5\}\}} + 3 X_{\{\{5\}, \{1\}, \{3, 4\}, \{2\}\}} \\ &- 3 X_{\{\{1\}, \{3, 5\}, \{4\}, \{2\}\}} + X_{\{\{5\}, \{4\}, \{1, 2\}, \{3\}\}} - 6 X_{\{\{5\}, \{1\}, \{2, 4\}, \{3\}\}} + X_{\{\{4, 5\}, \{1\}, \{2\}, \{3\}\}} \\ &+ 3 X_{\{\{1\}, \{2, 5\}, \{4\}, \{3\}\}} \end{aligned}$$

$$\begin{aligned} S(X_{\{\{4\}, \{2, 3\}, \{1, 5\}\}}) &= -4 X_{\{\{5\}, \{1\}, \{4\}, \{2\}, \{3\}\}} + 4 X_{\{\{5\}, \{1, 3\}, \{4\}, \{2\}\}} - X_{\{\{5\}, \{3, 4\}, \{1, 2\}\}} \\ &- 2 X_{\{\{5\}, \{2\}, \{3\}, \{1, 4\}\}} + X_{\{\{1\}, \{2, 5\}, \{3, 4\}\}} - 5 X_{\{\{5\}, \{1\}, \{4\}, \{2, 3\}\}} - X_{\{\{4, 5\}, \{1, 2\}, \{3\}\}} \\ &+ X_{\{\{5\}, \{2, 3\}, \{1, 4\}\}} - 4 X_{\{\{5\}, \{1\}, \{3, 4\}, \{2\}\}} + X_{\{\{3, 5\}, \{4\}, \{1, 2\}\}} + 4 X_{\{\{1\}, \{3, 5\}, \{4\}, \{2\}\}} \\ &+ X_{\{\{4, 5\}, \{1, 3\}, \{2\}\}} - 2 X_{\{\{5\}, \{4\}, \{1, 2\}, \{3\}\}} + 6 X_{\{\{5\}, \{1\}, \{2, 4\}, \{3\}\}} - X_{\{\{4\}, \{2, 3\}, \{1, 5\}\}} \\ &- 3 X_{\{\{4, 5\}, \{1\}, \{2\}, \{3\}\}} - 2 X_{\{\{4, 5\}, \{1\}, \{2, 3\}\}} - 2 X_{\{\{1\}, \{2, 5\}, \{4\}, \{3\}\}} \end{aligned}$$

$$\begin{aligned} S(X_{\{\{2, 4\}, \{3\}, \{1, 5\}\}}) &= -X_{\{\{2, 4\}, \{3\}, \{1, 5\}\}} - 4 X_{\{\{5\}, \{1\}, \{4\}, \{2\}, \{3\}\}} + 5 X_{\{\{5\}, \{1, 3\}, \{4\}, \{2\}\}} \\ &- 2 X_{\{\{5\}, \{3, 4\}, \{1, 2\}\}} - 2 X_{\{\{5\}, \{2\}, \{3\}, \{1, 4\}\}} + X_{\{\{1\}, \{2, 5\}, \{3, 4\}\}} - 5 X_{\{\{5\}, \{1\}, \{4\}, \{2, 3\}\}} \\ &- 2 X_{\{\{4, 5\}, \{1, 2\}, \{3\}\}} + X_{\{\{5\}, \{2, 3\}, \{1, 4\}\}} - 5 X_{\{\{5\}, \{1\}, \{3, 4\}, \{2\}\}} + 2 X_{\{\{3, 5\}, \{4\}, \{1, 2\}\}} \\ &+ 5 X_{\{\{1\}, \{3, 5\}, \{4\}, \{2\}\}} + 2 X_{\{\{4, 5\}, \{1, 3\}, \{2\}\}} - 3 X_{\{\{5\}, \{4\}, \{1, 2\}, \{3\}\}} + 6 X_{\{\{5\}, \{1\}, \{2, 4\}, \{3\}\}} \\ &- 3 X_{\{\{4, 5\}, \{1\}, \{2\}, \{3\}\}} - 2 X_{\{\{4, 5\}, \{1\}, \{2, 3\}\}} - 2 X_{\{\{1\}, \{2, 5\}, \{4\}, \{3\}\}} \end{aligned}$$

$$\begin{aligned} S(X_{\{\{2, 3, 4\}, \{1, 5\}\}}) &= 12 X_{\{\{5\}, \{1\}, \{4\}, \{2\}, \{3\}\}} + X_{\{\{4, 5\}, \{1, 2, 3\}\}} - 9 X_{\{\{5\}, \{1, 3\}, \{4\}, \{2\}\}} \\ &+ 6 X_{\{\{5\}, \{3, 4\}, \{1, 2\}\}} + 3 X_{\{\{5\}, \{2\}, \{3\}, \{1, 4\}\}} - 3 X_{\{\{1\}, \{2, 5\}, \{3, 4\}\}} + 12 X_{\{\{5\}, \{1\}, \{4\}, \{2, 3\}\}} \\ &+ 6 X_{\{\{4, 5\}, \{1, 2\}, \{3\}\}} - 3 X_{\{\{5\}, \{2, 3\}, \{1, 4\}\}} + 12 X_{\{\{5\}, \{1\}, \{3, 4\}, \{2\}\}} - 3 X_{\{\{3, 5\}, \{4\}, \{1, 2\}\}} \\ &- 9 X_{\{\{1\}, \{3, 5\}, \{4\}, \{2\}\}} - 3 X_{\{\{4, 5\}, \{1, 3\}, \{2\}\}} + 9 X_{\{\{5\}, \{4\}, \{1, 2\}, \{3\}\}} - 12 X_{\{\{5\}, \{1\}, \{2, 4\}, \{3\}\}} \\ &+ 9 X_{\{\{4, 5\}, \{1\}, \{2\}, \{3\}\}} + 6 X_{\{\{4, 5\}, \{1\}, \{2, 3\}\}} + X_{\{\{3, 4, 5\}, \{1\}, \{2\}\}} + X_{\{\{3, 4, 5\}, \{1, 2\}\}} \\ &+ 3 X_{\{\{1\}, \{2, 5\}, \{4\}, \{3\}\}} + X_{\{\{5\}, \{4\}, \{1, 2, 3\}\}} - X_{\{\{2, 3, 4\}, \{1, 5\}\}} \end{aligned}$$

$$\begin{aligned} S(X_{\{\{3, 4\}, \{2\}, \{1, 5\}\}}) &= -4 X_{\{\{5\}, \{1\}, \{4\}, \{2\}, \{3\}\}} + 4 X_{\{\{5\}, \{1, 3\}, \{4\}, \{2\}\}} - 2 X_{\{\{5\}, \{3, 4\}, \{1, 2\}\}} \\ &- X_{\{\{3, 4\}, \{2\}, \{1, 5\}\}} - 2 X_{\{\{5\}, \{2\}, \{3\}, \{1, 4\}\}} + X_{\{\{1\}, \{2, 5\}, \{3, 4\}\}} - 4 X_{\{\{5\}, \{1\}, \{4\}, \{2, 3\}\}} \\ &- X_{\{\{4, 5\}, \{1, 2\}, \{3\}\}} + X_{\{\{5\}, \{2, 3\}, \{1, 4\}\}} - 5 X_{\{\{5\}, \{1\}, \{3, 4\}, \{2\}\}} + X_{\{\{3, 5\}, \{4\}, \{1, 2\}\}} \\ &+ 4 X_{\{\{1\}, \{3, 5\}, \{4\}, \{2\}\}} + X_{\{\{4, 5\}, \{1, 3\}, \{2\}\}} - 3 X_{\{\{5\}, \{4\}, \{1, 2\}, \{3\}\}} + 6 X_{\{\{5\}, \{1\}, \{2, 4\}, \{3\}\}} \\ &- 2 X_{\{\{4, 5\}, \{1\}, \{2\}, \{3\}\}} - X_{\{\{4, 5\}, \{1\}, \{2, 3\}\}} - 2 X_{\{\{1\}, \{2, 5\}, \{4\}, \{3\}\}} \end{aligned}$$

$$\begin{aligned} S(X_{\{\{1, 3, 5\}, \{4\}, \{2\}\}}) &= -6 X_{\{\{5\}, \{1\}, \{4\}, \{2\}, \{3\}\}} + 3 X_{\{\{5\}, \{1, 3\}, \{4\}, \{2\}\}} - 2 X_{\{\{5\}, \{1\}, \{2, 3, 4\}\}} \\ &- X_{\{\{5\}, \{2\}, \{3\}, \{1, 4\}\}} + X_{\{\{5\}, \{1, 2, 4\}, \{3\}\}} + X_{\{\{2, 3, 5\}, \{1\}, \{4\}\}} - 6 X_{\{\{5\}, \{1\}, \{4\}, \{2, 3\}\}} \\ &+ X_{\{\{2, 4, 5\}, \{1\}, \{3\}\}} - X_{\{\{1, 3, 5\}, \{4\}, \{2\}\}} - 6 X_{\{\{5\}, \{1\}, \{3, 4\}, \{2\}\}} + 3 X_{\{\{1\}, \{3, 5\}, \{4\}, \{2\}\}} \\ &+ X_{\{\{5\}, \{1, 3, 4\}, \{2\}\}} - 2 X_{\{\{5\}, \{4\}, \{1, 2\}, \{3\}\}} + 6 X_{\{\{5\}, \{1\}, \{2, 4\}, \{3\}\}} - 2 X_{\{\{4, 5\}, \{1\}, \{2\}, \{3\}\}} \\ &- X_{\{\{3, 4, 5\}, \{1\}, \{2\}\}} - X_{\{\{1\}, \{2, 5\}, \{4\}, \{3\}\}} - X_{\{\{5\}, \{4\}, \{1, 2, 3\}\}} \end{aligned}$$

$$\begin{aligned} S(X_{\{\{1, 3, 5\}, \{2, 4\}\}}) &= 12 X_{\{\{5\}, \{1\}, \{4\}, \{2\}, \{3\}\}} + X_{\{\{4, 5\}, \{1, 2, 3\}\}} - 6 X_{\{\{5\}, \{1, 3\}, \{4\}, \{2\}\}} \\ &+ 2 X_{\{\{5\}, \{1\}, \{2, 3, 4\}\}} + 5 X_{\{\{5\}, \{3, 4\}, \{1, 2\}\}} - X_{\{\{1\}, \{2, 5\}, \{3, 4\}\}} - X_{\{\{5\}, \{1, 2, 4\}, \{3\}\}} \\ &- 2 X_{\{\{5\}, \{1, 3\}, \{2, 4\}\}} - X_{\{\{2, 3, 5\}, \{1\}, \{4\}\}} + 12 X_{\{\{5\}, \{1\}, \{4\}, \{2, 3\}\}} + 4 X_{\{\{4, 5\}, \{1, 2\}, \{3\}\}} \\ &- X_{\{\{2, 4, 5\}, \{1\}, \{3\}\}} - X_{\{\{5\}, \{2, 3\}, \{1, 4\}\}} + 12 X_{\{\{5\}, \{1\}, \{3, 4\}, \{2\}\}} - X_{\{\{3, 5\}, \{4\}, \{1, 2\}\}} \\ &- 6 X_{\{\{1\}, \{3, 5\}, \{4\}, \{2\}\}} - X_{\{\{4, 5\}, \{1, 3\}, \{2\}\}} - X_{\{\{5\}, \{1, 3, 4\}, \{2\}\}} + 8 X_{\{\{5\}, \{4\}, \{1, 2\}, \{3\}\}} \\ &- X_{\{\{1, 3, 5\}, \{2, 4\}\}} - 2 X_{\{\{1\}, \{3, 5\}, \{2, 4\}\}} - 10 X_{\{\{5\}, \{1\}, \{2, 4\}, \{3\}\}} + 8 X_{\{\{4, 5\}, \{1\}, \{2\}, \{3\}\}} \\ &+ 5 X_{\{\{4, 5\}, \{1\}, \{2, 3\}\}} + 2 X_{\{\{3, 4, 5\}, \{1\}, \{2\}\}} + X_{\{\{3, 4, 5\}, \{1, 2\}\}} + 2 X_{\{\{5\}, \{4\}, \{1, 2, 3\}\}} \end{aligned}$$

$$\begin{aligned}
& S(X_{\{\{4,5\}, \{1\}, \{2\}, \{3\}\}}) = 2 X_{\{\{5\}, \{1\}, \{4\}, \{2\}, \{3\}\}} + X_{\{\{5\}, \{4\}, \{1,2\}, \{3\}\}} \\
& S(X_{\{\{4,5\}, \{1,2\}, \{3\}\}}) = \\
& \quad -4 X_{\{\{5\}, \{1\}, \{4\}, \{2\}, \{3\}\}} - X_{\{\{4,5\}, \{1,2\}, \{3\}\}} - 2 X_{\{\{5\}, \{4\}, \{1,2\}, \{3\}\}} - 2 X_{\{\{4,5\}, \{1\}, \{2\}, \{3\}\}} \\
& S(X_{\{\{4,5\}, \{1,3\}, \{2\}\}}) = -4 X_{\{\{5\}, \{1\}, \{4\}, \{2\}, \{3\}\}} - X_{\{\{5\}, \{3,4\}, \{1,2\}\}} - X_{\{\{4,5\}, \{1,2\}, \{3\}\}} \\
& \quad - 2 X_{\{\{5\}, \{1\}, \{3,4\}, \{2\}\}} + X_{\{\{3,5\}, \{4\}, \{1,2\}\}} + 2 X_{\{\{1\}, \{3,5\}, \{4\}, \{2\}\}} - 2 X_{\{\{5\}, \{4\}, \{1,2\}, \{3\}\}} \\
& \quad - 2 X_{\{\{4,5\}, \{1\}, \{2\}, \{3\}\}} \\
& S(X_{\{\{4,5\}, \{1\}, \{2,3\}\}}) = \\
& \quad -4 X_{\{\{5\}, \{1\}, \{4\}, \{2\}, \{3\}\}} - X_{\{\{5\}, \{3,4\}, \{1,2\}\}} - 2 X_{\{\{5\}, \{1\}, \{3,4\}, \{2\}\}} - 2 X_{\{\{5\}, \{4\}, \{1,2\}, \{3\}\}} \\
& S(X_{\{\{4,5\}, \{1,2,3\}\}}) = 12 X_{\{\{5\}, \{1\}, \{4\}, \{2\}, \{3\}\}} + 3 X_{\{\{5\}, \{3,4\}, \{1,2\}\}} + 3 X_{\{\{4,5\}, \{1,2\}, \{3\}\}} \\
& \quad + 6 X_{\{\{5\}, \{1\}, \{3,4\}, \{2\}\}} + 6 X_{\{\{5\}, \{4\}, \{1,2\}, \{3\}\}} + 6 X_{\{\{4,5\}, \{1\}, \{2\}, \{3\}\}} + 2 X_{\{\{3,4,5\}, \{1\}, \{2\}\}} \\
& \quad + X_{\{\{3,4,5\}, \{1,2\}\}} \\
& S(X_{\{\{2,3,4,5\}, \{1\}\}}) = 26 X_{\{\{5\}, \{1\}, \{4\}, \{2\}, \{3\}\}} - 4 X_{\{\{5\}, \{1,3\}, \{4\}, \{2\}\}} + 4 X_{\{\{5\}, \{1\}, \{2,3,4\}\}} \\
& \quad + 6 X_{\{\{5\}, \{3,4\}, \{1,2\}\}} + X_{\{\{5\}, \{1,2,3,4\}\}} + 16 X_{\{\{5\}, \{1\}, \{4\}, \{2,3\}\}} + 14 X_{\{\{5\}, \{1\}, \{3,4\}, \{2\}\}} \\
& \quad + 14 X_{\{\{5\}, \{4\}, \{1,2\}, \{3\}\}} - 4 X_{\{\{5\}, \{1\}, \{2,4\}, \{3\}\}} + 4 X_{\{\{5\}, \{4\}, \{1,2,3\}\}} \\
& S(X_{\{\{1,2,4,5\}, \{3\}\}}) = 26 X_{\{\{5\}, \{1\}, \{4\}, \{2\}, \{3\}\}} - 14 X_{\{\{5\}, \{1,3\}, \{4\}, \{2\}\}} + 8 X_{\{\{5\}, \{1\}, \{2,3,4\}\}} \\
& \quad + 5 X_{\{\{5\}, \{3,4\}, \{1,2\}\}} - X_{\{\{1,2,4,5\}, \{3\}\}} + 4 X_{\{\{5\}, \{2\}, \{3\}, \{1,4\}\}} + X_{\{\{5\}, \{1,2,3,4\}\}} \\
& \quad - 2 X_{\{\{5\}, \{1,2,4\}, \{3\}\}} - 2 X_{\{\{2,3,5\}, \{1\}, \{4\}\}} + 25 X_{\{\{5\}, \{1\}, \{4\}, \{2,3\}\}} + 4 X_{\{\{4,5\}, \{1,2\}, \{3\}\}} \\
& \quad - 2 X_{\{\{2,4,5\}, \{1\}, \{3\}\}} + 25 X_{\{\{5\}, \{1\}, \{3,4\}, \{2\}\}} - 4 X_{\{\{3,5\}, \{4\}, \{1,2\}\}} - 14 X_{\{\{1\}, \{3,5\}, \{4\}, \{2\}\}} \\
& \quad - 4 X_{\{\{4,5\}, \{1,3\}, \{2\}\}} - 2 X_{\{\{5\}, \{1,3,4\}, \{2\}\}} + 13 X_{\{\{5\}, \{4\}, \{1,2\}, \{3\}\}} - 20 X_{\{\{5\}, \{1\}, \{2,4\}, \{3\}\}} \\
& \quad + 13 X_{\{\{4,5\}, \{1\}, \{2\}, \{3\}\}} + 5 X_{\{\{4,5\}, \{1\}, \{2,3\}\}} + 4 X_{\{\{3,4,5\}, \{1\}, \{2\}\}} + 4 X_{\{\{1\}, \{2,5\}, \{4\}, \{3\}\}} \\
& \quad + 4 X_{\{\{5\}, \{4\}, \{1,2,3\}\}} + X_{\{\{2,3,4,5\}, \{1\}\}} \\
& S(X_{\{\{2,4,5\}, \{1\}, \{3\}\}}) = -6 X_{\{\{5\}, \{1\}, \{4\}, \{2\}, \{3\}\}} + 2 X_{\{\{5\}, \{1,3\}, \{4\}, \{2\}\}} - X_{\{\{5\}, \{1\}, \{2,3,4\}\}} \\
& \quad - 5 X_{\{\{5\}, \{1\}, \{4\}, \{2,3\}\}} - 2 X_{\{\{5\}, \{1\}, \{3,4\}, \{2\}\}} + X_{\{\{5\}, \{1,3,4\}, \{2\}\}} - 3 X_{\{\{5\}, \{4\}, \{1,2\}, \{3\}\}} \\
& \quad + 2 X_{\{\{5\}, \{1\}, \{2,4\}, \{3\}\}} - X_{\{\{5\}, \{4\}, \{1,2,3\}\}} \\
& S(X_{\{\{2,4,5\}, \{1,3\}\}}) = 12 X_{\{\{5\}, \{1\}, \{4\}, \{2\}, \{3\}\}} + X_{\{\{4,5\}, \{1,2,3\}\}} - 3 X_{\{\{5\}, \{1,3\}, \{4\}, \{2\}\}} \\
& \quad + X_{\{\{5\}, \{1\}, \{2,3,4\}\}} + 3 X_{\{\{5\}, \{3,4\}, \{1,2\}\}} - 2 X_{\{\{5\}, \{1,3\}, \{2,4\}\}} + 10 X_{\{\{5\}, \{1\}, \{4\}, \{2,3\}\}} \\
& \quad + 4 X_{\{\{4,5\}, \{1,2\}, \{3\}\}} - X_{\{\{2,4,5\}, \{1\}, \{3\}\}} - X_{\{\{2,4,5\}, \{1,3\}\}} + 7 X_{\{\{5\}, \{1\}, \{3,4\}, \{2\}\}} \\
& \quad - X_{\{\{3,5\}, \{4\}, \{1,2\}\}} - 5 X_{\{\{1\}, \{3,5\}, \{4\}, \{2\}\}} - X_{\{\{4,5\}, \{1,3\}, \{2\}\}} - X_{\{\{5\}, \{1,3,4\}, \{2\}\}} \\
& \quad + 7 X_{\{\{5\}, \{4\}, \{1,2\}, \{3\}\}} - 2 X_{\{\{1\}, \{3,5\}, \{2,4\}\}} - 6 X_{\{\{5\}, \{1\}, \{2,4\}, \{3\}\}} + 8 X_{\{\{4,5\}, \{1\}, \{2\}, \{3\}\}} \\
& \quad + 5 X_{\{\{4,5\}, \{1\}, \{2,3\}\}} + X_{\{\{3,4,5\}, \{1\}, \{2\}\}} + X_{\{\{3,4,5\}, \{1,2\}\}} + 2 X_{\{\{5\}, \{4\}, \{1,2,3\}\}} \\
& S(X_{\{\{1,2,3,4,5\}\}}) = -150 X_{\{\{5\}, \{1\}, \{4\}, \{2\}, \{3\}\}} - 10 X_{\{\{4,5\}, \{1,2,3\}\}} + 45 X_{\{\{5\}, \{1,3\}, \{4\}, \{2\}\}} \\
& \quad - 30 X_{\{\{5\}, \{1\}, \{2,3,4\}\}} - 45 X_{\{\{5\}, \{3,4\}, \{1,2\}\}} - 5 X_{\{\{5\}, \{2\}, \{3\}, \{1,4\}\}} + 5 X_{\{\{1\}, \{2,5\}, \{3,4\}\}} \\
& \quad - 5 X_{\{\{5\}, \{1,2,3,4\}\}} + 5 X_{\{\{5\}, \{1,2,4\}, \{3\}\}} + 5 X_{\{\{5\}, \{1,3\}, \{2,4\}\}} + 5 X_{\{\{2,3,5\}, \{1\}, \{4\}\}} \\
& \quad - 120 X_{\{\{5\}, \{1\}, \{4\}, \{2,3\}\}} - 40 X_{\{\{4,5\}, \{1,2\}, \{3\}\}} + 5 X_{\{\{2,4,5\}, \{1\}, \{3\}\}} + 5 X_{\{\{5\}, \{2,3\}, \{1,4\}\}} \\
& \quad - 120 X_{\{\{5\}, \{1\}, \{3,4\}, \{2\}\}} + 10 X_{\{\{3,5\}, \{4\}, \{1,2\}\}} + 45 X_{\{\{1\}, \{3,5\}, \{4\}, \{2\}\}} \\
& \quad + 10 X_{\{\{4,5\}, \{1,3\}, \{2\}\}} + 5 X_{\{\{5\}, \{1,3,4\}, \{2\}\}} - 85 X_{\{\{5\}, \{4\}, \{1,2\}, \{3\}\}} + 5 X_{\{\{1\}, \{3,5\}, \{2,4\}\}} \\
& \quad + 70 X_{\{\{5\}, \{1\}, \{2,4\}, \{3\}\}} - X_{\{\{1,2,3,4,5\}\}} - 85 X_{\{\{4,5\}, \{1\}, \{2\}, \{3\}\}} - 45 X_{\{\{4,5\}, \{1\}, \{2,3\}\}} \\
& \quad - 25 X_{\{\{3,4,5\}, \{1\}, \{2\}\}} - 10 X_{\{\{3,4,5\}, \{1,2\}\}} - 5 X_{\{\{1\}, \{2,5\}, \{4\}, \{3\}\}} - 25 X_{\{\{5\}, \{4\}, \{1,2,3\}\}} \\
& \quad - 5 X_{\{\{2,3,4,5\}, \{1\}\}} \\
& S(X_{\{\{1,3,4,5\}, \{2\}\}}) = 26 X_{\{\{5\}, \{1\}, \{4\}, \{2\}, \{3\}\}} - 10 X_{\{\{5\}, \{1,3\}, \{4\}, \{2\}\}} + 7 X_{\{\{5\}, \{1\}, \{2,3,4\}\}}
\end{aligned}$$

$$\begin{aligned}
& + 6 X_{\{\{5\}, \{3,4\}, \{1,2\}\}} + 3 X_{\{\{5\}, \{2\}, \{3\}, \{1,4\}\}} + X_{\{\{5\}, \{1,2,3,4\}\}} + 22 X_{\{\{5\}, \{1\}, \{4\}, \{2,3\}\}} \\
& + 3 X_{\{\{4,5\}, \{1,2\}, \{3\}\}} - 3 X_{\{\{2,4,5\}, \{1\}, \{3\}\}} + 23 X_{\{\{5\}, \{1\}, \{3,4\}, \{2\}\}} - 3 X_{\{\{3,5\}, \{4\}, \{1,2\}\}} \\
& - 12 X_{\{\{1\}, \{3,5\}, \{4\}, \{2\}\}} - 3 X_{\{\{4,5\}, \{1,3\}, \{2\}\}} - 3 X_{\{\{5\}, \{1,3,4\}, \{2\}\}} + 14 X_{\{\{5\}, \{4\}, \{1,2\}, \{3\}\}} \\
& - 16 X_{\{\{5\}, \{1\}, \{2,4\}, \{3\}\}} - X_{\{\{1,3,4,5\}, \{2\}\}} + 9 X_{\{\{4,5\}, \{1\}, \{2\}, \{3\}\}} + 3 X_{\{\{4,5\}, \{1\}, \{2,3\}\}} \\
& + 3 X_{\{\{3,4,5\}, \{1\}, \{2\}\}} + 3 X_{\{\{1\}, \{2,5\}, \{4\}, \{3\}\}} + 4 X_{\{\{5\}, \{4\}, \{1,2,3\}\}} + X_{\{\{2,3,4,5\}, \{1\}\}} \\
\mathbf{S}(X_{\{\{3,4,5\}, \{1\}, \{2\}\}}) = & \\
& - 6 X_{\{\{5\}, \{1\}, \{4\}, \{2\}, \{3\}\}} - 3 X_{\{\{5\}, \{1\}, \{4\}, \{2,3\}\}} - 3 X_{\{\{5\}, \{4\}, \{1,2\}, \{3\}\}} - X_{\{\{5\}, \{4\}, \{1,2,3\}\}} \\
\mathbf{S}(X_{\{\{3,4,5\}, \{1,2\}\}}) = & 12 X_{\{\{5\}, \{1\}, \{4\}, \{2\}, \{3\}\}} + X_{\{\{4,5\}, \{1,2,3\}\}} + 6 X_{\{\{5\}, \{1\}, \{4\}, \{2,3\}\}} \\
& + 3 X_{\{\{4,5\}, \{1,2\}, \{3\}\}} + 6 X_{\{\{5\}, \{4\}, \{1,2\}, \{3\}\}} + 6 X_{\{\{4,5\}, \{1\}, \{2\}, \{3\}\}} + 3 X_{\{\{4,5\}, \{1\}, \{2,3\}\}} \\
& + 2 X_{\{\{5\}, \{4\}, \{1,2,3\}\}} \\
\mathbf{S}(X_{\{\{1\}, \{2,5\}, \{4\}, \{3\}\}}) = & 2 X_{\{\{5\}, \{1\}, \{4\}, \{2\}, \{3\}\}} - 2 X_{\{\{5\}, \{1,3\}, \{4\}, \{2\}\}} + X_{\{\{5\}, \{2\}, \{3\}, \{1,4\}\}} \\
& + 2 X_{\{\{5\}, \{1\}, \{4\}, \{2,3\}\}} + X_{\{\{5\}, \{1\}, \{3,4\}, \{2\}\}} + X_{\{\{5\}, \{4\}, \{1,2\}, \{3\}\}} - 2 X_{\{\{5\}, \{1\}, \{2,4\}, \{3\}\}} \\
\mathbf{S}(X_{\{\{2,5\}, \{1,3\}, \{4\}\}}) = & -4 X_{\{\{5\}, \{1\}, \{4\}, \{2\}, \{3\}\}} + 3 X_{\{\{5\}, \{1,3\}, \{4\}, \{2\}\}} - X_{\{\{2,5\}, \{1,3\}, \{4\}\}} \\
& - X_{\{\{5\}, \{3,4\}, \{1,2\}\}} - X_{\{\{5\}, \{2\}, \{3\}, \{1,4\}\}} + X_{\{\{5\}, \{1,3\}, \{2,4\}\}} - 4 X_{\{\{5\}, \{1\}, \{4\}, \{2,3\}\}} \\
& - X_{\{\{4,5\}, \{1,2\}, \{3\}\}} - 3 X_{\{\{5\}, \{1\}, \{3,4\}, \{2\}\}} + X_{\{\{3,5\}, \{4\}, \{1,2\}\}} + 3 X_{\{\{1\}, \{3,5\}, \{4\}, \{2\}\}} \\
& + X_{\{\{4,5\}, \{1,3\}, \{2\}\}} - 2 X_{\{\{5\}, \{4\}, \{1,2\}, \{3\}\}} + X_{\{\{1\}, \{3,5\}, \{2,4\}\}} + 4 X_{\{\{5\}, \{1\}, \{2,4\}, \{3\}\}} \\
& - 3 X_{\{\{4,5\}, \{1\}, \{2\}, \{3\}\}} - 2 X_{\{\{4,5\}, \{1\}, \{2,3\}\}} - X_{\{\{1\}, \{2,5\}, \{4\}, \{3\}\}} \\
\mathbf{S}(X_{\{\{2,5\}, \{3\}, \{1,4\}\}}) = & -4 X_{\{\{5\}, \{1\}, \{4\}, \{2\}, \{3\}\}} + 5 X_{\{\{5\}, \{1,3\}, \{4\}, \{2\}\}} - 2 X_{\{\{5\}, \{3,4\}, \{1,2\}\}} \\
& - X_{\{\{2,5\}, \{3\}, \{1,4\}\}} - 2 X_{\{\{5\}, \{2\}, \{3\}, \{1,4\}\}} + X_{\{\{5\}, \{1,3\}, \{2,4\}\}} - 5 X_{\{\{5\}, \{1\}, \{4\}, \{2,3\}\}} \\
& - 2 X_{\{\{4,5\}, \{1,2\}, \{3\}\}} - 5 X_{\{\{5\}, \{1\}, \{3,4\}, \{2\}\}} + 2 X_{\{\{3,5\}, \{4\}, \{1,2\}\}} + 5 X_{\{\{1\}, \{3,5\}, \{4\}, \{2\}\}} \\
& + 2 X_{\{\{4,5\}, \{1,3\}, \{2\}\}} - 3 X_{\{\{5\}, \{4\}, \{1,2\}, \{3\}\}} + X_{\{\{1\}, \{3,5\}, \{2,4\}\}} + 6 X_{\{\{5\}, \{1\}, \{2,4\}, \{3\}\}} \\
& - 3 X_{\{\{4,5\}, \{1\}, \{2\}, \{3\}\}} - 2 X_{\{\{4,5\}, \{1\}, \{2,3\}\}} - 2 X_{\{\{1\}, \{2,5\}, \{4\}, \{3\}\}} \\
\mathbf{S}(X_{\{\{2,5\}, \{1,3,4\}\}}) = & 12 X_{\{\{5\}, \{1\}, \{4\}, \{2\}, \{3\}\}} + X_{\{\{4,5\}, \{1,2,3\}\}} - 8 X_{\{\{5\}, \{1,3\}, \{4\}, \{2\}\}} \\
& + X_{\{\{5\}, \{1\}, \{2,3,4\}\}} + 6 X_{\{\{5\}, \{3,4\}, \{1,2\}\}} + X_{\{\{5\}, \{2\}, \{3\}, \{1,4\}\}} - X_{\{\{2,5\}, \{1,3,4\}\}} \\
& - X_{\{\{1\}, \{2,5\}, \{3,4\}\}} - 2 X_{\{\{5\}, \{1,3\}, \{2,4\}\}} + 11 X_{\{\{5\}, \{1\}, \{4\}, \{2,3\}\}} + 5 X_{\{\{4,5\}, \{1,2\}, \{3\}\}} \\
& - X_{\{\{2,4,5\}, \{1\}, \{3\}\}} - X_{\{\{5\}, \{2,3\}, \{1,4\}\}} + 12 X_{\{\{5\}, \{1\}, \{3,4\}, \{2\}\}} - 2 X_{\{\{3,5\}, \{4\}, \{1,2\}\}} \\
& - 6 X_{\{\{1\}, \{3,5\}, \{4\}, \{2\}\}} - 2 X_{\{\{4,5\}, \{1,3\}, \{2\}\}} - X_{\{\{5\}, \{1,3,4\}, \{2\}\}} + 9 X_{\{\{5\}, \{4\}, \{1,2\}, \{3\}\}} \\
& - 2 X_{\{\{1\}, \{3,5\}, \{2,4\}\}} - 10 X_{\{\{5\}, \{1\}, \{2,4\}, \{3\}\}} + 8 X_{\{\{4,5\}, \{1\}, \{2\}, \{3\}\}} + 5 X_{\{\{4,5\}, \{1\}, \{2,3\}\}} \\
& + 2 X_{\{\{3,4,5\}, \{1\}, \{2\}\}} + X_{\{\{3,4,5\}, \{1,2\}\}} + X_{\{\{1\}, \{2,5\}, \{4\}, \{3\}\}} + X_{\{\{5\}, \{4\}, \{1,2,3\}\}} \\
\mathbf{S}(X_{\{\{1\}, \{2,5\}, \{3,4\}\}}) = & -4 X_{\{\{5\}, \{1\}, \{4\}, \{2\}, \{3\}\}} + 2 X_{\{\{5\}, \{1,3\}, \{4\}, \{2\}\}} - 2 X_{\{\{5\}, \{3,4\}, \{1,2\}\}} \\
& - 2 X_{\{\{5\}, \{1\}, \{4\}, \{2,3\}\}} + X_{\{\{5\}, \{2,3\}, \{1,4\}\}} - 3 X_{\{\{5\}, \{1\}, \{3,4\}, \{2\}\}} - 3 X_{\{\{5\}, \{4\}, \{1,2\}, \{3\}\}} \\
& + 2 X_{\{\{5\}, \{1\}, \{2,4\}, \{3\}\}} \\
\mathbf{S}(X_{\{\{2,3,5\}, \{1\}, \{4\}\}}) = & -6 X_{\{\{5\}, \{1\}, \{4\}, \{2\}, \{3\}\}} + 2 X_{\{\{5\}, \{1,3\}, \{4\}, \{2\}\}} - X_{\{\{5\}, \{1\}, \{2,3,4\}\}} \\
& + X_{\{\{5\}, \{1,2,4\}, \{3\}\}} - 5 X_{\{\{5\}, \{1\}, \{4\}, \{2,3\}\}} - 3 X_{\{\{5\}, \{1\}, \{3,4\}, \{2\}\}} - 2 X_{\{\{5\}, \{4\}, \{1,2\}, \{3\}\}} \\
& + 2 X_{\{\{5\}, \{1\}, \{2,4\}, \{3\}\}} - X_{\{\{5\}, \{4\}, \{1,2,3\}\}} \\
\mathbf{S}(X_{\{\{2,3,5\}, \{1,4\}\}}) = & 12 X_{\{\{5\}, \{1\}, \{4\}, \{2\}, \{3\}\}} + X_{\{\{4,5\}, \{1,2,3\}\}} - 6 X_{\{\{5\}, \{1,3\}, \{4\}, \{2\}\}} \\
& + X_{\{\{5\}, \{1\}, \{2,3,4\}\}} + 5 X_{\{\{5\}, \{3,4\}, \{1,2\}\}} + X_{\{\{5\}, \{2\}, \{3\}, \{1,4\}\}} - X_{\{\{1\}, \{2,5\}, \{3,4\}\}} \\
& - X_{\{\{5\}, \{1,2,4\}, \{3\}\}} - 2 X_{\{\{5\}, \{1,3\}, \{2,4\}\}} - X_{\{\{2,3,5\}, \{1\}, \{4\}\}} + 12 X_{\{\{5\}, \{1\}, \{4\}, \{2,3\}\}} \\
& + 5 X_{\{\{4,5\}, \{1,2\}, \{3\}\}} - X_{\{\{2,3,5\}, \{1,4\}\}} - X_{\{\{5\}, \{2,3\}, \{1,4\}\}} + 11 X_{\{\{5\}, \{1\}, \{3,4\}, \{2\}\}} \\
& - 2 X_{\{\{3,5\}, \{4\}, \{1,2\}\}} - 8 X_{\{\{1\}, \{3,5\}, \{4\}, \{2\}\}} - 2 X_{\{\{4,5\}, \{1,3\}, \{2\}\}} + 8 X_{\{\{5\}, \{4\}, \{1,2\}, \{3\}\}}
\end{aligned}$$

$$\begin{aligned}
& -2 X_{\{\{1\}, \{3,5\}, \{2,4\}\}} - 10 X_{\{\{5\}, \{1\}, \{2,4\}, \{3\}\}} + 9 X_{\{\{4,5\}, \{1\}, \{2\}, \{3\}\}} + 6 X_{\{\{4,5\}, \{1\}, \{2,3\}\}} \\
& + X_{\{\{3,4,5\}, \{1\}, \{2\}\}} + X_{\{\{3,4,5\}, \{1,2\}\}} + X_{\{\{1\}, \{2,5\}, \{4\}, \{3\}\}} + 2 X_{\{\{5\}, \{4\}, \{1,2,3\}\}} \\
\mathbf{S}(X_{\{\{1,2,5\}, \{4\}, \{3\}\}}) &= -6 X_{\{\{5\}, \{1\}, \{4\}, \{2\}, \{3\}\}} + 4 X_{\{\{5\}, \{1,3\}, \{4\}, \{2\}\}} - 2 X_{\{\{5\}, \{1\}, \{2,3,4\}\}} \\
& - 2 X_{\{\{5\}, \{2\}, \{3\}, \{1,4\}\}} + 2 X_{\{\{5\}, \{1,2,4\}, \{3\}\}} + 2 X_{\{\{2,3,5\}, \{1\}, \{4\}\}} - 6 X_{\{\{5\}, \{1\}, \{4\}, \{2,3\}\}} \\
& - X_{\{\{1,2,5\}, \{4\}, \{3\}\}} - 7 X_{\{\{5\}, \{1\}, \{3,4\}, \{2\}\}} + 4 X_{\{\{1\}, \{3,5\}, \{4\}, \{2\}\}} - 2 X_{\{\{5\}, \{4\}, \{1,2\}, \{3\}\}} \\
& + 8 X_{\{\{5\}, \{1\}, \{2,4\}, \{3\}\}} - 3 X_{\{\{4,5\}, \{1\}, \{2\}, \{3\}\}} - X_{\{\{3,4,5\}, \{1\}, \{2\}\}} - 2 X_{\{\{1\}, \{2,5\}, \{4\}, \{3\}\}} \\
& - X_{\{\{5\}, \{4\}, \{1,2,3\}\}} \\
\mathbf{S}(X_{\{\{1,2,5\}, \{3,4\}\}}) &= 12 X_{\{\{5\}, \{1\}, \{4\}, \{2\}, \{3\}\}} + X_{\{\{4,5\}, \{1,2,3\}\}} - 4 X_{\{\{5\}, \{1,3\}, \{4\}, \{2\}\}} \\
& + 2 X_{\{\{5\}, \{1\}, \{2,3,4\}\}} + 4 X_{\{\{5\}, \{3,4\}, \{1,2\}\}} - 2 X_{\{\{1\}, \{2,5\}, \{3,4\}\}} - 2 X_{\{\{5\}, \{1,2,4\}, \{3\}\}} \\
& - 2 X_{\{\{2,3,5\}, \{1\}, \{4\}\}} + 9 X_{\{\{5\}, \{1\}, \{4\}, \{2,3\}\}} + 3 X_{\{\{4,5\}, \{1,2\}, \{3\}\}} - 2 X_{\{\{5\}, \{2,3\}, \{1,4\}\}} \\
& - X_{\{\{1,2,5\}, \{3,4\}\}} + 11 X_{\{\{5\}, \{1\}, \{3,4\}, \{2\}\}} - 4 X_{\{\{1\}, \{3,5\}, \{4\}, \{2\}\}} + 7 X_{\{\{5\}, \{4\}, \{1,2\}, \{3\}\}} \\
& - 8 X_{\{\{5\}, \{1\}, \{2,4\}, \{3\}\}} + 7 X_{\{\{4,5\}, \{1\}, \{2\}, \{3\}\}} + 3 X_{\{\{4,5\}, \{1\}, \{2,3\}\}} + 2 X_{\{\{3,4,5\}, \{1\}, \{2\}\}} \\
& + X_{\{\{3,4,5\}, \{1,2\}\}} + 2 X_{\{\{5\}, \{4\}, \{1,2,3\}\}} \\
\mathbf{S}(X_{\{\{1\}, \{3,5\}, \{4\}, \{2\}\}}) &= \\
& 2 X_{\{\{5\}, \{1\}, \{4\}, \{2\}, \{3\}\}} - X_{\{\{5\}, \{1,3\}, \{4\}, \{2\}\}} + X_{\{\{5\}, \{1\}, \{4\}, \{2,3\}\}} + X_{\{\{5\}, \{4\}, \{1,2\}, \{3\}\}} \\
\mathbf{S}(X_{\{\{3,5\}, \{4\}, \{1,2\}\}}) &= -4 X_{\{\{5\}, \{1\}, \{4\}, \{2\}, \{3\}\}} + 2 X_{\{\{5\}, \{1,3\}, \{4\}, \{2\}\}} \\
& - 2 X_{\{\{5\}, \{1\}, \{4\}, \{2,3\}\}} - X_{\{\{4,5\}, \{1,2\}, \{3\}\}} + X_{\{\{4,5\}, \{1,3\}, \{2\}\}} - 2 X_{\{\{5\}, \{4\}, \{1,2\}, \{3\}\}} \\
& - 2 X_{\{\{4,5\}, \{1\}, \{2\}, \{3\}\}} - X_{\{\{4,5\}, \{1\}, \{2,3\}\}} \\
\mathbf{S}(X_{\{\{3,5\}, \{2\}, \{1,4\}\}}) &= -4 X_{\{\{5\}, \{1\}, \{4\}, \{2\}, \{3\}\}} + 3 X_{\{\{5\}, \{1,3\}, \{4\}, \{2\}\}} - 2 X_{\{\{5\}, \{3,4\}, \{1,2\}\}} \\
& - X_{\{\{5\}, \{2\}, \{3\}, \{1,4\}\}} + X_{\{\{5\}, \{1,3\}, \{2,4\}\}} - 3 X_{\{\{5\}, \{1\}, \{4\}, \{2,3\}\}} - X_{\{\{4,5\}, \{1,2\}, \{3\}\}} \\
& - 4 X_{\{\{5\}, \{1\}, \{3,4\}, \{2\}\}} + X_{\{\{3,5\}, \{4\}, \{1,2\}\}} + 3 X_{\{\{1\}, \{3,5\}, \{4\}, \{2\}\}} - X_{\{\{3,5\}, \{2\}, \{1,4\}\}} \\
& + X_{\{\{4,5\}, \{1,3\}, \{2\}\}} - 3 X_{\{\{5\}, \{4\}, \{1,2\}, \{3\}\}} + X_{\{\{1\}, \{3,5\}, \{2,4\}\}} + 4 X_{\{\{5\}, \{1\}, \{2,4\}, \{3\}\}} \\
& - 2 X_{\{\{4,5\}, \{1\}, \{2\}, \{3\}\}} - X_{\{\{4,5\}, \{1\}, \{2,3\}\}} - X_{\{\{1\}, \{2,5\}, \{4\}, \{3\}\}} \\
\mathbf{S}(X_{\{\{1\}, \{3,5\}, \{2,4\}\}}) &= -4 X_{\{\{5\}, \{1\}, \{4\}, \{2\}, \{3\}\}} + 2 X_{\{\{5\}, \{1,3\}, \{4\}, \{2\}\}} - 2 X_{\{\{5\}, \{3,4\}, \{1,2\}\}} \\
& + X_{\{\{5\}, \{1,3\}, \{2,4\}\}} - 2 X_{\{\{5\}, \{1\}, \{4\}, \{2,3\}\}} - 3 X_{\{\{5\}, \{1\}, \{3,4\}, \{2\}\}} - 3 X_{\{\{5\}, \{4\}, \{1,2\}, \{3\}\}} \\
& + 2 X_{\{\{5\}, \{1\}, \{2,4\}, \{3\}\}} \\
\mathbf{S}(X_{\{\{3,5\}, \{1,2,4\}\}}) &= 12 X_{\{\{5\}, \{1\}, \{4\}, \{2\}, \{3\}\}} + X_{\{\{4,5\}, \{1,2,3\}\}} - 5 X_{\{\{5\}, \{1,3\}, \{4\}, \{2\}\}} \\
& + X_{\{\{5\}, \{1\}, \{2,3,4\}\}} + 5 X_{\{\{5\}, \{3,4\}, \{1,2\}\}} - X_{\{\{5\}, \{1,2,4\}, \{3\}\}} - 2 X_{\{\{5\}, \{1,3\}, \{2,4\}\}} \\
& - X_{\{\{2,3,5\}, \{1\}, \{4\}\}} + 7 X_{\{\{5\}, \{1\}, \{4\}, \{2,3\}\}} + 4 X_{\{\{4,5\}, \{1,2\}, \{3\}\}} + 10 X_{\{\{5\}, \{1\}, \{3,4\}, \{2\}\}} \\
& - X_{\{\{3,5\}, \{4\}, \{1,2\}\}} - 3 X_{\{\{1\}, \{3,5\}, \{4\}, \{2\}\}} - X_{\{\{4,5\}, \{1,3\}, \{2\}\}} + 8 X_{\{\{5\}, \{4\}, \{1,2\}, \{3\}\}} \\
& - X_{\{\{3,5\}, \{1,2,4\}\}} - 2 X_{\{\{1\}, \{3,5\}, \{2,4\}\}} - 6 X_{\{\{5\}, \{1\}, \{2,4\}, \{3\}\}} + 7 X_{\{\{4,5\}, \{1\}, \{2\}, \{3\}\}} \\
& + 3 X_{\{\{4,5\}, \{1\}, \{2,3\}\}} + 2 X_{\{\{3,4,5\}, \{1\}, \{2\}\}} + X_{\{\{3,4,5\}, \{1,2\}\}} + X_{\{\{5\}, \{4\}, \{1,2,3\}\}} \\
\mathbf{S}(X_{\{\{1,4,5\}, \{2\}, \{3\}\}}) &= -6 X_{\{\{5\}, \{1\}, \{4\}, \{2\}, \{3\}\}} + 4 X_{\{\{5\}, \{1,3\}, \{4\}, \{2\}\}} - 2 X_{\{\{5\}, \{1\}, \{2,3,4\}\}} \\
& - 2 X_{\{\{5\}, \{2\}, \{3\}, \{1,4\}\}} - 7 X_{\{\{5\}, \{1\}, \{4\}, \{2,3\}\}} + 2 X_{\{\{2,4,5\}, \{1\}, \{3\}\}} - 6 X_{\{\{5\}, \{1\}, \{3,4\}, \{2\}\}} \\
& + 4 X_{\{\{1\}, \{3,5\}, \{4\}, \{2\}\}} + 2 X_{\{\{5\}, \{1,3,4\}, \{2\}\}} - 3 X_{\{\{5\}, \{4\}, \{1,2\}, \{3\}\}} + 8 X_{\{\{5\}, \{1\}, \{2,4\}, \{3\}\}} \\
& - X_{\{\{1,4,5\}, \{2\}, \{3\}\}} - 2 X_{\{\{4,5\}, \{1\}, \{2\}, \{3\}\}} - X_{\{\{3,4,5\}, \{1\}, \{2\}\}} - 2 X_{\{\{1\}, \{2,5\}, \{4\}, \{3\}\}} \\
& - X_{\{\{5\}, \{4\}, \{1,2,3\}\}} \\
\mathbf{S}(X_{\{\{1,4,5\}, \{2,3\}\}}) &= 12 X_{\{\{5\}, \{1\}, \{4\}, \{2\}, \{3\}\}} + X_{\{\{4,5\}, \{1,2,3\}\}} - 4 X_{\{\{5\}, \{1,3\}, \{4\}, \{2\}\}} \\
& + 2 X_{\{\{5\}, \{1\}, \{2,3,4\}\}} + 3 X_{\{\{5\}, \{3,4\}, \{1,2\}\}} - 2 X_{\{\{1\}, \{2,5\}, \{3,4\}\}} + 11 X_{\{\{5\}, \{1\}, \{4\}, \{2,3\}\}} \\
& + 3 X_{\{\{4,5\}, \{1,2\}, \{3\}\}} - 2 X_{\{\{2,4,5\}, \{1\}, \{3\}\}} - 2 X_{\{\{5\}, \{2,3\}, \{1,4\}\}} + 9 X_{\{\{5\}, \{1\}, \{3,4\}, \{2\}\}}
\end{aligned}$$

```

- 4 X_{\{1\},\{3,5\},\{4\},\{2\}} - 2 X_{\{5\},\{1,3,4\},\{2\}} + 7 X_{\{5\},\{4\},\{1,2\},\{3\}} - 8 X_{\{5\},\{1\},\{2,4\},\{3\}}
- X_{\{1,4,5\},\{2,3\}} + 7 X_{\{4,5\},\{1\},\{2\},\{3\}} + 4 X_{\{4,5\},\{1\},\{2,3\}} + 2 X_{\{3,4,5\},\{1\},\{2\}}
+ X_{\{3,4,5\},\{1,2\}} + 2 X_{\{5\},\{4\},\{1,2,3\}}
S(X_{\{1,2,3,5\},\{4\}}) = 26 X_{\{5\},\{1\},\{4\},\{2\},\{3\}} - 12 X_{\{5\},\{1,3\},\{4\},\{2\}} + 7 X_{\{5\},\{1\},\{2,3,4\}}
+ 3 X_{\{5\},\{3,4\},\{1,2\}} + 3 X_{\{5\},\{2\},\{3\},\{1,4\}} + X_{\{5\},\{1,2,3,4\}} - 3 X_{\{5\},\{1,2,4\},\{3\}}
- 3 X_{\{2,3,5\},\{1\},\{4\}} + 23 X_{\{5\},\{1\},\{4\},\{2,3\}} + 3 X_{\{4,5\},\{1,2\},\{3\}} + 22 X_{\{5\},\{1\},\{3,4\},\{2\}}
- 3 X_{\{3,5\},\{4\},\{1,2\}} - 10 X_{\{1\},\{3,5\},\{4\},\{2\}} - 3 X_{\{4,5\},\{1,3\},\{2\}} + 9 X_{\{5\},\{4\},\{1,2\},\{3\}}
- 16 X_{\{5\},\{1\},\{2,4\},\{3\}} - X_{\{1,2,3,5\},\{4\}} + 14 X_{\{4,5\},\{1\},\{2\},\{3\}} + 6 X_{\{4,5\},\{1\},\{2,3\}}
+ 4 X_{\{3,4,5\},\{1\},\{2\}} + 3 X_{\{1\},\{2,5\},\{4\},\{3\}} + 3 X_{\{5\},\{4\},\{1,2,3\}} + X_{\{2,3,4,5\},\{1\}}

```

[-] looking for a pattern for antipode on the P-basis

```

> for i from 1 to 5 do
  print(S(P[{seq({j},j=1..i)]})=
    ToP(antipode(P[{seq({j},j=1..i)]))));
od;

          S(P_{\{1\}}) = -P_{\{1\}}
          S(P_{\{1\},\{2\}}) = P_{\{1\},\{2\}}
          S(P_{\{1\},\{2\},\{3\}}) = -P_{\{1\},\{2\},\{3\}}
          S(P_{\{1\},\{4\},\{2\},\{3\}}) = P_{\{1\},\{4\},\{2\},\{3\}}
          S(P_{\{5\},\{1\},\{4\},\{2\},\{3\}}) = -P_{\{5\},\{1\},\{4\},\{2\},\{3\}}
> for i from 1 to 5 do
  print(S(P[{{seq(j,j=1..i)}}])=
    ToP(antipode(P[{{seq(j,j=1..i)}}])));
od;

          S(P_{\{1\}}) = -P_{\{1\}}
          S(P_{\{1,2\}}) = -P_{\{1,2\}}
          S(P_{\{1,2,3\}}) = -P_{\{1,2,3\}}
          S(P_{\{1,2,3,4\}}) = -P_{\{1,2,3,4\}}
          S(P_{\{1,2,3,4,5\}}) = -P_{\{1,2,3,4,5\}}

```

[-] n=2 antipode P-basis

```

> for A in listallsp(2) do
  print(S(P[A]) = ToP(antipode(P[A])));
od;

          S(P_{\{1\},\{2\}}) = P_{\{1\},\{2\}}
          S(P_{\{1,2\}}) = -P_{\{1,2\}}

```

[-] n=3 antipode P-basis

```

> for A in listallsp(3) do
  print(S(P[A]) = ToP(antipode(P[A])));
od;

          S(P_{\{1\},\{2\},\{3\}}) = -P_{\{1\},\{2\},\{3\}}

```

$$\begin{aligned}
S(P_{\{\{1,2\},\{3\}\}}) &= P_{\{\{1\},\{2,3\}\}} \\
S(P_{\{\{1,3\},\{2\}\}}) &= -P_{\{\{1,3\},\{2\}\}} + P_{\{\{1,2\},\{3\}\}} + P_{\{\{1\},\{2,3\}\}} \\
S(P_{\{\{1\},\{2,3\}\}}) &= P_{\{\{1,2\},\{3\}\}} \\
S(P_{\{\{1,2,3\}\}}) &= -P_{\{\{1,2,3\}\}}
\end{aligned}$$

>

n=4 antipode P-basis

```

> for A in listallsp(4) do
  print(S(P[A]) = ToP(antipode(P[A])));
od;

```

$$\begin{aligned}
S(P_{\{\{1\},\{4\},\{2\},\{3\}\}}) &= P_{\{\{1\},\{4\},\{2\},\{3\}\}} \\
S(P_{\{\{4\},\{1,2\},\{3\}\}}) &= -P_{\{\{1\},\{3,4\},\{2\}\}} \\
S(P_{\{\{1,3\},\{4\},\{2\}\}}) &= -P_{\{\{1\},\{4\},\{2,3\}\}} - P_{\{\{1\},\{3,4\},\{2\}\}} + P_{\{\{1\},\{2,4\},\{3\}\}} \\
S(P_{\{\{1\},\{4\},\{2,3\}\}}) &= -P_{\{\{1\},\{4\},\{2,3\}\}} \\
S(P_{\{\{4\},\{1,2,3\}\}}) &= P_{\{\{1\},\{2,3,4\}\}} \\
S(P_{\{\{2\},\{3\},\{1,4\}\}}) &= 2 P_{\{\{1,3\},\{4\},\{2\}\}} - P_{\{\{4\},\{1,2\},\{3\}\}} - 2 P_{\{\{1\},\{4\},\{2,3\}\}} \\
&\quad - P_{\{\{1\},\{3,4\},\{2\}\}} - P_{\{\{2\},\{3\},\{1,4\}\}} + 2 P_{\{\{1\},\{2,4\},\{3\}\}} \\
S(P_{\{\{2,3\},\{1,4\}\}}) &= -P_{\{\{2,3\},\{1,4\}\}} + 2 P_{\{\{3,4\},\{1,2\}\}} \\
S(P_{\{\{1,3,4\},\{2\}\}}) &= -P_{\{\{1,3,4\},\{2\}\}} + P_{\{\{4\},\{1,2,3\}\}} + P_{\{\{1\},\{2,3,4\}\}} \\
S(P_{\{\{1\},\{2,4\},\{3\}\}}) &= P_{\{\{1,3\},\{4\},\{2\}\}} - P_{\{\{4\},\{1,2\},\{3\}\}} - P_{\{\{1\},\{4\},\{2,3\}\}} \\
S(P_{\{\{1,3\},\{2,4\}\}}) &= -P_{\{\{1,3\},\{2,4\}\}} + 2 P_{\{\{3,4\},\{1,2\}\}} \\
S(P_{\{\{1\},\{2,3,4\}\}}) &= P_{\{\{4\},\{1,2,3\}\}} \\
S(P_{\{\{1,2,4\},\{3\}\}}) &= -P_{\{\{1,2,4\},\{3\}\}} + P_{\{\{4\},\{1,2,3\}\}} + P_{\{\{1\},\{2,3,4\}\}} \\
S(P_{\{\{1\},\{3,4\},\{2\}\}}) &= -P_{\{\{4\},\{1,2\},\{3\}\}} \\
S(P_{\{\{3,4\},\{1,2\}\}}) &= P_{\{\{3,4\},\{1,2\}\}} \\
S(P_{\{\{1,2,3,4\}\}}) &= -P_{\{\{1,2,3,4\}\}}
\end{aligned}$$

>

n=5 antipode P-basis

```

> for A in listallsp(5) do
  print(S(P[A]) = ToP(antipode(P[A])));
od;

```

$$\begin{aligned}
S(P_{\{\{5\},\{1\},\{4\},\{2\},\{3\}\}}) &= -P_{\{\{5\},\{1\},\{4\},\{2\},\{3\}\}} \\
S(P_{\{\{5\},\{4\},\{1,2\},\{3\}\}}) &= P_{\{\{4,5\},\{1\},\{2\},\{3\}\}} \\
S(P_{\{\{5\},\{1,3\},\{4\},\{2\}\}}) &= -P_{\{\{1\},\{3,5\},\{4\},\{2\}\}} + P_{\{\{4,5\},\{1\},\{2\},\{3\}\}} + P_{\{\{5\},\{1\},\{3,4\},\{2\}\}} \\
S(P_{\{\{5\},\{1\},\{4\},\{2,3\}\}}) &= P_{\{\{5\},\{1\},\{3,4\},\{2\}\}} \\
S(P_{\{\{5\},\{4\},\{1,2,3\}\}}) &= -P_{\{\{3,4,5\},\{1\},\{2\}\}} \\
S(P_{\{\{5\},\{2\},\{3\},\{1,4\}\}}) &= -2 P_{\{\{1\},\{3,5\},\{4\},\{2\}\}} + P_{\{\{4,5\},\{1\},\{2\},\{3\}\}} + P_{\{\{1\},\{2,5\},\{4\},\{3\}\}} \\
&\quad - 2 P_{\{\{5\},\{1\},\{2,4\},\{3\}\}} + P_{\{\{5\},\{1\},\{4\},\{2,3\}\}} + 2 P_{\{\{5\},\{1\},\{3,4\},\{2\}\}} \\
S(P_{\{\{5\},\{2,3\},\{1,4\}\}}) &= P_{\{\{1\},\{2,5\},\{3,4\}\}} - 2 P_{\{\{4,5\},\{1\},\{2,3\}\}} \\
S(P_{\{\{5\},\{1,3,4\},\{2\}\}}) &= -P_{\{\{5\},\{1\},\{2,3,4\}\}} - P_{\{\{3,4,5\},\{1\},\{2\}\}} + P_{\{\{2,4,5\},\{1\},\{3\}\}} \\
S(P_{\{\{5\},\{1\},\{2,4\},\{3\}\}}) &= P_{\{\{5\},\{1\},\{4\},\{2,3\}\}} - P_{\{\{5\},\{1\},\{2,4\},\{3\}\}} + P_{\{\{5\},\{1\},\{3,4\},\{2\}\}} \\
S(P_{\{\{5\},\{1,3\},\{2,4\}\}}) &= P_{\{\{1\},\{3,5\},\{2,4\}\}} - 2 P_{\{\{4,5\},\{1\},\{2,3\}\}}
\end{aligned}$$

$$\begin{aligned}
& S(P_{\{\{5\}, \{1\}, \{2,3,4\}\}}) = -P_{\{\{5\}, \{1\}, \{2,3,4\}\}} \\
& S(P_{\{\{5\}, \{1,2,4\}, \{3\}\}}) = -P_{\{\{5\}, \{1\}, \{2,3,4\}\}} - P_{\{\{3,4,5\}, \{1\}, \{2\}\}} + P_{\{\{2,3,5\}, \{1\}, \{4\}\}} \\
& S(P_{\{\{5\}, \{1\}, \{3,4\}, \{2\}\}}) = P_{\{\{5\}, \{1\}, \{4\}, \{2,3\}\}} \\
& S(P_{\{\{5\}, \{3,4\}, \{1,2\}\}}) = -P_{\{\{4,5\}, \{1\}, \{2,3\}\}} \\
& S(P_{\{\{5\}, \{1,2,3,4\}\}}) = P_{\{\{2,3,4,5\}, \{1\}\}} \\
& S(P_{\{\{4\}, \{2\}, \{3\}, \{1,5\}\}}) = -3 P_{\{\{1\}, \{3,5\}, \{4\}, \{2\}\}} + 3 P_{\{\{5\}, \{1\}, \{4\}, \{2,3\}\}} \\
& \quad + 3 P_{\{\{5\}, \{1\}, \{3,4\}, \{2\}\}} + 3 P_{\{\{1\}, \{2,5\}, \{4\}, \{3\}\}} - 6 P_{\{\{5\}, \{1\}, \{2,4\}, \{3\}\}} - 3 P_{\{\{5\}, \{1,3\}, \{4\}, \{2\}\}} \\
& \quad + 3 P_{\{\{5\}, \{2\}, \{3\}, \{1,4\}\}} + P_{\{\{5\}, \{4\}, \{1,2\}, \{3\}\}} - P_{\{\{4\}, \{2\}, \{3\}, \{1,5\}\}} + P_{\{\{4,5\}, \{1\}, \{2\}, \{3\}\}} \\
& S(P_{\{\{4\}, \{2,3\}, \{1,5\}\}}) = P_{\{\{4,5\}, \{1,3\}, \{2\}\}} + P_{\{\{3,5\}, \{4\}, \{1,2\}\}} - P_{\{\{4,5\}, \{1,2\}, \{3\}\}} \\
& \quad - P_{\{\{5\}, \{3,4\}, \{1,2\}\}} + P_{\{\{1\}, \{2,5\}, \{3,4\}\}} - 2 P_{\{\{4,5\}, \{1\}, \{2,3\}\}} + P_{\{\{5\}, \{2,3\}, \{1,4\}\}} \\
& \quad - P_{\{\{4\}, \{2,3\}, \{1,5\}\}} \\
& S(P_{\{\{2,4\}, \{3\}, \{1,5\}\}}) = 2 P_{\{\{4,5\}, \{1,3\}, \{2\}\}} + 2 P_{\{\{3,5\}, \{4\}, \{1,2\}\}} - P_{\{\{2,4\}, \{3\}, \{1,5\}\}} \\
& \quad - 2 P_{\{\{4,5\}, \{1,2\}, \{3\}\}} - 2 P_{\{\{5\}, \{3,4\}, \{1,2\}\}} + P_{\{\{1\}, \{2,5\}, \{3,4\}\}} - 2 P_{\{\{4,5\}, \{1\}, \{2,3\}\}} \\
& \quad + P_{\{\{5\}, \{2,3\}, \{1,4\}\}} \\
& \quad S(P_{\{\{2,3,4\}, \{1,5\}\}}) = -P_{\{\{2,3,4\}, \{1,5\}\}} + P_{\{\{3,4,5\}, \{1,2\}\}} + P_{\{\{4,5\}, \{1,2,3\}\}} \\
& S(P_{\{\{3,4\}, \{2\}, \{1,5\}\}}) = -P_{\{\{3,4\}, \{2\}, \{1,5\}\}} - 2 P_{\{\{5\}, \{3,4\}, \{1,2\}\}} - P_{\{\{4,5\}, \{1,2\}, \{3\}\}} \\
& \quad + P_{\{\{5\}, \{2,3\}, \{1,4\}\}} + P_{\{\{1\}, \{2,5\}, \{3,4\}\}} - P_{\{\{4,5\}, \{1\}, \{2,3\}\}} + P_{\{\{3,5\}, \{4\}, \{1,2\}\}} \\
& \quad + P_{\{\{4,5\}, \{1,3\}, \{2\}\}} \\
& S(P_{\{\{1,3,5\}, \{4\}, \{2\}\}}) = P_{\{\{2,4,5\}, \{1\}, \{3\}\}} - 2 P_{\{\{5\}, \{1\}, \{2,3,4\}\}} - P_{\{\{1,3,5\}, \{4\}, \{2\}\}} \\
& \quad + P_{\{\{2,3,5\}, \{1\}, \{4\}\}} - P_{\{\{5\}, \{4\}, \{1,2,3\}\}} + P_{\{\{5\}, \{1,2,4\}, \{3\}\}} + P_{\{\{5\}, \{1,3,4\}, \{2\}\}} \\
& \quad - P_{\{\{3,4,5\}, \{1\}, \{2\}\}} \\
& \quad S(P_{\{\{1,3,5\}, \{2,4\}\}}) = -P_{\{\{1,3,5\}, \{2,4\}\}} + P_{\{\{3,4,5\}, \{1,2\}\}} + P_{\{\{4,5\}, \{1,2,3\}\}} \\
& \quad S(P_{\{\{4,5\}, \{1\}, \{2\}, \{3\}\}}) = P_{\{\{5\}, \{4\}, \{1,2\}, \{3\}\}} \\
& \quad S(P_{\{\{4,5\}, \{1,2\}, \{3\}\}}) = -P_{\{\{4,5\}, \{1,2\}, \{3\}\}} \\
& S(P_{\{\{4,5\}, \{1,3\}, \{2\}\}}) = -P_{\{\{5\}, \{3,4\}, \{1,2\}\}} + P_{\{\{3,5\}, \{4\}, \{1,2\}\}} - P_{\{\{4,5\}, \{1,2\}, \{3\}\}} \\
& \quad S(P_{\{\{4,5\}, \{1\}, \{2,3\}\}}) = -P_{\{\{5\}, \{3,4\}, \{1,2\}\}} \\
& \quad S(P_{\{\{4,5\}, \{1,2,3\}\}}) = P_{\{\{3,4,5\}, \{1,2\}\}} \\
& \quad S(P_{\{\{2,3,4,5\}, \{1\}\}}) = P_{\{\{5\}, \{1,2,3,4\}\}} \\
& S(P_{\{\{1,2,4,5\}, \{3\}\}}) = -P_{\{\{1,2,4,5\}, \{3\}\}} + P_{\{\{2,3,4,5\}, \{1\}\}} + P_{\{\{5\}, \{1,2,3,4\}\}} \\
& S(P_{\{\{2,4,5\}, \{1\}, \{3\}\}}) = -P_{\{\{5\}, \{4\}, \{1,2,3\}\}} + P_{\{\{5\}, \{1,3,4\}, \{2\}\}} - P_{\{\{5\}, \{1\}, \{2,3,4\}\}} \\
& S(P_{\{\{2,4,5\}, \{1,3\}\}}) = -P_{\{\{2,4,5\}, \{1,3\}\}} + P_{\{\{3,4,5\}, \{1,2\}\}} + P_{\{\{4,5\}, \{1,2,3\}\}} \\
& \quad S(P_{\{\{1,2,3,4,5\}\}}) = -P_{\{\{1,2,3,4,5\}\}} \\
& S(P_{\{\{1,3,4,5\}, \{2\}\}}) = -P_{\{\{1,3,4,5\}, \{2\}\}} + P_{\{\{2,3,4,5\}, \{1\}\}} + P_{\{\{5\}, \{1,2,3,4\}\}} \\
& \quad S(P_{\{\{3,4,5\}, \{1\}, \{2\}\}}) = -P_{\{\{5\}, \{4\}, \{1,2,3\}\}} \\
& \quad S(P_{\{\{3,4,5\}, \{1,2\}\}}) = P_{\{\{4,5\}, \{1,2,3\}\}} \\
& S(P_{\{\{1\}, \{2,5\}, \{4\}, \{3\}\}}) = 2 P_{\{\{5\}, \{1\}, \{4\}, \{2,3\}\}} - 2 P_{\{\{5\}, \{1,3\}, \{4\}, \{2\}\}} + P_{\{\{5\}, \{2\}, \{3\}, \{1,4\}\}} \\
& \quad - 2 P_{\{\{5\}, \{1\}, \{2,4\}, \{3\}\}} + P_{\{\{5\}, \{4\}, \{1,2\}, \{3\}\}} + P_{\{\{5\}, \{1\}, \{3,4\}, \{2\}\}} \\
& S(P_{\{\{2,5\}, \{1,3\}, \{4\}\}}) = P_{\{\{5\}, \{1,3\}, \{2,4\}\}} + P_{\{\{3,5\}, \{4\}, \{1,2\}\}} + P_{\{\{4,5\}, \{1,3\}, \{2\}\}} \\
& \quad + P_{\{\{1\}, \{3,5\}, \{2,4\}\}} - P_{\{\{2,5\}, \{1,3\}, \{4\}\}} - P_{\{\{4,5\}, \{1,2\}, \{3\}\}} - P_{\{\{5\}, \{3,4\}, \{1,2\}\}} \\
& \quad - 2 P_{\{\{4,5\}, \{1\}, \{2,3\}\}}
\end{aligned}$$

$$\begin{aligned}
S(P_{\{\{2,5\},\{3\},\{1,4\}\}}) &= P_{\{\{5\},\{1,3\},\{2,4\}\}} + 2 P_{\{\{3,5\},\{4\},\{1,2\}\}} + 2 P_{\{\{4,5\},\{1,3\},\{2\}\}} \\
&+ P_{\{\{1\},\{3,5\},\{2,4\}\}} - 2 P_{\{\{4,5\},\{1,2\},\{3\}\}} - 2 P_{\{\{5\},\{3,4\},\{1,2\}\}} - P_{\{\{2,5\},\{3\},\{1,4\}\}} \\
&- 2 P_{\{\{4,5\},\{1\},\{2,3\}\}} \\
S(P_{\{\{2,5\},\{1,3,4\}\}}) &= -P_{\{\{2,5\},\{1,3,4\}\}} + P_{\{\{3,4,5\},\{1,2\}\}} + P_{\{\{4,5\},\{1,2,3\}\}} \\
S(P_{\{\{1\},\{2,5\},\{3,4\}\}}) &= -2 P_{\{\{5\},\{3,4\},\{1,2\}\}} + P_{\{\{5\},\{2,3\},\{1,4\}\}} \\
S(P_{\{\{2,3,5\},\{1\},\{4\}\}}) &= P_{\{\{5\},\{1,2,4\},\{3\}\}} - P_{\{\{5\},\{4\},\{1,2,3\}\}} - P_{\{\{5\},\{1\},\{2,3,4\}\}} \\
S(P_{\{\{2,3,5\},\{1,4\}\}}) &= -P_{\{\{2,3,5\},\{1,4\}\}} + P_{\{\{3,4,5\},\{1,2\}\}} + P_{\{\{4,5\},\{1,2,3\}\}} \\
S(P_{\{\{1,2,5\},\{4\},\{3\}\}}) &= 2 P_{\{\{5\},\{1,2,4\},\{3\}\}} - P_{\{\{5\},\{4\},\{1,2,3\}\}} - P_{\{\{1,2,5\},\{4\},\{3\}\}} \\
&+ 2 P_{\{\{2,3,5\},\{1\},\{4\}\}} - 2 P_{\{\{5\},\{1\},\{2,3,4\}\}} - P_{\{\{3,4,5\},\{1\},\{2\}\}} \\
S(P_{\{\{1,2,5\},\{3,4\}\}}) &= -P_{\{\{1,2,5\},\{3,4\}\}} + P_{\{\{3,4,5\},\{1,2\}\}} + P_{\{\{4,5\},\{1,2,3\}\}} \\
S(P_{\{\{1\},\{3,5\},\{4\},\{2\}\}}) &= P_{\{\{5\},\{1\},\{4\},\{2,3\}\}} - P_{\{\{5\},\{1,3\},\{4\},\{2\}\}} + P_{\{\{5\},\{4\},\{1,2\},\{3\}\}} \\
S(P_{\{\{3,5\},\{4\},\{1,2\}\}}) &= -P_{\{\{4,5\},\{1\},\{2,3\}\}} - P_{\{\{4,5\},\{1,2\},\{3\}\}} + P_{\{\{4,5\},\{1,3\},\{2\}\}} \\
S(P_{\{\{3,5\},\{2\},\{1,4\}\}}) &= -P_{\{\{4,5\},\{1\},\{2,3\}\}} + P_{\{\{1\},\{3,5\},\{2,4\}\}} - 2 P_{\{\{5\},\{3,4\},\{1,2\}\}} \\
&- P_{\{\{3,5\},\{2\},\{1,4\}\}} + P_{\{\{3,5\},\{4\},\{1,2\}\}} - P_{\{\{4,5\},\{1,2\},\{3\}\}} + P_{\{\{4,5\},\{1,3\},\{2\}\}} \\
&+ P_{\{\{5\},\{1,3\},\{2,4\}\}} \\
S(P_{\{\{1\},\{3,5\},\{2,4\}\}}) &= -2 P_{\{\{5\},\{3,4\},\{1,2\}\}} + P_{\{\{5\},\{1,3\},\{2,4\}\}} \\
S(P_{\{\{3,5\},\{1,2,4\}\}}) &= -P_{\{\{3,5\},\{1,2,4\}\}} + P_{\{\{4,5\},\{1,2,3\}\}} + P_{\{\{3,4,5\},\{1,2\}\}} \\
S(P_{\{\{1,4,5\},\{2\},\{3\}\}}) &= 2 P_{\{\{5\},\{1,3,4\},\{2\}\}} + 2 P_{\{\{2,4,5\},\{1\},\{3\}\}} - P_{\{\{1,4,5\},\{2\},\{3\}\}} \\
&- 2 P_{\{\{5\},\{1\},\{2,3,4\}\}} - P_{\{\{5\},\{4\},\{1,2,3\}\}} - P_{\{\{3,4,5\},\{1\},\{2\}\}} \\
S(P_{\{\{1,4,5\},\{2,3\}\}}) &= -P_{\{\{1,4,5\},\{2,3\}\}} + P_{\{\{4,5\},\{1,2,3\}\}} + P_{\{\{3,4,5\},\{1,2\}\}} \\
S(P_{\{\{1,2,3,5\},\{4\}\}}) &= -P_{\{\{1,2,3,5\},\{4\}\}} + P_{\{\{2,3,4,5\},\{1\}\}} + P_{\{\{5\},\{1,2,3,4\}\}}
\end{aligned}$$

>

— looking for a pattern for antipode on the M-basis

```

> for i from 1 to 5 do
  print(S(M[seq({j},j=1..i)]))=
  ToM(antipode(M[seq({j},j=1..i)]));
od;

```

$$\begin{aligned}
S(M_{\{\{1\}\}}) &= -M_{\{\{1\}\}} \\
S(M_{\{\{1\},\{2\}\}}) &= M_{\{\{1\},\{2\}\}} + 2 M_{\{\{1,2\}\}} \\
S(M_{\{\{1\},\{2\},\{3\}\}}) &= -M_{\{\{1\},\{2\},\{3\}\}} - 3 M_{\{\{1\},\{2,3\}\}} - 3 M_{\{\{1,2\},\{3\}\}} - 6 M_{\{\{1,2,3\}\}} \\
S(M_{\{\{1\},\{4\},\{2\},\{3\}\}}) &= M_{\{\{1\},\{4\},\{2\},\{3\}\}} - 2 M_{\{\{1\},\{2,4\},\{3\}\}} + 2 M_{\{\{2\},\{3\},\{1,4\}\}} \\
&+ 4 M_{\{\{1\},\{3,4\},\{2\}\}} + 4 M_{\{\{4\},\{1,2\},\{3\}\}} - 2 M_{\{\{1,3\},\{4\},\{2\}\}} + 12 M_{\{\{3,4\},\{1,2\}\}} \\
&+ 12 M_{\{\{4\},\{1,2,3\}\}} + 6 M_{\{\{1\},\{4\},\{2,3\}\}} + 6 M_{\{\{2,3\},\{1,4\}\}} + 12 M_{\{\{1\},\{2,3,4\}\}} \\
&+ 24 M_{\{\{1,2,3,4\}\}} - 6 M_{\{\{1,3\},\{2,4\}\}} \\
S(M_{\{\{5\},\{1\},\{4\},\{2\},\{3\}\}}) &= -10 M_{\{\{5\},\{1\},\{4\},\{2,3\}\}} + 5 M_{\{\{5\},\{1,3\},\{4\},\{2\}\}} \\
&- 5 M_{\{\{5\},\{4\},\{1,2\},\{3\}\}} - M_{\{\{5\},\{1\},\{4\},\{2\},\{3\}\}} - 5 M_{\{\{5\},\{2\},\{3\},\{1,4\}\}} \\
&- 20 M_{\{\{5\},\{4\},\{1,2,3\}\}} - 10 M_{\{\{5\},\{2,3\},\{1,4\}\}} + 20 M_{\{\{5\},\{1,3\},\{2,4\}\}} \\
&+ 10 M_{\{\{5\},\{1\},\{2,4\},\{3\}\}} + 10 M_{\{\{5\},\{1,2,4\},\{3\}\}} - 30 M_{\{\{5\},\{1\},\{2,3,4\}\}} \\
&+ 10 M_{\{\{2,4\},\{3\},\{1,5\}\}} - 10 M_{\{\{4\},\{2,3\},\{1,5\}\}} - 60 M_{\{\{5\},\{1,2,3,4\}\}} - 30 M_{\{\{5\},\{3,4\},\{1,2\}\}} \\
&- 10 M_{\{\{5\},\{1\},\{3,4\},\{2\}\}} - 30 M_{\{\{2,3,4\},\{1,5\}\}} + 10 M_{\{\{1,3,5\},\{4\},\{2\}\}} - 10 M_{\{\{3,4\},\{2\},\{1,5\}\}} \\
&- 5 M_{\{\{4,5\},\{1\},\{2\},\{3\}\}} + 30 M_{\{\{1,3,5\},\{2,4\}\}} - 20 M_{\{\{4,5\},\{1,2\},\{3\}\}} - 30 M_{\{\{4,5\},\{1\},\{2,3\}\}}
\end{aligned}$$

$$\begin{aligned}
& + 10 M_{\{\{4,5\}, \{1,3\}, \{2\}\}} - 60 M_{\{\{2,3,4,5\}, \{1\}\}} - 60 M_{\{\{4,5\}, \{1,2,3\}\}} + 10 M_{\{\{2,4,5\}, \{1\}, \{3\}\}} \\
& + 30 M_{\{\{2,4,5\}, \{1,3\}\}} - 120 M_{\{\{1,2,3,4,5\}\}} - 20 M_{\{\{3,4,5\}, \{1\}, \{2\}\}} - 5 M_{\{\{1\}, \{2,5\}, \{4\}, \{3\}\}} \\
& - 60 M_{\{\{3,4,5\}, \{1,2\}\}} - 10 M_{\{\{2,5\}, \{3\}, \{1,4\}\}} - 10 M_{\{\{1\}, \{2,5\}, \{3,4\}\}} - 30 M_{\{\{1,2,5\}, \{3,4\}\}} \\
& - 10 M_{\{\{1,2,5\}, \{4\}, \{3\}\}} + 10 M_{\{\{3,5\}, \{4\}, \{1,2\}\}} + 5 M_{\{\{1\}, \{3,5\}, \{4\}, \{2\}\}} + 20 M_{\{\{1\}, \{3,5\}, \{2,4\}\}} \\
& - 30 M_{\{\{1,4,5\}, \{2,3\}\}} - 10 M_{\{\{1,4,5\}, \{2\}, \{3\}\}} + 30 M_{\{\{3,5\}, \{1,2,4\}\}}
\end{aligned}$$

>

- n=2 antipode M-basis

```
> for A in listallsp(2) do
  print(S(M[A]) = ToM(antipode(M[A])));
od;
```

$$S(M_{\{\{1\}, \{2\}\}}) = M_{\{\{1\}, \{2\}\}} + 2 M_{\{\{1,2\}\}}$$

$$S(M_{\{\{1,2\}\}}) = -M_{\{\{1,2\}\}}$$

>

- n=3 antipode M-basis

```
> for A in listallsp(3) do
  print(S(M[A]) = ToM(antipode(M[A])));
od;
```

$$S(M_{\{\{1\}, \{2\}, \{3\}\}}) = -M_{\{\{1\}, \{2\}, \{3\}\}} - 3 M_{\{\{1\}, \{2,3\}\}} - 3 M_{\{\{1,2\}, \{3\}\}} - 6 M_{\{\{1,2,3\}\}}$$

$$S(M_{\{\{1,2\}, \{3\}\}}) = 2 M_{\{\{1,2,3\}\}} + M_{\{\{1\}, \{2,3\}\}}$$

$$S(M_{\{\{1,3\}, \{2\}\}}) = -M_{\{\{1,3\}, \{2\}\}} + M_{\{\{1,2\}, \{3\}\}} + 2 M_{\{\{1,2,3\}\}} + M_{\{\{1\}, \{2,3\}\}}$$

$$S(M_{\{\{1\}, \{2,3\}\}}) = M_{\{\{1,2\}, \{3\}\}} + 2 M_{\{\{1,2,3\}\}}$$

$$S(M_{\{\{1,2,3\}\}}) = -M_{\{\{1,2,3\}\}}$$

>

- n=4 antipode M-basis

```
> for A in listallsp(4) do
  print(S(M[A]) = ToM(antipode(M[A])));
od;
```

$$S(M_{\{\{1\}, \{4\}, \{2\}, \{3\}\}}) = M_{\{\{1\}, \{4\}, \{2\}, \{3\}\}} - 2 M_{\{\{1\}, \{2,4\}, \{3\}\}} + 2 M_{\{\{2\}, \{3\}, \{1,4\}\}}$$

$$+ 4 M_{\{\{1\}, \{3,4\}, \{2\}\}} + 4 M_{\{\{4\}, \{1,2\}, \{3\}\}} - 2 M_{\{\{1,3\}, \{4\}, \{2\}\}} + 12 M_{\{\{3,4\}, \{1,2\}\}}$$

$$+ 12 M_{\{\{4\}, \{1,2,3\}\}} + 6 M_{\{\{1\}, \{4\}, \{2,3\}\}} + 6 M_{\{\{2,3\}, \{1,4\}\}} + 12 M_{\{\{1\}, \{2,3,4\}\}}$$

$$+ 24 M_{\{\{1,2,3,4\}\}} - 6 M_{\{\{1,3\}, \{2,4\}\}}$$

$$S(M_{\{\{4\}, \{1,2\}, \{3\}\}}) = M_{\{\{1,2,4\}, \{3\}\}} - M_{\{\{4\}, \{1,2,3\}\}} - 6 M_{\{\{1,2,3,4\}\}} - 3 M_{\{\{1\}, \{2,3,4\}\}}$$

$$- 2 M_{\{\{3,4\}, \{1,2\}\}} - M_{\{\{1\}, \{3,4\}, \{2\}\}} - M_{\{\{1,3,4\}, \{2\}\}}$$

$$S(M_{\{\{1,3\}, \{4\}, \{2\}\}}) = 2 M_{\{\{1,3\}, \{2,4\}\}} - 3 M_{\{\{3,4\}, \{1,2\}\}} - 2 M_{\{\{4\}, \{1,2,3\}\}} - 6 M_{\{\{1,2,3,4\}\}}$$

$$+ M_{\{\{1\}, \{2,4\}, \{3\}\}} + M_{\{\{1,2,4\}, \{3\}\}} - M_{\{\{1\}, \{4\}, \{2,3\}\}} - M_{\{\{2,3\}, \{1,4\}\}} - 3 M_{\{\{1\}, \{2,3,4\}\}}$$

$$- M_{\{\{1\}, \{3,4\}, \{2\}\}}$$

$$S(M_{\{\{1\}, \{4\}, \{2,3\}\}}) =$$

$$-2 M_{\{\{1\}, \{2,3,4\}\}} - 2 M_{\{\{3,4\}, \{1,2\}\}} - 2 M_{\{\{4\}, \{1,2,3\}\}} - 6 M_{\{\{1,2,3,4\}\}} - M_{\{\{1\}, \{4\}, \{2,3\}\}}$$

$$S(M_{\{\{4\}, \{1,2,3\}\}}) = 2 M_{\{\{1,2,3,4\}\}} + M_{\{\{1\}, \{2,3,4\}\}}$$

$$S(M_{\{\{2\}, \{3\}, \{1,4\}\}}) = -M_{\{\{2\}, \{3\}, \{1,4\}\}} + 2 M_{\{\{1,3\}, \{4\}, \{2\}\}} + 4 M_{\{\{1,3\}, \{2,4\}\}}$$

$$- 3 M_{\{\{4\}, \{1,2,3\}\}} - M_{\{\{4\}, \{1,2\}, \{3\}\}} + M_{\{\{1,2,4\}, \{3\}\}} - 4 M_{\{\{3,4\}, \{1,2\}\}} - 6 M_{\{\{1,2,3,4\}\}}$$

$$\begin{aligned}
& + 2 M_{\{\{1\}, \{2, 4\}, \{3\}\}} - 2 M_{\{\{1\}, \{4\}, \{2, 3\}\}} - 2 M_{\{\{2, 3\}, \{1, 4\}\}} - 3 M_{\{\{1\}, \{2, 3, 4\}\}} \\
& - M_{\{\{1\}, \{3, 4\}, \{2\}\}} + M_{\{\{1, 3, 4\}, \{2\}\}} \\
& \quad S(M_{\{\{2, 3\}, \{1, 4\}\}}) = -M_{\{\{2, 3\}, \{1, 4\}\}} + 2 M_{\{\{3, 4\}, \{1, 2\}\}} + 2 M_{\{\{1, 2, 3, 4\}\}} \\
& \quad S(M_{\{\{1, 3, 4\}, \{2\}\}}) = -M_{\{\{1, 3, 4\}, \{2\}\}} + M_{\{\{4\}, \{1, 2, 3\}\}} + 2 M_{\{\{1, 2, 3, 4\}\}} + M_{\{\{1\}, \{2, 3, 4\}\}} \\
S(M_{\{\{1\}, \{2, 4\}, \{3\}\}}) &= M_{\{\{1, 3\}, \{4\}, \{2\}\}} + 2 M_{\{\{1, 3\}, \{2, 4\}\}} + M_{\{\{1, 3, 4\}, \{2\}\}} - 3 M_{\{\{4\}, \{1, 2, 3\}\}} \\
& - 6 M_{\{\{1, 2, 3, 4\}\}} - 3 M_{\{\{3, 4\}, \{1, 2\}\}} - M_{\{\{1\}, \{4\}, \{2, 3\}\}} - M_{\{\{2, 3\}, \{1, 4\}\}} - 2 M_{\{\{1\}, \{2, 3, 4\}\}} \\
& - M_{\{\{4\}, \{1, 2\}, \{3\}\}} \\
& \quad S(M_{\{\{1, 3\}, \{2, 4\}\}}) = -M_{\{\{1, 3\}, \{2, 4\}\}} + 2 M_{\{\{3, 4\}, \{1, 2\}\}} + 2 M_{\{\{1, 2, 3, 4\}\}} \\
& \quad S(M_{\{\{1\}, \{2, 3, 4\}\}}) = M_{\{\{4\}, \{1, 2, 3\}\}} + 2 M_{\{\{1, 2, 3, 4\}\}} \\
& \quad S(M_{\{\{1, 2, 4\}, \{3\}\}}) = -M_{\{\{1, 2, 4\}, \{3\}\}} + M_{\{\{4\}, \{1, 2, 3\}\}} + 2 M_{\{\{1, 2, 3, 4\}\}} + M_{\{\{1\}, \{2, 3, 4\}\}} \\
S(M_{\{\{1\}, \{3, 4\}, \{2\}\}}) &= -M_{\{\{1\}, \{2, 3, 4\}\}} + M_{\{\{1, 3, 4\}, \{2\}\}} - 3 M_{\{\{4\}, \{1, 2, 3\}\}} - 6 M_{\{\{1, 2, 3, 4\}\}} \\
& - M_{\{\{4\}, \{1, 2\}, \{3\}\}} - M_{\{\{1, 2, 4\}, \{3\}\}} - 2 M_{\{\{3, 4\}, \{1, 2\}\}} \\
& \quad S(M_{\{\{3, 4\}, \{1, 2\}\}}) = M_{\{\{3, 4\}, \{1, 2\}\}} + 2 M_{\{\{1, 2, 3, 4\}\}} \\
& \quad S(M_{\{\{1, 2, 3, 4\}\}}) = -M_{\{\{1, 2, 3, 4\}\}}
\end{aligned}$$

>

- n=5 antipode M-basis

```

> for A in listallsp(5) do
  print(S(M[A]) = ToM(antipode(M[A])));
od;

```

$$\begin{aligned}
S(M_{\{\{5\}, \{1\}, \{4\}, \{2\}, \{3\}\}}) &= -10 M_{\{\{5\}, \{1\}, \{4\}, \{2, 3\}\}} + 5 M_{\{\{5\}, \{1, 3\}, \{4\}, \{2\}\}} \\
& - 5 M_{\{\{5\}, \{4\}, \{1, 2\}, \{3\}\}} - M_{\{\{5\}, \{1\}, \{4\}, \{2\}, \{3\}\}} - 5 M_{\{\{5\}, \{2\}, \{3\}, \{1, 4\}\}} \\
& - 20 M_{\{\{5\}, \{4\}, \{1, 2, 3\}\}} - 10 M_{\{\{5\}, \{2, 3\}, \{1, 4\}\}} + 20 M_{\{\{5\}, \{1, 3\}, \{2, 4\}\}} \\
& + 10 M_{\{\{5\}, \{1\}, \{2, 4\}, \{3\}\}} + 10 M_{\{\{5\}, \{1, 2, 4\}, \{3\}\}} - 30 M_{\{\{5\}, \{1\}, \{2, 3, 4\}\}} \\
& + 10 M_{\{\{2, 4\}, \{3\}, \{1, 5\}\}} - 10 M_{\{\{4\}, \{2, 3\}, \{1, 5\}\}} - 60 M_{\{\{5\}, \{1, 2, 3, 4\}\}} - 30 M_{\{\{5\}, \{3, 4\}, \{1, 2\}\}} \\
& - 10 M_{\{\{5\}, \{1\}, \{3, 4\}, \{2\}\}} - 30 M_{\{\{2, 3, 4\}, \{1, 5\}\}} + 10 M_{\{\{1, 3, 5\}, \{4\}, \{2\}\}} - 10 M_{\{\{3, 4\}, \{2\}, \{1, 5\}\}} \\
& - 5 M_{\{\{4, 5\}, \{1\}, \{2\}, \{3\}\}} + 30 M_{\{\{1, 3, 5\}, \{2, 4\}\}} - 20 M_{\{\{4, 5\}, \{1, 2\}, \{3\}\}} - 30 M_{\{\{4, 5\}, \{1\}, \{2, 3\}\}} \\
& + 10 M_{\{\{4, 5\}, \{1, 3\}, \{2\}\}} - 60 M_{\{\{2, 3, 4, 5\}, \{1\}\}} - 60 M_{\{\{4, 5\}, \{1, 2, 3\}\}} + 10 M_{\{\{2, 4, 5\}, \{1\}, \{3\}\}} \\
& + 30 M_{\{\{2, 4, 5\}, \{1, 3\}\}} - 120 M_{\{\{1, 2, 3, 4, 5\}\}} - 20 M_{\{\{3, 4, 5\}, \{1\}, \{2\}\}} - 5 M_{\{\{1\}, \{2, 5\}, \{4\}, \{3\}\}} \\
& - 60 M_{\{\{3, 4, 5\}, \{1, 2\}\}} - 10 M_{\{\{2, 5\}, \{3\}, \{1, 4\}\}} - 10 M_{\{\{1\}, \{2, 5\}, \{3, 4\}\}} - 30 M_{\{\{1, 2, 5\}, \{3, 4\}\}} \\
& - 10 M_{\{\{1, 2, 5\}, \{4\}, \{3\}\}} + 10 M_{\{\{3, 5\}, \{4\}, \{1, 2\}\}} + 5 M_{\{\{1\}, \{3, 5\}, \{4\}, \{2\}\}} + 20 M_{\{\{1\}, \{3, 5\}, \{2, 4\}\}} \\
& - 30 M_{\{\{1, 4, 5\}, \{2, 3\}\}} - 10 M_{\{\{1, 4, 5\}, \{2\}, \{3\}\}} + 30 M_{\{\{3, 5\}, \{1, 2, 4\}\}} \\
S(M_{\{\{5\}, \{4\}, \{1, 2\}, \{3\}\}}) &= M_{\{\{5\}, \{4\}, \{1, 2, 3\}\}} - 2 M_{\{\{5\}, \{1, 2, 4\}, \{3\}\}} + 3 M_{\{\{5\}, \{1\}, \{2, 3, 4\}\}} \\
& + 6 M_{\{\{5\}, \{1, 2, 3, 4\}\}} + 3 M_{\{\{2, 3, 4\}, \{1, 5\}\}} + 3 M_{\{\{4, 5\}, \{1, 2\}, \{3\}\}} + 3 M_{\{\{4, 5\}, \{1\}, \{2, 3\}\}} \\
& + 12 M_{\{\{2, 3, 4, 5\}, \{1\}\}} + 9 M_{\{\{4, 5\}, \{1, 2, 3\}\}} + 24 M_{\{\{1, 2, 3, 4, 5\}\}} + 3 M_{\{\{1, 3, 4, 5\}, \{2\}\}} \\
& + 4 M_{\{\{3, 4, 5\}, \{1\}, \{2\}\}} + 9 M_{\{\{3, 4, 5\}, \{1, 2\}\}} + M_{\{\{1, 2, 5\}, \{4\}, \{3\}\}} - 3 M_{\{\{2, 3, 5\}, \{1, 4\}\}} \\
& - 3 M_{\{\{2, 3, 5\}, \{1\}, \{4\}\}} + 3 M_{\{\{1, 4, 5\}, \{2, 3\}\}} - 3 M_{\{\{3, 5\}, \{1, 2, 4\}\}} - 3 M_{\{\{1, 2, 3, 5\}, \{4\}\}} \\
& + M_{\{\{4, 5\}, \{1\}, \{2\}, \{3\}\}} + M_{\{\{2, 4, 5\}, \{1\}, \{3\}\}} + M_{\{\{1, 4, 5\}, \{2\}, \{3\}\}} \\
S(M_{\{\{5\}, \{1, 3\}, \{4\}, \{2\}\}}) &= M_{\{\{5\}, \{4\}, \{1, 2, 3\}\}} - M_{\{\{5\}, \{1, 3\}, \{2, 4\}\}} - M_{\{\{5\}, \{1, 2, 4\}, \{3\}\}} \\
& + 4 M_{\{\{5\}, \{1\}, \{2, 3, 4\}\}} + 8 M_{\{\{5\}, \{1, 2, 3, 4\}\}} + 3 M_{\{\{5\}, \{3, 4\}, \{1, 2\}\}} + M_{\{\{5\}, \{1\}, \{3, 4\}, \{2\}\}} \\
& + 4 M_{\{\{2, 3, 4\}, \{1, 5\}\}} + M_{\{\{3, 4\}, \{2\}, \{1, 5\}\}} + M_{\{\{4, 5\}, \{1\}, \{2\}, \{3\}\}} - 4 M_{\{\{1, 3, 5\}, \{2, 4\}\}}
\end{aligned}$$

$$\begin{aligned}
& + 3 M_{\{\{4,5\}, \{1,2\}, \{3\}\}} + 5 M_{\{\{4,5\}, \{1\}, \{2,3\}\}} + 12 M_{\{\{2,3,4,5\}, \{1\}\}} + 11 M_{\{\{4,5\}, \{1,2,3\}\}} \\
& - M_{\{\{2,4,5\}, \{1\}, \{3\}\}} - 4 M_{\{\{2,4,5\}, \{1,3\}\}} + 24 M_{\{\{1,2,3,4,5\}\}} + M_{\{\{1,3,4,5\}, \{2\}\}} \\
& + 4 M_{\{\{3,4,5\}, \{1\}, \{2\}\}} + 11 M_{\{\{3,4,5\}, \{1,2\}\}} + M_{\{\{1\}, \{2,5\}, \{3,4\}\}} + M_{\{\{2,5\}, \{1,3\}, \{4\}\}} \\
& + 3 M_{\{\{1,2,5\}, \{3,4\}\}} - 2 M_{\{\{2,3,5\}, \{1,4\}\}} - 2 M_{\{\{2,3,5\}, \{1\}, \{4\}\}} - 3 M_{\{\{3,5\}, \{4\}, \{1,2\}\}} \\
& - M_{\{\{1\}, \{3,5\}, \{4\}, \{2\}\}} - 3 M_{\{\{1\}, \{3,5\}, \{2,4\}\}} - M_{\{\{3,5\}, \{2\}, \{1,4\}\}} + 5 M_{\{\{1,4,5\}, \{2,3\}\}} \\
& + M_{\{\{1,4,5\}, \{2\}, \{3\}\}} - 6 M_{\{\{3,5\}, \{1,2,4\}\}} - 3 M_{\{\{1,2,3,5\}, \{4\}\}} \\
S(M_{\{\{5\}, \{1\}, \{4\}, \{2,3\}\}}) & = M_{\{\{5\}, \{4\}, \{1,2,3\}\}} + M_{\{\{5\}, \{1,3,4\}, \{2\}\}} - M_{\{\{5\}, \{2,3\}, \{1,4\}\}} \\
& - M_{\{\{5\}, \{1,2,4\}, \{3\}\}} + 3 M_{\{\{5\}, \{1\}, \{2,3,4\}\}} + M_{\{\{4\}, \{2,3\}, \{1,5\}\}} + 8 M_{\{\{5\}, \{1,2,3,4\}\}} \\
& + 3 M_{\{\{5\}, \{3,4\}, \{1,2\}\}} + M_{\{\{5\}, \{1\}, \{3,4\}, \{2\}\}} + 3 M_{\{\{2,3,4\}, \{1,5\}\}} + M_{\{\{3,4\}, \{2\}, \{1,5\}\}} \\
& + M_{\{\{4,5\}, \{1,2\}, \{3\}\}} + 4 M_{\{\{4,5\}, \{1\}, \{2,3\}\}} - M_{\{\{4,5\}, \{1,3\}, \{2\}\}} + 10 M_{\{\{2,3,4,5\}, \{1\}\}} \\
& + 9 M_{\{\{4,5\}, \{1,2,3\}\}} - M_{\{\{2,4,5\}, \{1,3\}\}} + 24 M_{\{\{1,2,3,4,5\}\}} + M_{\{\{1,3,4,5\}, \{2\}\}} \\
& + 2 M_{\{\{3,4,5\}, \{1\}, \{2\}\}} + 9 M_{\{\{3,4,5\}, \{1,2\}\}} - M_{\{\{2,5\}, \{1,3,4\}\}} - M_{\{\{1\}, \{2,5\}, \{3,4\}\}} \\
& + M_{\{\{1,2,5\}, \{3,4\}\}} - 2 M_{\{\{2,3,5\}, \{1,4\}\}} - M_{\{\{3,5\}, \{4\}, \{1,2\}\}} + 2 M_{\{\{1,4,5\}, \{2,3\}\}} \\
& - 2 M_{\{\{3,5\}, \{1,2,4\}\}} - M_{\{\{1,2,3,5\}, \{4\}\}} \\
S(M_{\{\{5\}, \{4\}, \{1,2,3\}\}}) & = M_{\{\{1,2,3,5\}, \{4\}\}} - M_{\{\{5\}, \{1,2,3,4\}\}} - 6 M_{\{\{1,2,3,4,5\}\}} \\
& - 3 M_{\{\{2,3,4,5\}, \{1\}\}} - M_{\{\{3,4,5\}, \{1\}, \{2\}\}} - 2 M_{\{\{3,4,5\}, \{1,2\}\}} - M_{\{\{1,3,4,5\}, \{2\}\}} \\
S(M_{\{\{5\}, \{2\}, \{3\}, \{1,4\}\}}) & = M_{\{\{5\}, \{1\}, \{4\}, \{2,3\}\}} + 2 M_{\{\{5\}, \{4\}, \{1,2,3\}\}} + M_{\{\{5\}, \{2,3\}, \{1,4\}\}} \\
& - 4 M_{\{\{5\}, \{1,3\}, \{2,4\}\}} - 2 M_{\{\{5\}, \{1\}, \{2,4\}, \{3\}\}} - 2 M_{\{\{5\}, \{1,2,4\}, \{3\}\}} + 5 M_{\{\{5\}, \{1\}, \{2,3,4\}\}} \\
& - 2 M_{\{\{2,4\}, \{3\}, \{1,5\}\}} + M_{\{\{4\}, \{2,3\}, \{1,5\}\}} + 10 M_{\{\{5\}, \{1,2,3,4\}\}} + 6 M_{\{\{5\}, \{3,4\}, \{1,2\}\}} \\
& + 2 M_{\{\{5\}, \{1\}, \{3,4\}, \{2\}\}} + 5 M_{\{\{2,3,4\}, \{1,5\}\}} - 2 M_{\{\{1,3,5\}, \{4\}, \{2\}\}} + 2 M_{\{\{3,4\}, \{2\}, \{1,5\}\}} \\
& + M_{\{\{4,5\}, \{1\}, \{2\}, \{3\}\}} - 8 M_{\{\{1,3,5\}, \{2,4\}\}} + 4 M_{\{\{4,5\}, \{1,2\}, \{3\}\}} + 7 M_{\{\{4,5\}, \{1\}, \{2,3\}\}} \\
& - 2 M_{\{\{4,5\}, \{1,3\}, \{2\}\}} + 12 M_{\{\{2,3,4,5\}, \{1\}\}} + 13 M_{\{\{4,5\}, \{1,2,3\}\}} - 3 M_{\{\{2,4,5\}, \{1\}, \{3\}\}} \\
& - 8 M_{\{\{2,4,5\}, \{1,3\}\}} + 24 M_{\{\{1,2,3,4,5\}\}} - M_{\{\{1,3,4,5\}, \{2\}\}} + 4 M_{\{\{3,4,5\}, \{1\}, \{2\}\}} \\
& + M_{\{\{1\}, \{2,5\}, \{4\}, \{3\}\}} + 13 M_{\{\{3,4,5\}, \{1,2\}\}} + 2 M_{\{\{2,5\}, \{3\}, \{1,4\}\}} + 2 M_{\{\{1\}, \{2,5\}, \{3,4\}\}} \\
& + M_{\{\{2,5\}, \{1,3\}, \{4\}\}} + 6 M_{\{\{1,2,5\}, \{3,4\}\}} + M_{\{\{1,2,5\}, \{4\}, \{3\}\}} - M_{\{\{2,3,5\}, \{1,4\}\}} \\
& - M_{\{\{2,3,5\}, \{1\}, \{4\}\}} - 5 M_{\{\{3,5\}, \{4\}, \{1,2\}\}} - 2 M_{\{\{1\}, \{3,5\}, \{4\}, \{2\}\}} - 6 M_{\{\{1\}, \{3,5\}, \{2,4\}\}} \\
& - M_{\{\{3,5\}, \{2\}, \{1,4\}\}} + 7 M_{\{\{1,4,5\}, \{2,3\}\}} + 2 M_{\{\{1,4,5\}, \{2\}, \{3\}\}} - 9 M_{\{\{3,5\}, \{1,2,4\}\}} \\
& - 3 M_{\{\{1,2,3,5\}, \{4\}\}} \\
S(M_{\{\{5\}, \{2,3\}, \{1,4\}\}}) & = -M_{\{\{1,4,5\}, \{2,3\}\}} + M_{\{\{2,3,5\}, \{1,4\}\}} + M_{\{\{1\}, \{2,5\}, \{3,4\}\}} \\
& + M_{\{\{2,5\}, \{1,3,4\}\}} + M_{\{\{1,2,5\}, \{3,4\}\}} - 2 M_{\{\{4,5\}, \{1\}, \{2,3\}\}} - 4 M_{\{\{4,5\}, \{1,2,3\}\}} \\
& - 2 M_{\{\{2,3,4,5\}, \{1\}\}} - 6 M_{\{\{1,2,3,4,5\}\}} - 2 M_{\{\{3,4,5\}, \{1,2\}\}} \\
S(M_{\{\{5\}, \{1,3,4\}, \{2\}\}}) & = M_{\{\{2,5\}, \{1,3,4\}\}} - M_{\{\{4,5\}, \{1,2,3\}\}} - 2 M_{\{\{5\}, \{1,2,3,4\}\}} \\
& - 6 M_{\{\{1,2,3,4,5\}\}} - 3 M_{\{\{2,3,4,5\}, \{1\}\}} - 2 M_{\{\{3,4,5\}, \{1,2\}\}} + M_{\{\{2,4,5\}, \{1\}, \{3\}\}} \\
& + M_{\{\{2,4,5\}, \{1,3\}\}} + M_{\{\{1,2,4,5\}, \{3\}\}} - M_{\{\{5\}, \{1\}, \{2,3,4\}\}} - M_{\{\{2,3,4\}, \{1,5\}\}} \\
& - M_{\{\{3,4,5\}, \{1\}, \{2\}\}} \\
S(M_{\{\{5\}, \{1\}, \{2,4\}, \{3\}\}}) & = M_{\{\{5\}, \{1\}, \{4\}, \{2,3\}\}} + 2 M_{\{\{5\}, \{4\}, \{1,2,3\}\}} - 2 M_{\{\{5\}, \{1,3\}, \{2,4\}\}} \\
& - M_{\{\{5\}, \{1\}, \{2,4\}, \{3\}\}} - M_{\{\{5\}, \{1,2,4\}, \{3\}\}} + 4 M_{\{\{5\}, \{1\}, \{2,3,4\}\}} + M_{\{\{4\}, \{2,3\}, \{1,5\}\}} \\
& + 10 M_{\{\{5\}, \{1,2,3,4\}\}} + 5 M_{\{\{5\}, \{3,4\}, \{1,2\}\}} + M_{\{\{5\}, \{1\}, \{3,4\}, \{2\}\}} + 4 M_{\{\{2,3,4\}, \{1,5\}\}} \\
& + M_{\{\{3,4\}, \{2\}, \{1,5\}\}} - 4 M_{\{\{1,3,5\}, \{2,4\}\}} + 2 M_{\{\{4,5\}, \{1,2\}, \{3\}\}} + 5 M_{\{\{4,5\}, \{1\}, \{2,3\}\}} \\
& - 2 M_{\{\{4,5\}, \{1,3\}, \{2\}\}} + 10 M_{\{\{2,3,4,5\}, \{1\}\}} + 11 M_{\{\{4,5\}, \{1,2,3\}\}} - M_{\{\{2,4,5\}, \{1\}, \{3\}\}}
\end{aligned}$$

$$\begin{aligned}
& -5 M_{\{\{2,4,5\},\{1,3\}\}} + 24 M_{\{\{1,2,3,4,5\}\}} - M_{\{\{1,3,4,5\},\{2\}\}} + 2 M_{\{\{3,4,5\},\{1\},\{2\}\}} \\
& + 11 M_{\{\{3,4,5\},\{1,2\}\}} - M_{\{\{2,5\},\{1,3,4\}\}} + 4 M_{\{\{1,2,5\},\{3,4\}\}} - M_{\{\{2,3,5\},\{1,4\}\}} \\
& - 2 M_{\{\{3,5\},\{4\},\{1,2\}\}} - 2 M_{\{\{1\},\{3,5\},\{2,4\}\}} + 4 M_{\{\{1,4,5\},\{2,3\}\}} - 5 M_{\{\{3,5\},\{1,2,4\}\}} \\
& - M_{\{\{1,2,3,5\},\{4\}\}} \\
S(M_{\{\{5\},\{1,3\},\{2,4\}\}}) &= M_{\{\{2,4,5\},\{1,3\}\}} + 2 M_{\{\{1,3,5\},\{2,4\}\}} - 2 M_{\{\{3,4,5\},\{1,2\}\}} \\
& - 6 M_{\{\{1,2,3,4,5\}\}} - 4 M_{\{\{4,5\},\{1,2,3\}\}} + M_{\{\{1\},\{3,5\},\{2,4\}\}} + M_{\{\{3,5\},\{1,2,4\}\}} \\
& - 2 M_{\{\{4,5\},\{1\},\{2,3\}\}} - 2 M_{\{\{1,4,5\},\{2,3\}\}} - 2 M_{\{\{2,3,4,5\},\{1\}\}} \\
S(M_{\{\{5\},\{1\},\{2,3,4\}\}}) &= -M_{\{\{4,5\},\{1,2,3\}\}} - 6 M_{\{\{1,2,3,4,5\}\}} - M_{\{\{3,4,5\},\{1,2\}\}} \\
& - M_{\{\{5\},\{1\},\{2,3,4\}\}} - 2 M_{\{\{5\},\{1,2,3,4\}\}} - 2 M_{\{\{2,3,4,5\},\{1\}\}} \\
S(M_{\{\{5\},\{1,2,4\},\{3\}\}}) &= M_{\{\{3,5\},\{1,2,4\}\}} + M_{\{\{1,2,4,5\},\{3\}\}} + M_{\{\{1,2,3,5\},\{4\}\}} - M_{\{\{4,5\},\{1,2,3\}\}} \\
& - 3 M_{\{\{2,3,4,5\},\{1\}\}} - 6 M_{\{\{1,2,3,4,5\}\}} - 2 M_{\{\{3,4,5\},\{1,2\}\}} + M_{\{\{2,3,5\},\{1\},\{4\}\}} \\
& + M_{\{\{2,3,5\},\{1,4\}\}} - M_{\{\{5\},\{1\},\{2,3,4\}\}} - 2 M_{\{\{5\},\{1,2,3,4\}\}} - M_{\{\{2,3,4\},\{1,5\}\}} \\
& - M_{\{\{3,4,5\},\{1\},\{2\}\}} - M_{\{\{1,3,4,5\},\{2\}\}} \\
S(M_{\{\{5\},\{1\},\{3,4\},\{2\}\}}) &= M_{\{\{5\},\{1\},\{4\},\{2,3\}\}} + 2 M_{\{\{5\},\{4\},\{1,2,3\}\}} - M_{\{\{5\},\{2,3\},\{1,4\}\}} \\
& + 3 M_{\{\{5\},\{1\},\{2,3,4\}\}} + M_{\{\{4\},\{2,3\},\{1,5\}\}} + 10 M_{\{\{5\},\{1,2,3,4\}\}} + 4 M_{\{\{5\},\{3,4\},\{1,2\}\}} \\
& + 3 M_{\{\{2,3,4\},\{1,5\}\}} + M_{\{\{3,4\},\{2\},\{1,5\}\}} + M_{\{\{4,5\},\{1,2\},\{3\}\}} + 3 M_{\{\{4,5\},\{1\},\{2,3\}\}} \\
& - M_{\{\{4,5\},\{1,3\},\{2\}\}} + 8 M_{\{\{2,3,4,5\},\{1\}\}} + 9 M_{\{\{4,5\},\{1,2,3\}\}} - M_{\{\{2,4,5\},\{1\},\{3\}\}} \\
& - 2 M_{\{\{2,4,5\},\{1,3\}\}} + 24 M_{\{\{1,2,3,4,5\}\}} - M_{\{\{1,3,4,5\},\{2\}\}} + M_{\{\{3,4,5\},\{1\},\{2\}\}} \\
& + 9 M_{\{\{3,4,5\},\{1,2\}\}} - 2 M_{\{\{2,5\},\{1,3,4\}\}} - M_{\{\{1\},\{2,5\},\{3,4\}\}} + 2 M_{\{\{1,2,5\},\{3,4\}\}} \\
& - M_{\{\{2,3,5\},\{1,4\}\}} + M_{\{\{2,3,5\},\{1\},\{4\}\}} - M_{\{\{3,5\},\{4\},\{1,2\}\}} + M_{\{\{1,4,5\},\{2,3\}\}} \\
& - M_{\{\{3,5\},\{1,2,4\}\}} + M_{\{\{1,2,3,5\},\{4\}\}} \\
S(M_{\{\{5\},\{3,4\},\{1,2\}\}}) &= M_{\{\{1,2,5\},\{3,4\}\}} - M_{\{\{3,4,5\},\{1,2\}\}} - 6 M_{\{\{1,2,3,4,5\}\}} \\
& - 3 M_{\{\{4,5\},\{1,2,3\}\}} - M_{\{\{4,5\},\{1\},\{2,3\}\}} - M_{\{\{1,4,5\},\{2,3\}\}} - 2 M_{\{\{2,3,4,5\},\{1\}\}} \\
& S(M_{\{\{5\},\{1,2,3,4\}\}}) = 2 M_{\{\{1,2,3,4,5\}\}} + M_{\{\{2,3,4,5\},\{1\}\}} \\
S(M_{\{\{4\},\{2\},\{3\},\{1,5\}\}}) &= 3 M_{\{\{5\},\{1\},\{4\},\{2,3\}\}} - 3 M_{\{\{5\},\{1,3\},\{4\},\{2\}\}} + M_{\{\{5\},\{4\},\{1,2\},\{3\}\}} \\
& + 3 M_{\{\{5\},\{2\},\{3\},\{1,4\}\}} + 4 M_{\{\{5\},\{4\},\{1,2,3\}\}} + 3 M_{\{\{5\},\{2,3\},\{1,4\}\}} - 9 M_{\{\{5\},\{1,3\},\{2,4\}\}} \\
& - 6 M_{\{\{5\},\{1\},\{2,4\},\{3\}\}} - 5 M_{\{\{5\},\{1,2,4\},\{3\}\}} + 6 M_{\{\{5\},\{1\},\{2,3,4\}\}} - 6 M_{\{\{2,4\},\{3\},\{1,5\}\}} \\
& + 3 M_{\{\{4\},\{2,3\},\{1,5\}\}} - M_{\{\{4\},\{2\},\{3\},\{1,5\}\}} + 12 M_{\{\{5\},\{1,2,3,4\}\}} + 9 M_{\{\{5\},\{3,4\},\{1,2\}\}} \\
& + 3 M_{\{\{5\},\{1\},\{3,4\},\{2\}\}} + 6 M_{\{\{2,3,4\},\{1,5\}\}} - 6 M_{\{\{1,3,5\},\{4\},\{2\}\}} + 3 M_{\{\{3,4\},\{2\},\{1,5\}\}} \\
& + M_{\{\{4,5\},\{1\},\{2\},\{3\}\}} - 12 M_{\{\{1,3,5\},\{2,4\}\}} + 6 M_{\{\{4,5\},\{1,2\},\{3\}\}} + 9 M_{\{\{4,5\},\{1\},\{2,3\}\}} \\
& - 6 M_{\{\{4,5\},\{1,3\},\{2\}\}} + 12 M_{\{\{2,3,4,5\},\{1\}\}} + 15 M_{\{\{4,5\},\{1,2,3\}\}} - 5 M_{\{\{2,4,5\},\{1\},\{3\}\}} \\
& - 12 M_{\{\{2,4,5\},\{1,3\}\}} + 24 M_{\{\{1,2,3,4,5\}\}} - 3 M_{\{\{1,3,4,5\},\{2\}\}} + 4 M_{\{\{3,4,5\},\{1\},\{2\}\}} \\
& + 3 M_{\{\{1\},\{2,5\},\{4\},\{3\}\}} + 15 M_{\{\{3,4,5\},\{1,2\}\}} + 6 M_{\{\{2,5\},\{3\},\{1,4\}\}} + 3 M_{\{\{1\},\{2,5\},\{3,4\}\}} \\
& + 9 M_{\{\{1,2,5\},\{3,4\}\}} + 4 M_{\{\{1,2,5\},\{4\},\{3\}\}} - 6 M_{\{\{3,5\},\{4\},\{1,2\}\}} - 3 M_{\{\{1\},\{3,5\},\{4\},\{2\}\}} \\
& - 9 M_{\{\{1\},\{3,5\},\{2,4\}\}} + 9 M_{\{\{1,4,5\},\{2,3\}\}} + 4 M_{\{\{1,4,5\},\{2\},\{3\}\}} - 12 M_{\{\{3,5\},\{1,2,4\}\}} \\
& - 3 M_{\{\{1,2,3,5\},\{4\}\}} \\
S(M_{\{\{4\},\{2,3\},\{1,5\}\}}) &= M_{\{\{5\},\{2,3\},\{1,4\}\}} - M_{\{\{4\},\{2,3\},\{1,5\}\}} - M_{\{\{5\},\{1,2,3,4\}\}} \\
& - M_{\{\{5\},\{3,4\},\{1,2\}\}} - M_{\{\{4,5\},\{1,2\},\{3\}\}} - 2 M_{\{\{4,5\},\{1\},\{2,3\}\}} + M_{\{\{4,5\},\{1,3\},\{2\}\}} \\
& - 2 M_{\{\{2,3,4,5\},\{1\}\}} - 4 M_{\{\{4,5\},\{1,2,3\}\}} - M_{\{\{1,2,4,5\},\{3\}\}} + M_{\{\{2,4,5\},\{1,3\}\}} \\
& - 6 M_{\{\{1,2,3,4,5\}\}} + M_{\{\{1,3,4,5\},\{2\}\}} - 3 M_{\{\{3,4,5\},\{1,2\}\}} + M_{\{\{2,5\},\{1,3,4\}\}}
\end{aligned}$$

$$\begin{aligned}
& + M_{\{\{1\}, \{2,5\}, \{3,4\}\}} + M_{\{\{2,3,5\}, \{1,4\}\}} + M_{\{\{3,5\}, \{4\}, \{1,2\}\}} - M_{\{\{1,4,5\}, \{2,3\}\}} \\
& + M_{\{\{3,5\}, \{1,2,4\}\}} + M_{\{\{1,2,3,5\}, \{4\}\}} \\
S(M_{\{\{2,4\}, \{3\}, \{1,5\}\}}) &= M_{\{\{5\}, \{2,3\}, \{1,4\}\}} - M_{\{\{2,4\}, \{3\}, \{1,5\}\}} - 2 M_{\{\{5\}, \{1,2,3,4\}\}} \\
& - 2 M_{\{\{5\}, \{3,4\}, \{1,2\}\}} - 2 M_{\{\{4,5\}, \{1,2\}, \{3\}\}} - 2 M_{\{\{4,5\}, \{1\}, \{2,3\}\}} + 2 M_{\{\{4,5\}, \{1,3\}, \{2\}\}} \\
& - 2 M_{\{\{2,3,4,5\}, \{1\}\}} - 4 M_{\{\{4,5\}, \{1,2,3\}\}} - 2 M_{\{\{1,2,4,5\}, \{3\}\}} + 2 M_{\{\{2,4,5\}, \{1,3\}\}} \\
& - 6 M_{\{\{1,2,3,4,5\}\}} + 2 M_{\{\{1,3,4,5\}, \{2\}\}} - 4 M_{\{\{3,4,5\}, \{1,2\}\}} + M_{\{\{2,5\}, \{1,3,4\}\}} \\
& + M_{\{\{1\}, \{2,5\}, \{3,4\}\}} - M_{\{\{1,2,5\}, \{3,4\}\}} + M_{\{\{2,3,5\}, \{1,4\}\}} + 2 M_{\{\{3,5\}, \{4\}, \{1,2\}\}} \\
& - M_{\{\{1,4,5\}, \{2,3\}\}} + 2 M_{\{\{3,5\}, \{1,2,4\}\}} + 2 M_{\{\{1,2,3,5\}, \{4\}\}} \\
S(M_{\{\{2,3,4\}, \{1,5\}\}}) &= -M_{\{\{2,3,4\}, \{1,5\}\}} + M_{\{\{3,4,5\}, \{1,2\}\}} + 2 M_{\{\{1,2,3,4,5\}\}} + M_{\{\{4,5\}, \{1,2,3\}\}} \\
S(M_{\{\{3,4\}, \{2\}, \{1,5\}\}}) &= M_{\{\{5\}, \{2,3\}, \{1,4\}\}} - 2 M_{\{\{5\}, \{1,2,3,4\}\}} - 2 M_{\{\{5\}, \{3,4\}, \{1,2\}\}} \\
& - M_{\{\{3,4\}, \{2\}, \{1,5\}\}} - M_{\{\{4,5\}, \{1,2\}, \{3\}\}} - M_{\{\{4,5\}, \{1\}, \{2,3\}\}} + M_{\{\{4,5\}, \{1,3\}, \{2\}\}} \\
& - M_{\{\{2,3,4,5\}, \{1\}\}} - 3 M_{\{\{4,5\}, \{1,2,3\}\}} - M_{\{\{1,2,4,5\}, \{3\}\}} + M_{\{\{2,4,5\}, \{1,3\}\}} - 6 M_{\{\{1,2,3,4,5\}\}} \\
& + M_{\{\{1,3,4,5\}, \{2\}\}} - 4 M_{\{\{3,4,5\}, \{1,2\}\}} + M_{\{\{2,5\}, \{1,3,4\}\}} + M_{\{\{1\}, \{2,5\}, \{3,4\}\}} \\
& - M_{\{\{1,2,5\}, \{3,4\}\}} + M_{\{\{2,3,5\}, \{1,4\}\}} + M_{\{\{3,5\}, \{4\}, \{1,2\}\}} + M_{\{\{3,5\}, \{1,2,4\}\}} + M_{\{\{1,2,3,5\}, \{4\}\}} \\
S(M_{\{\{1,3,5\}, \{4\}, \{2\}\}}) &= -M_{\{\{5\}, \{4\}, \{1,2,3\}\}} + M_{\{\{5\}, \{1,3,4\}, \{2\}\}} + M_{\{\{5\}, \{1,2,4\}, \{3\}\}} \\
& - 2 M_{\{\{5\}, \{1\}, \{2,3,4\}\}} - 3 M_{\{\{5\}, \{1,2,3,4\}\}} - 2 M_{\{\{2,3,4\}, \{1,5\}\}} - M_{\{\{1,3,5\}, \{4\}, \{2\}\}} \\
& - 3 M_{\{\{2,3,4,5\}, \{1\}\}} - 2 M_{\{\{4,5\}, \{1,2,3\}\}} + M_{\{\{2,4,5\}, \{1\}, \{3\}\}} + 2 M_{\{\{1,2,4,5\}, \{3\}\}} \\
& + M_{\{\{2,4,5\}, \{1,3\}\}} - 6 M_{\{\{1,2,3,4,5\}\}} - M_{\{\{3,4,5\}, \{1\}, \{2\}\}} - 2 M_{\{\{3,4,5\}, \{1,2\}\}} \\
& + M_{\{\{2,5\}, \{1,3,4\}\}} + M_{\{\{2,3,5\}, \{1,4\}\}} + M_{\{\{2,3,5\}, \{1\}, \{4\}\}} + M_{\{\{3,5\}, \{1,2,4\}\}} \\
S(M_{\{\{1,3,5\}, \{2,4\}\}}) &= -M_{\{\{1,3,5\}, \{2,4\}\}} + M_{\{\{3,4,5\}, \{1,2\}\}} + 2 M_{\{\{1,2,3,4,5\}\}} + M_{\{\{4,5\}, \{1,2,3\}\}} \\
S(M_{\{\{4,5\}, \{1\}, \{2\}, \{3\}\}}) &= M_{\{\{5\}, \{4\}, \{1,2\}, \{3\}\}} + 4 M_{\{\{5\}, \{4\}, \{1,2,3\}\}} - 3 M_{\{\{5\}, \{1,3,4\}, \{2\}\}} \\
& + M_{\{\{5\}, \{1,2,4\}, \{3\}\}} + 3 M_{\{\{5\}, \{1\}, \{2,3,4\}\}} + 12 M_{\{\{5\}, \{1,2,3,4\}\}} + 3 M_{\{\{5\}, \{3,4\}, \{1,2\}\}} \\
& + 3 M_{\{\{2,3,4\}, \{1,5\}\}} + 3 M_{\{\{4,5\}, \{1,2\}, \{3\}\}} + 6 M_{\{\{2,3,4,5\}, \{1\}\}} + 9 M_{\{\{4,5\}, \{1,2,3\}\}} \\
& - 2 M_{\{\{2,4,5\}, \{1\}, \{3\}\}} - 3 M_{\{\{2,4,5\}, \{1,3\}\}} + 24 M_{\{\{1,2,3,4,5\}\}} - 3 M_{\{\{1,3,4,5\}, \{2\}\}} \\
& + M_{\{\{3,4,5\}, \{1\}, \{2\}\}} + 9 M_{\{\{3,4,5\}, \{1,2\}\}} - 3 M_{\{\{2,5\}, \{1,3,4\}\}} + 3 M_{\{\{1,2,5\}, \{3,4\}\}} \\
& + M_{\{\{1,2,5\}, \{4\}, \{3\}\}} + M_{\{\{1,4,5\}, \{2\}, \{3\}\}} + 3 M_{\{\{1,2,3,5\}, \{4\}\}} \\
S(M_{\{\{4,5\}, \{1,2\}, \{3\}\}}) &= -M_{\{\{5\}, \{1,2,3,4\}\}} - 6 M_{\{\{1,2,3,4,5\}\}} - M_{\{\{2,3,4,5\}, \{1\}\}} \\
& - 2 M_{\{\{3,4,5\}, \{1,2\}\}} - M_{\{\{4,5\}, \{1,2\}, \{3\}\}} - 2 M_{\{\{4,5\}, \{1,2,3\}\}} \\
S(M_{\{\{4,5\}, \{1,3\}, \{2\}\}}) &= -3 M_{\{\{3,4,5\}, \{1,2\}\}} - 2 M_{\{\{5\}, \{1,2,3,4\}\}} - 6 M_{\{\{1,2,3,4,5\}\}} \\
& + M_{\{\{3,5\}, \{4\}, \{1,2\}\}} + M_{\{\{3,5\}, \{1,2,4\}\}} + M_{\{\{1,2,3,5\}, \{4\}\}} - M_{\{\{5\}, \{3,4\}, \{1,2\}\}} \\
& - M_{\{\{1,2,5\}, \{3,4\}\}} - M_{\{\{4,5\}, \{1,2\}, \{3\}\}} - 2 M_{\{\{4,5\}, \{1,2,3\}\}} - M_{\{\{1,2,4,5\}, \{3\}\}} \\
& + M_{\{\{2,4,5\}, \{1,3\}\}} + M_{\{\{1,3,4,5\}, \{2\}\}} - M_{\{\{2,3,4,5\}, \{1\}\}} \\
S(M_{\{\{4,5\}, \{1\}, \{2,3\}\}}) &= -M_{\{\{5\}, \{3,4\}, \{1,2\}\}} - M_{\{\{1,2,5\}, \{3,4\}\}} - 3 M_{\{\{3,4,5\}, \{1,2\}\}} \\
& - 2 M_{\{\{5\}, \{1,2,3,4\}\}} - 6 M_{\{\{1,2,3,4,5\}\}} + M_{\{\{1,4,5\}, \{2,3\}\}} - M_{\{\{4,5\}, \{1,2,3\}\}} \\
& \quad S(M_{\{\{4,5\}, \{1,2,3\}\}}) = M_{\{\{3,4,5\}, \{1,2\}\}} + 2 M_{\{\{1,2,3,4,5\}\}} \\
& \quad S(M_{\{\{2,3,4,5\}, \{1\}\}}) = M_{\{\{5\}, \{1,2,3,4\}\}} + 2 M_{\{\{1,2,3,4,5\}\}} \\
S(M_{\{\{1,2,4,5\}, \{3\}\}}) &= -M_{\{\{1,2,4,5\}, \{3\}\}} + M_{\{\{2,3,4,5\}, \{1\}\}} + 2 M_{\{\{1,2,3,4,5\}\}} + M_{\{\{5\}, \{1,2,3,4\}\}} \\
S(M_{\{\{2,4,5\}, \{1\}, \{3\}\}}) &= M_{\{\{2,4,5\}, \{1,3\}\}} + M_{\{\{1,2,4,5\}, \{3\}\}} - M_{\{\{5\}, \{1\}, \{2,3,4\}\}} \\
& - 3 M_{\{\{5\}, \{1,2,3,4\}\}} - M_{\{\{2,3,4\}, \{1,5\}\}} - 2 M_{\{\{2,3,4,5\}, \{1\}\}} - 6 M_{\{\{1,2,3,4,5\}\}} \\
& - M_{\{\{3,4,5\}, \{1,2\}\}} - M_{\{\{5\}, \{4\}, \{1,2,3\}\}} - M_{\{\{1,2,3,5\}, \{4\}\}} - 2 M_{\{\{4,5\}, \{1,2,3\}\}}
\end{aligned}$$

$$\begin{aligned}
& + M_{\{\{5\}, \{1, 3, 4\}, \{2\}\}} + M_{\{\{2, 5\}, \{1, 3, 4\}\}} + M_{\{\{1, 3, 4, 5\}, \{2\}\}} \\
S(M_{\{\{2, 4, 5\}, \{1, 3\}\}}) &= -M_{\{\{2, 4, 5\}, \{1, 3\}\}} + M_{\{\{3, 4, 5\}, \{1, 2\}\}} + 2 M_{\{\{1, 2, 3, 4, 5\}\}} + M_{\{\{4, 5\}, \{1, 2, 3\}\}} \\
& S(M_{\{\{1, 2, 3, 4, 5\}\}}) = -M_{\{\{1, 2, 3, 4, 5\}\}} \\
S(M_{\{\{1, 3, 4, 5\}, \{2\}\}}) &= -M_{\{\{1, 3, 4, 5\}, \{2\}\}} + M_{\{\{2, 3, 4, 5\}, \{1\}\}} + 2 M_{\{\{1, 2, 3, 4, 5\}\}} + M_{\{\{5\}, \{1, 2, 3, 4\}\}} \\
S(M_{\{\{3, 4, 5\}, \{1\}, \{2\}\}}) &= -3 M_{\{\{5\}, \{1, 2, 3, 4\}\}} - M_{\{\{2, 3, 4, 5\}, \{1\}\}} - 6 M_{\{\{1, 2, 3, 4, 5\}\}} \\
& - M_{\{\{5\}, \{4\}, \{1, 2, 3\}\}} - M_{\{\{1, 2, 3, 5\}, \{4\}\}} - 2 M_{\{\{4, 5\}, \{1, 2, 3\}\}} + M_{\{\{1, 3, 4, 5\}, \{2\}\}} \\
& S(M_{\{\{3, 4, 5\}, \{1, 2\}\}}) = 2 M_{\{\{1, 2, 3, 4, 5\}\}} + M_{\{\{4, 5\}, \{1, 2, 3\}\}} \\
S(M_{\{\{1\}, \{2, 5\}, \{4\}, \{3\}\}}) &= 2 M_{\{\{5\}, \{1\}, \{4\}, \{2, 3\}\}} - 2 M_{\{\{5\}, \{1, 3\}, \{4\}, \{2\}\}} + M_{\{\{5\}, \{4\}, \{1, 2\}, \{3\}\}} \\
& + M_{\{\{5\}, \{2\}, \{3\}, \{1, 4\}\}} + 4 M_{\{\{5\}, \{4\}, \{1, 2, 3\}\}} - M_{\{\{5\}, \{1, 3, 4\}, \{2\}\}} + 2 M_{\{\{5\}, \{2, 3\}, \{1, 4\}\}} \\
& - 6 M_{\{\{5\}, \{1, 3\}, \{2, 4\}\}} - 2 M_{\{\{5\}, \{1\}, \{2, 4\}, \{3\}\}} - 3 M_{\{\{5\}, \{1, 2, 4\}, \{3\}\}} + 5 M_{\{\{5\}, \{1\}, \{2, 3, 4\}\}} \\
& - 2 M_{\{\{2, 4\}, \{3\}, \{1, 5\}\}} + 2 M_{\{\{4\}, \{2, 3\}, \{1, 5\}\}} + 12 M_{\{\{5\}, \{1, 2, 3, 4\}\}} + 7 M_{\{\{5\}, \{3, 4\}, \{1, 2\}\}} \\
& + M_{\{\{5\}, \{1\}, \{3, 4\}, \{2\}\}} + 5 M_{\{\{2, 3, 4\}, \{1, 5\}\}} - 2 M_{\{\{1, 3, 5\}, \{4\}, \{2\}\}} + M_{\{\{3, 4\}, \{2\}, \{1, 5\}\}} \\
& - 8 M_{\{\{1, 3, 5\}, \{2, 4\}\}} + 4 M_{\{\{4, 5\}, \{1, 2\}, \{3\}\}} + 6 M_{\{\{4, 5\}, \{1\}, \{2, 3\}\}} - 5 M_{\{\{4, 5\}, \{1, 3\}, \{2\}\}} \\
& + 10 M_{\{\{2, 3, 4, 5\}, \{1\}\}} + 13 M_{\{\{4, 5\}, \{1, 2, 3\}\}} - 2 M_{\{\{2, 4, 5\}, \{1\}, \{3\}\}} - 9 M_{\{\{2, 4, 5\}, \{1, 3\}\}} \\
& + 24 M_{\{\{1, 2, 3, 4, 5\}\}} - 3 M_{\{\{1, 3, 4, 5\}, \{2\}\}} + 2 M_{\{\{3, 4, 5\}, \{1\}, \{2\}\}} + 13 M_{\{\{3, 4, 5\}, \{1, 2\}\}} \\
& - M_{\{\{2, 5\}, \{1, 3, 4\}\}} + 2 M_{\{\{2, 5\}, \{3\}, \{1, 4\}\}} + M_{\{\{1\}, \{2, 5\}, \{3, 4\}\}} - M_{\{\{2, 5\}, \{1, 3\}, \{4\}\}} \\
& + 7 M_{\{\{1, 2, 5\}, \{3, 4\}\}} + 2 M_{\{\{1, 2, 5\}, \{4\}, \{3\}\}} - 2 M_{\{\{3, 5\}, \{4\}, \{1, 2\}\}} - 4 M_{\{\{1\}, \{3, 5\}, \{2, 4\}\}} \\
& + M_{\{\{3, 5\}, \{2\}, \{1, 4\}\}} + 6 M_{\{\{1, 4, 5\}, \{2, 3\}\}} + M_{\{\{1, 4, 5\}, \{2\}, \{3\}\}} - 8 M_{\{\{3, 5\}, \{1, 2, 4\}\}} \\
& - M_{\{\{1, 2, 3, 5\}, \{4\}\}} \\
S(M_{\{\{2, 5\}, \{1, 3\}, \{4\}\}}) &= -M_{\{\{2, 5\}, \{1, 3\}, \{4\}\}} + M_{\{\{3, 5\}, \{4\}, \{1, 2\}\}} + 2 M_{\{\{3, 5\}, \{1, 2, 4\}\}} \\
& + M_{\{\{1, 2, 3, 5\}, \{4\}\}} - M_{\{\{5\}, \{3, 4\}, \{1, 2\}\}} - M_{\{\{5\}, \{1, 2, 3, 4\}\}} - M_{\{\{1, 2, 5\}, \{3, 4\}\}} \\
& - 3 M_{\{\{3, 4, 5\}, \{1, 2\}\}} - 6 M_{\{\{1, 2, 3, 4, 5\}\}} - M_{\{\{4, 5\}, \{1, 2\}, \{3\}\}} - 4 M_{\{\{4, 5\}, \{1, 2, 3\}\}} \\
& - M_{\{\{1, 2, 4, 5\}, \{3\}\}} + M_{\{\{4, 5\}, \{1, 3\}, \{2\}\}} + M_{\{\{1, 3, 4, 5\}, \{2\}\}} + 2 M_{\{\{2, 4, 5\}, \{1, 3\}\}} \\
& + M_{\{\{1\}, \{3, 5\}, \{2, 4\}\}} + 2 M_{\{\{1, 3, 5\}, \{2, 4\}\}} - 2 M_{\{\{4, 5\}, \{1\}, \{2, 3\}\}} - 2 M_{\{\{1, 4, 5\}, \{2, 3\}\}} \\
& - 2 M_{\{\{2, 3, 4, 5\}, \{1\}\}} + M_{\{\{5\}, \{1, 3\}, \{2, 4\}\}} \\
S(M_{\{\{2, 5\}, \{3\}, \{1, 4\}\}}) &= -M_{\{\{2, 5\}, \{3\}, \{1, 4\}\}} + 2 M_{\{\{3, 5\}, \{4\}, \{1, 2\}\}} + 3 M_{\{\{3, 5\}, \{1, 2, 4\}\}} \\
& + 2 M_{\{\{1, 2, 3, 5\}, \{4\}\}} - 2 M_{\{\{5\}, \{3, 4\}, \{1, 2\}\}} - 2 M_{\{\{5\}, \{1, 2, 3, 4\}\}} - 2 M_{\{\{1, 2, 5\}, \{3, 4\}\}} \\
& - 4 M_{\{\{3, 4, 5\}, \{1, 2\}\}} - 6 M_{\{\{1, 2, 3, 4, 5\}\}} - 2 M_{\{\{4, 5\}, \{1, 2\}, \{3\}\}} - 4 M_{\{\{4, 5\}, \{1, 2, 3\}\}} \\
& - 2 M_{\{\{1, 2, 4, 5\}, \{3\}\}} + 2 M_{\{\{4, 5\}, \{1, 3\}, \{2\}\}} + 2 M_{\{\{1, 3, 4, 5\}, \{2\}\}} + 3 M_{\{\{2, 4, 5\}, \{1, 3\}\}} \\
& + M_{\{\{1\}, \{3, 5\}, \{2, 4\}\}} + 2 M_{\{\{1, 3, 5\}, \{2, 4\}\}} - 2 M_{\{\{4, 5\}, \{1\}, \{2, 3\}\}} - 2 M_{\{\{1, 4, 5\}, \{2, 3\}\}} \\
& - 2 M_{\{\{2, 3, 4, 5\}, \{1\}\}} + M_{\{\{5\}, \{1, 3\}, \{2, 4\}\}} \\
S(M_{\{\{2, 5\}, \{1, 3, 4\}\}}) &= -M_{\{\{2, 5\}, \{1, 3, 4\}\}} + M_{\{\{3, 4, 5\}, \{1, 2\}\}} + 2 M_{\{\{1, 2, 3, 4, 5\}\}} + M_{\{\{4, 5\}, \{1, 2, 3\}\}} \\
S(M_{\{\{1\}, \{2, 5\}, \{3, 4\}\}}) &= -2 M_{\{\{5\}, \{3, 4\}, \{1, 2\}\}} - M_{\{\{1, 2, 5\}, \{3, 4\}\}} - 2 M_{\{\{5\}, \{1, 2, 3, 4\}\}} \\
& - 4 M_{\{\{3, 4, 5\}, \{1, 2\}\}} - 6 M_{\{\{1, 2, 3, 4, 5\}\}} + M_{\{\{2, 5\}, \{1, 3, 4\}\}} - 2 M_{\{\{4, 5\}, \{1, 2, 3\}\}} \\
& + M_{\{\{5\}, \{2, 3\}, \{1, 4\}\}} + M_{\{\{2, 3, 5\}, \{1, 4\}\}} + M_{\{\{1, 4, 5\}, \{2, 3\}\}} \\
S(M_{\{\{2, 3, 5\}, \{1\}, \{4\}\}}) &= -2 M_{\{\{2, 3, 4, 5\}, \{1\}\}} + M_{\{\{2, 3, 5\}, \{1, 4\}\}} - M_{\{\{5\}, \{1\}, \{2, 3, 4\}\}} \\
& - M_{\{\{2, 3, 4\}, \{1, 5\}\}} - 3 M_{\{\{5\}, \{1, 2, 3, 4\}\}} - 6 M_{\{\{1, 2, 3, 4, 5\}\}} - M_{\{\{3, 4, 5\}, \{1, 2\}\}} \\
& - M_{\{\{5\}, \{4\}, \{1, 2, 3\}\}} - 2 M_{\{\{4, 5\}, \{1, 2, 3\}\}} + M_{\{\{5\}, \{1, 2, 4\}, \{3\}\}} + M_{\{\{1, 2, 4, 5\}, \{3\}\}} \\
& + M_{\{\{3, 5\}, \{1, 2, 4\}\}} \\
S(M_{\{\{2, 3, 5\}, \{1, 4\}\}}) &= -M_{\{\{2, 3, 5\}, \{1, 4\}\}} + M_{\{\{3, 4, 5\}, \{1, 2\}\}} + 2 M_{\{\{1, 2, 3, 4, 5\}\}} + M_{\{\{4, 5\}, \{1, 2, 3\}\}}
\end{aligned}$$

$$\begin{aligned}
S(M_{\{\{1,2,5\},\{4\},\{3\}\}}) &= -M_{\{\{1,2,5\},\{4\},\{3\}\}} - M_{\{\{3,4,5\},\{1\},\{2\}\}} - M_{\{\{1,3,4,5\},\{2\}\}} \\
&\quad - 3 M_{\{\{2,3,4,5\},\{1\}\}} + 2 M_{\{\{2,3,5\},\{1\},\{4\}\}} + M_{\{\{1,2,3,5\},\{4\}\}} + 2 M_{\{\{2,3,5\},\{1,4\}\}} \\
&\quad - 2 M_{\{\{5\},\{1\},\{2,3,4\}\}} - 2 M_{\{\{2,3,4\},\{1,5\}\}} - 3 M_{\{\{5\},\{1,2,3,4\}\}} - 6 M_{\{\{1,2,3,4,5\}\}} \\
&\quad - 2 M_{\{\{3,4,5\},\{1,2\}\}} - M_{\{\{5\},\{4\},\{1,2,3\}\}} - 2 M_{\{\{4,5\},\{1,2,3\}\}} + 2 M_{\{\{5\},\{1,2,4\},\{3\}\}} \\
&\quad + 2 M_{\{\{1,2,4,5\},\{3\}\}} + 2 M_{\{\{3,5\},\{1,2,4\}\}} \\
S(M_{\{\{1,2,5\},\{3,4\}\}}) &= -M_{\{\{1,2,5\},\{3,4\}\}} + M_{\{\{3,4,5\},\{1,2\}\}} + 2 M_{\{\{1,2,3,4,5\}\}} + M_{\{\{4,5\},\{1,2,3\}\}} \\
S(M_{\{\{1\},\{3,5\},\{4\},\{2\}\}}) &= M_{\{\{5\},\{1\},\{4\},\{2,3\}\}} - M_{\{\{5\},\{1,3\},\{4\},\{2\}\}} + M_{\{\{5\},\{4\},\{1,2\},\{3\}\}} \\
&\quad + 4 M_{\{\{5\},\{4\},\{1,2,3\}\}} - 2 M_{\{\{5\},\{1,3,4\},\{2\}\}} + M_{\{\{5\},\{2,3\},\{1,4\}\}} - 3 M_{\{\{5\},\{1,3\},\{2,4\}\}} \\
&\quad - M_{\{\{5\},\{1,2,4\},\{3\}\}} + 4 M_{\{\{5\},\{1\},\{2,3,4\}\}} + M_{\{\{4\},\{2,3\},\{1,5\}\}} + 12 M_{\{\{5\},\{1,2,3,4\}\}} \\
&\quad + 5 M_{\{\{5\},\{3,4\},\{1,2\}\}} + 4 M_{\{\{2,3,4\},\{1,5\}\}} - 4 M_{\{\{1,3,5\},\{2,4\}\}} + 3 M_{\{\{4,5\},\{1,2\},\{3\}\}} \\
&\quad + 3 M_{\{\{4,5\},\{1\},\{2,3\}\}} - 3 M_{\{\{4,5\},\{1,3\},\{2\}\}} + 8 M_{\{\{2,3,4,5\},\{1\}\}} + 11 M_{\{\{4,5\},\{1,2,3\}\}} \\
&\quad - M_{\{\{2,4,5\},\{1\},\{3\}\}} - 6 M_{\{\{2,4,5\},\{1,3\}\}} + 24 M_{\{\{1,2,3,4,5\}\}} - 3 M_{\{\{1,3,4,5\},\{2\}\}} \\
&\quad + M_{\{\{3,4,5\},\{1\},\{2\}\}} + 11 M_{\{\{3,4,5\},\{1,2\}\}} - 2 M_{\{\{2,5\},\{1,3,4\}\}} - M_{\{\{2,5\},\{1,3\},\{4\}\}} \\
&\quad + 5 M_{\{\{1,2,5\},\{3,4\}\}} + M_{\{\{1,2,5\},\{4\},\{3\}\}} - M_{\{\{1\},\{3,5\},\{2,4\}\}} + M_{\{\{3,5\},\{2\},\{1,4\}\}} \\
&\quad + 3 M_{\{\{1,4,5\},\{2,3\}\}} - 4 M_{\{\{3,5\},\{1,2,4\}\}} + M_{\{\{1,2,3,5\},\{4\}\}} \\
S(M_{\{\{3,5\},\{4\},\{1,2\}\}}) &= -M_{\{\{4,5\},\{1\},\{2,3\}\}} - M_{\{\{1,4,5\},\{2,3\}\}} - 3 M_{\{\{4,5\},\{1,2,3\}\}} \\
&\quad - 2 M_{\{\{2,3,4,5\},\{1\}\}} - 6 M_{\{\{1,2,3,4,5\}\}} - 2 M_{\{\{3,4,5\},\{1,2\}\}} - M_{\{\{4,5\},\{1,2\},\{3\}\}} \\
&\quad - M_{\{\{1,2,4,5\},\{3\}\}} + M_{\{\{1,2,3,5\},\{4\}\}} + M_{\{\{3,5\},\{1,2,4\}\}} - M_{\{\{5\},\{1,2,3,4\}\}} + M_{\{\{4,5\},\{1,3\},\{2\}\}} \\
&\quad + M_{\{\{1,3,4,5\},\{2\}\}} + M_{\{\{2,4,5\},\{1,3\}\}} \\
S(M_{\{\{3,5\},\{2\},\{1,4\}\}}) &= -2 M_{\{\{5\},\{1,2,3,4\}\}} - 2 M_{\{\{5\},\{3,4\},\{1,2\}\}} + 2 M_{\{\{1,3,5\},\{2,4\}\}} \\
&\quad - M_{\{\{4,5\},\{1,2\},\{3\}\}} - M_{\{\{4,5\},\{1\},\{2,3\}\}} + M_{\{\{4,5\},\{1,3\},\{2\}\}} - M_{\{\{2,3,4,5\},\{1\}\}} \\
&\quad - 3 M_{\{\{4,5\},\{1,2,3\}\}} - M_{\{\{1,2,4,5\},\{3\}\}} + 2 M_{\{\{2,4,5\},\{1,3\}\}} - 6 M_{\{\{1,2,3,4,5\}\}} \\
&\quad + M_{\{\{1,3,4,5\},\{2\}\}} - 4 M_{\{\{3,4,5\},\{1,2\}\}} - 2 M_{\{\{1,2,5\},\{3,4\}\}} + M_{\{\{3,5\},\{4\},\{1,2\}\}} \\
&\quad + M_{\{\{1\},\{3,5\},\{2,4\}\}} - M_{\{\{3,5\},\{2\},\{1,4\}\}} - M_{\{\{1,4,5\},\{2,3\}\}} + 2 M_{\{\{3,5\},\{1,2,4\}\}} \\
&\quad + M_{\{\{1,2,3,5\},\{4\}\}} + M_{\{\{5\},\{1,3\},\{2,4\}\}} \\
S(M_{\{\{1\},\{3,5\},\{2,4\}\}}) &= 2 M_{\{\{1,3,5\},\{2,4\}\}} + M_{\{\{3,5\},\{1,2,4\}\}} - 2 M_{\{\{4,5\},\{1,2,3\}\}} \\
&\quad - 6 M_{\{\{1,2,3,4,5\}\}} - 2 M_{\{\{5\},\{3,4\},\{1,2\}\}} - 2 M_{\{\{1,2,5\},\{3,4\}\}} - 2 M_{\{\{5\},\{1,2,3,4\}\}} \\
&\quad - 4 M_{\{\{3,4,5\},\{1,2\}\}} + M_{\{\{5\},\{1,3\},\{2,4\}\}} + M_{\{\{2,4,5\},\{1,3\}\}} \\
S(M_{\{\{3,5\},\{1,2,4\}\}}) &= -M_{\{\{3,5\},\{1,2,4\}\}} + M_{\{\{4,5\},\{1,2,3\}\}} + 2 M_{\{\{1,2,3,4,5\}\}} + M_{\{\{3,4,5\},\{1,2\}\}} \\
S(M_{\{\{1,4,5\},\{2\},\{3\}\}}) &= -M_{\{\{1,4,5\},\{2\},\{3\}\}} - M_{\{\{5\},\{4\},\{1,2,3\}\}} - M_{\{\{1,2,3,5\},\{4\}\}} \\
&\quad - 3 M_{\{\{5\},\{1,2,3,4\}\}} - 2 M_{\{\{4,5\},\{1,2,3\}\}} - 6 M_{\{\{1,2,3,4,5\}\}} - M_{\{\{3,4,5\},\{1\},\{2\}\}} \\
&\quad + M_{\{\{1,3,4,5\},\{2\}\}} - 3 M_{\{\{2,3,4,5\},\{1\}\}} + 2 M_{\{\{2,4,5\},\{1\},\{3\}\}} + 2 M_{\{\{1,2,4,5\},\{3\}\}} \\
&\quad + 2 M_{\{\{2,4,5\},\{1,3\}\}} - 2 M_{\{\{5\},\{1\},\{2,3,4\}\}} - 2 M_{\{\{2,3,4\},\{1,5\}\}} - 2 M_{\{\{3,4,5\},\{1,2\}\}} \\
&\quad + 2 M_{\{\{5\},\{1,3,4\},\{2\}\}} + 2 M_{\{\{2,5\},\{1,3,4\}\}} \\
S(M_{\{\{1,4,5\},\{2,3\}\}}) &= -M_{\{\{1,4,5\},\{2,3\}\}} + M_{\{\{4,5\},\{1,2,3\}\}} + 2 M_{\{\{1,2,3,4,5\}\}} + M_{\{\{3,4,5\},\{1,2\}\}} \\
S(M_{\{\{1,2,3,5\},\{4\}\}}) &= -M_{\{\{1,2,3,5\},\{4\}\}} + M_{\{\{2,3,4,5\},\{1\}\}} + 2 M_{\{\{1,2,3,4,5\}\}} + M_{\{\{5\},\{1,2,3,4\}\}}
\end{aligned}$$

[>

[- commutative image of X_[n] basis elements

```
[ > for i from 1 to 5 do
      print(chi(X[{{seq(j,j=1..i)}}]) =
```

```
project(X[{{seq(j,j=1..i)}}]);
od;
```

$$\chi(X_{\{\{1\}\}}) = m_1$$

$$\chi(X_{\{\{1,2\}\}}) = -2 m_{1,1}$$

$$\chi(X_{\{\{1,2,3\}\}}) = 12 m_{1,1,1} + 3 m_{2,1}$$

$$\chi(X_{\{\{1,2,3,4\}\}}) = -144 m_{1,1,1,1} - 48 m_{2,1,1} - 4 m_{3,1} - 18 m_{2,2}$$

$$\chi(X_{\{\{1,2,3,4,5\}\}}) =$$

$$420 m_{2,2,1} + 160 m_{3,1,1} + 2880 m_{1,1,1,1,1} + 1080 m_{2,1,1,1} + 70 m_{3,2} + 5 m_{4,1}$$

```
> for i from 1 to 5 do
  print(chi(X[{{seq(j,j=1..i)}}])) =
  tos(project(X[{{seq(j,j=1..i)}}]));
od;
```

$$\chi(X_{\{\{1\}\}}) = s_1$$

$$\chi(X_{\{\{1,2\}\}}) = -2 s_{1,1}$$

$$\chi(X_{\{\{1,2,3\}\}}) = 3 s_{2,1} + 6 s_{1,1,1}$$

$$\chi(X_{\{\{1,2,3,4\}\}}) = -4 s_{3,1} - 14 s_{2,2} - 26 s_{2,1,1} - 26 s_{1,1,1,1}$$

$$\chi(X_{\{\{1,2,3,4,5\}\}}) = 5 s_{4,1} + 65 s_{3,2} + 85 s_{3,1,1} + 195 s_{2,2,1} + 225 s_{2,1,1,1} + 150 s_{1,1,1,1,1}$$

```
> for i from 1 to 5 do
  print(chi(X[{{seq(j,j=1..i)}}])) =
  toh(project(X[{{seq(j,j=1..i)}}]));
od;
```

$$\chi(X_{\{\{1\}\}}) = h_1$$

$$\chi(X_{\{\{1,2\}\}}) = 2 h_2 - 2 h_1^2$$

$$\chi(X_{\{\{1,2,3\}\}}) = 6 h_1^3 - 9 h_1 h_2 + 3 h_3$$

$$\chi(X_{\{\{1,2,3,4\}\}}) = -26 h_1^4 + 52 h_2 h_1^2 - 16 h_1 h_3 + 4 h_4 - 14 h_2^2$$

$$\chi(X_{\{\{1,2,3,4,5\}\}}) =$$

$$150 h_1^5 - 375 h_2 h_1^3 + 115 h_3 h_1^2 - 25 h_1 h_4 + 195 h_1 h_2^2 + 5 h_5 - 65 h_3 h_2$$

```
> for i from 1 to 5 do
  print(chi(X[{{seq(j,j=1..i)}}])) =
  toe(project(X[{{seq(j,j=1..i)}}]));
od;
```

$$\chi(X_{\{\{1\}\}}) = e_1$$

$$\chi(X_{\{\{1,2\}\}}) = -2 e_2$$

$$\chi(X_{\{\{1,2,3\}\}}) = 3 e_1 e_2 + 3 e_3$$

$$\chi(X_{\{\{1,2,3,4\}\}}) = -4 e_2 e_1^2 - 8 e_1 e_3 - 4 e_4 - 10 e_2^2$$

$$\chi(X_{\{\{1,2,3,4,5\}\}}) = 5 e_2 e_1^3 + 15 e_3 e_1^2 + 15 e_1 e_4 + 55 e_1 e_2^2 + 55 e_3 e_2 + 5 e_5$$

```
> for i from 1 to 5 do
  print(chi(X[{{seq(j,j=1..i)}}])) =
  top(project(X[{{seq(j,j=1..i)}}]));
od;
```

$$\chi(X_{\{\{1\}\}}) = p_1$$

$$\chi(X_{\{\{1,2\}\}}) = p_2 - p_1^2$$

$$\chi(X_{\{\{1,2,3\}\}}) = p_3 - 3 p_2 p_1 + 2 p_1^3$$

$$\chi(X_{\{\{1,2,3,4\}\}}) = p_4 - 3 p_2^2 - 4 p_3 p_1 + 12 p_2 p_1^2 - 6 p_1^4$$


```

 $\chi(X_{\{\{1,2,3,4,5\}\}}) = p^5 + 20 p^3 p l^2 + 30 p^2^2 p l - 60 p^2 p l^3 - 10 p^3 p^2 - 5 p^4 p l + 24 p l^5$ 
> for i from 1 to 5 do
  print(chi(X[{{seq(j,j=1..i)}}]) =
  subs(m=f,tom(omega(project(X[{{seq(j,j=1..i)}}]))));
od;

 $\chi(X_{\{\{1\}\}}) = f_1$ 
 $\chi(X_{\{\{1,2\}\}}) = -2 f_2 - 2 f_{1,1}$ 
 $\chi(X_{\{\{1,2,3\}\}}) = 6 f_3 + 9 f_{2,1} + 12 f_{1,1,1}$ 
 $\chi(X_{\{\{1,2,3,4\}\}}) = -26 f_4 - 52 f_{3,1} - 96 f_{2,1,1} - 66 f_{2,2} - 144 f_{1,1,1,1}$ 
 $\chi(X_{\{\{1,2,3,4,5\}\}}) =$ 
 $1140 f_{2,2,1} + 150 f_5 + 570 f_{3,2} + 375 f_{4,1} + 1800 f_{2,1,1,1} + 880 f_{3,1,1} + 2880 f_{1,1,1,1,1}$ 
>

```

- commutative image of X_A basis elements n=2,3,4,5

I think that once we know what happens on X_[n] then the rest of the set partitions are determined.

```

> for A in listallsp(2) do
  print(chi(X[A]) = factor(toe(project(X[A]))));
od;

 $\chi(X_{\{\{1\},\{2\}\}}) = e l^2$ 
 $\chi(X_{\{\{1,2\}\}}) = -2 e 2$ 

> for A in listallsp(3) do
  print(chi(X[A]) = factor(toe(project(X[A]))));
od;

 $\chi(X_{\{\{1\},\{2\},\{3\}\}}) = e l^3$ 
 $\chi(X_{\{\{1,2\},\{3\}\}}) = -2 e 2 e l$ 
 $\chi(X_{\{\{1,3\},\{2\}\}}) = -2 e 2 e l$ 
 $\chi(X_{\{\{1\},\{2,3\}\}}) = -2 e 2 e l$ 
 $\chi(X_{\{\{1,2,3\}\}}) = 3 e 2 e l + 3 e 3$ 

> for A in listallsp(4) do
  print(chi(X[A]) = factor(toe(project(X[A]))));
od;

 $\chi(X_{\{\{1\},\{4\},\{2\},\{3\}\}}) = e l^4$ 
 $\chi(X_{\{\{4\},\{1,2\},\{3\}\}}) = -2 e 2 e l^2$ 
 $\chi(X_{\{\{1,3\},\{4\},\{2\}\}}) = -2 e 2 e l^2$ 
 $\chi(X_{\{\{1\},\{4\},\{2,3\}\}}) = -2 e 2 e l^2$ 
 $\chi(X_{\{\{4\},\{1,2,3\}\}}) = 3 e l (e l e 2 + e 3)$ 
 $\chi(X_{\{\{2\},\{3\},\{1,4\}\}}) = -2 e 2 e l^2$ 
 $\chi(X_{\{\{2,3\},\{1,4\}\}}) = 4 e 2^2$ 
 $\chi(X_{\{\{1,3,4\},\{2\}\}}) = 3 e l (e l e 2 + e 3)$ 
 $\chi(X_{\{\{1\},\{2,4\},\{3\}\}}) = -2 e 2 e l^2$ 
 $\chi(X_{\{\{1,3\},\{2,4\}\}}) = 4 e 2^2$ 
 $\chi(X_{\{\{1\},\{2,3,4\}\}}) = 3 e l (e l e 2 + e 3)$ 

```

$$\chi(X_{\{\{1,2,4\},\{3\}\}}) = 3 e1 (e1 e2 + e3)$$

$$\chi(X_{\{\{1\},\{3,4\},\{2\}\}}) = -2 e2 e1^2$$

$$\chi(X_{\{\{3,4\},\{1,2\}\}}) = 4 e2^2$$

$$\chi(X_{\{\{1,2,3,4\}\}}) = -4 e2 e1^2 - 8 e3 e1 - 4 e4 - 10 e2^2$$

```

> for A in listallsp(5) do
  print(chi(X[A]) = factor(toe(project(X[A]))));
od;

```

$$\chi(X_{\{\{5\},\{1\},\{4\},\{2\},\{3\}\}}) = e1^5$$

$$\chi(X_{\{\{5\},\{4\},\{1,2\},\{3\}\}}) = -2 e2 e1^3$$

$$\chi(X_{\{\{5\},\{1,3\},\{4\},\{2\}\}}) = -2 e2 e1^3$$

$$\chi(X_{\{\{5\},\{1\},\{4\},\{2,3\}\}}) = -2 e2 e1^3$$

$$\chi(X_{\{\{5\},\{4\},\{1,2,3\}\}}) = 3 e1^2 (e1 e2 + e3)$$

$$\chi(X_{\{\{5\},\{2\},\{3\},\{1,4\}\}}) = -2 e2 e1^3$$

$$\chi(X_{\{\{5\},\{2,3\},\{1,4\}\}}) = 4 e1 e2^2$$

$$\chi(X_{\{\{5\},\{1,3,4\},\{2\}\}}) = 3 e1^2 (e1 e2 + e3)$$

$$\chi(X_{\{\{5\},\{1\},\{2,4\},\{3\}\}}) = -2 e2 e1^3$$

$$\chi(X_{\{\{5\},\{1,3\},\{2,4\}\}}) = 4 e1 e2^2$$

$$\chi(X_{\{\{5\},\{1\},\{2,3,4\}\}}) = 3 e1^2 (e1 e2 + e3)$$

$$\chi(X_{\{\{5\},\{1,2,4\},\{3\}\}}) = 3 e1^2 (e1 e2 + e3)$$

$$\chi(X_{\{\{5\},\{1\},\{3,4\},\{2\}\}}) = -2 e2 e1^3$$

$$\chi(X_{\{\{5\},\{3,4\},\{1,2\}\}}) = 4 e1 e2^2$$

$$\chi(X_{\{\{5\},\{1,2,3,4\}\}}) = -2 e1 (2 e2 e1^2 + 4 e1 e3 + 5 e2^2 + 2 e4)$$

$$\chi(X_{\{\{4\},\{2\},\{3\},\{1,5\}\}}) = -2 e2 e1^3$$

$$\chi(X_{\{\{4\},\{2,3\},\{1,5\}\}}) = 4 e1 e2^2$$

$$\chi(X_{\{\{2,4\},\{3\},\{1,5\}\}}) = 4 e1 e2^2$$

$$\chi(X_{\{\{2,3,4\},\{1,5\}\}}) = -6 e2 (e1 e2 + e3)$$

$$\chi(X_{\{\{3,4\},\{2\},\{1,5\}\}}) = 4 e1 e2^2$$

$$\chi(X_{\{\{1,3,5\},\{4\},\{2\}\}}) = 3 e1^2 (e1 e2 + e3)$$

$$\chi(X_{\{\{1,3,5\},\{2,4\}\}}) = -6 e2 (e1 e2 + e3)$$

$$\chi(X_{\{\{4,5\},\{1\},\{2\},\{3\}\}}) = -2 e2 e1^3$$

$$\chi(X_{\{\{4,5\},\{1,2\},\{3\}\}}) = 4 e1 e2^2$$

$$\chi(X_{\{\{4,5\},\{1,3\},\{2\}\}}) = 4 e1 e2^2$$

$$\chi(X_{\{\{4,5\},\{1\},\{2,3\}\}}) = 4 e1 e2^2$$

$$\chi(X_{\{\{4,5\},\{1,2,3\}\}}) = -6 e2 (e1 e2 + e3)$$

$$\chi(X_{\{\{2,3,4,5\},\{1\}\}}) = -2 e1 (2 e2 e1^2 + 4 e1 e3 + 5 e2^2 + 2 e4)$$

$$\chi(X_{\{\{1,2,4,5\},\{3\}\}}) = -2 e1 (2 e2 e1^2 + 4 e1 e3 + 5 e2^2 + 2 e4)$$

$$\chi(X_{\{\{2,4,5\},\{1\},\{3\}\}}) = 3 e1^2 (e1 e2 + e3)$$

$$\chi(X_{\{\{2,4,5\},\{1,3\}\}}) = -6 e2 (e1 e2 + e3)$$

$$\begin{aligned}
\chi(X_{\{\{1,2,3,4,5\}\}}) &= 5 e2 e1^3 + 15 e3 e1^2 + 15 e1 e4 + 55 e1 e2^2 + 55 e3 e2 + 5 e5 \\
\chi(X_{\{\{1,3,4,5\},\{2\}\}}) &= -2 e1 (2 e2 e1^2 + 4 e1 e3 + 2 e4 + 5 e2^2) \\
\chi(X_{\{\{3,4,5\},\{1\},\{2\}\}}) &= 3 e1^2 (e1 e2 + e3) \\
\chi(X_{\{\{3,4,5\},\{1,2\}\}}) &= -6 e2 (e1 e2 + e3) \\
\chi(X_{\{\{1\},\{2,5\},\{4\},\{3\}\}}) &= -2 e2 e1^3 \\
\chi(X_{\{\{2,5\},\{1,3\},\{4\}\}}) &= 4 e1 e2^2 \\
\chi(X_{\{\{2,5\},\{3\},\{1,4\}\}}) &= 4 e1 e2^2 \\
\chi(X_{\{\{2,5\},\{1,3,4\}\}}) &= -6 e2 (e1 e2 + e3) \\
\chi(X_{\{\{1\},\{2,5\},\{3,4\}\}}) &= 4 e1 e2^2 \\
\chi(X_{\{\{2,3,5\},\{1\},\{4\}\}}) &= 3 e1^2 (e1 e2 + e3) \\
\chi(X_{\{\{2,3,5\},\{1,4\}\}}) &= -6 e2 (e1 e2 + e3) \\
\chi(X_{\{\{1,2,5\},\{4\},\{3\}\}}) &= 3 e1^2 (e1 e2 + e3) \\
\chi(X_{\{\{1,2,5\},\{3,4\}\}}) &= -6 e2 (e1 e2 + e3) \\
\chi(X_{\{\{1\},\{3,5\},\{4\},\{2\}\}}) &= -2 e2 e1^3 \\
\chi(X_{\{\{3,5\},\{4\},\{1,2\}\}}) &= 4 e1 e2^2 \\
\chi(X_{\{\{3,5\},\{2\},\{1,4\}\}}) &= 4 e1 e2^2 \\
\chi(X_{\{\{1\},\{3,5\},\{2,4\}\}}) &= 4 e1 e2^2 \\
\chi(X_{\{\{3,5\},\{1,2,4\}\}}) &= -6 e2 (e1 e2 + e3) \\
\chi(X_{\{\{1,4,5\},\{2\},\{3\}\}}) &= 3 e1^2 (e1 e2 + e3) \\
\chi(X_{\{\{1,4,5\},\{2,3\}\}}) &= -6 e2 (e1 e2 + e3) \\
\chi(X_{\{\{1,2,3,5\},\{4\}\}}) &= -2 e1 (2 e2 e1^2 + 4 e1 e3 + 2 e4 + 5 e2^2)
\end{aligned}$$

]