

There is an involution on  $\text{NCSym}$  which I will refer to as  $\text{omega2}$ .

$$\text{omega2}(\text{P}[A]) = (-1)^{|A| - \ell(A)} \text{P}[A]$$

This is likely to be the analogue of the involution  $\text{omega}$  on  $\text{Sym}$  (see Rosas/Sagan)

On  $\text{Sym}$  -  $\text{omega}$  and the antipode are very related so I compute here also

$\text{antipode}(\text{omega2}^*)$  in hopes of seeing a pattern

Note: we know the action of  $\text{omega2}$  on the  $\text{P}$ -basis and we do not know the action of the antipode on any basis

## = Data of $\text{omega2}$ on $\text{X}$ -basis

```
> for i from 1 to 6 do
    print(Omega(X[{{seq(j,j=1..i)}}]) = ToX(ToM(omega2(X[
    {{seq(j,j=1..i)}}])));
od;
```

$$\Omega(X_{\{\{1\}\}}) = X_{\{\{1\}\}}$$

$$\Omega(X_{\{\{1, 2\}\}}) = -X_{\{\{1, 2\}\}} - 2X_{\{\{2\}, \{1\}\}}$$

$$\Omega(X_{\{\{1, 2, 3\}\}}) = X_{\{\{1, 2, 3\}\}} + 2X_{\{\{2\}, \{1, 3\}\}} + 2X_{\{\{1\}, \{2, 3\}\}} + 2X_{\{\{1, 2\}, \{3\}\}} + 6X_{\{\{3\}, \{2\}, \{1\}\}}$$

$$\Omega(X_{\{\{1, 2, 3, 4\}\}}) = -6X_{\{\{2\}, \{1, 3\}, \{4\}\}} - 2X_{\{\{3\}, \{1, 2, 4\}\}} - 6X_{\{\{2\}, \{1\}, \{3, 4\}\}}$$

$$- 26X_{\{\{3\}, \{2\}, \{1\}, \{4\}\}} - 2X_{\{\{1, 2\}, \{3, 4\}\}} - 6X_{\{\{3\}, \{1\}, \{2, 4\}\}} - 2X_{\{\{1, 3\}, \{2, 4\}\}}$$

$$- 2X_{\{\{1, 2, 3\}, \{4\}\}} - 6X_{\{\{3\}, \{2\}, \{1, 4\}\}} - 2X_{\{\{2, 3\}, \{1, 4\}\}} - 2X_{\{\{1\}, \{2, 3, 4\}\}} - 2X_{\{\{2\}, \{1, 3, 4\}\}}$$

$$- X_{\{\{1, 2, 3, 4\}\}} - 6X_{\{\{1\}, \{2, 3\}, \{4\}\}} - 6X_{\{\{1, 2\}, \{3\}, \{4\}\}}$$

$$\Omega(X_{\{\{1, 2, 3, 4, 5\}\}}) = 6X_{\{\{2\}, \{4\}, \{1, 3, 5\}\}} + 26X_{\{\{2\}, \{1, 3\}, \{4\}, \{5\}\}} + 26X_{\{\{2\}, \{1\}, \{4\}, \{3, 5\}\}}$$

$$+ 6X_{\{\{3\}, \{1\}, \{2, 4, 5\}\}} + 2X_{\{\{2, 4\}, \{1, 3, 5\}\}} + 6X_{\{\{3\}, \{1, 2, 4\}, \{5\}\}} + 2X_{\{\{3, 4\}, \{1, 2, 5\}\}}$$

$$+ 6X_{\{\{2, 3\}, \{4\}, \{1, 5\}\}} + 26X_{\{\{2\}, \{1\}, \{3, 4\}, \{5\}\}} + 6X_{\{\{1, 2\}, \{4\}, \{3, 5\}\}}$$

$$+ 26X_{\{\{3\}, \{2\}, \{1\}, \{4, 5\}\}} + 26X_{\{\{3\}, \{2\}, \{4\}, \{1, 5\}\}} + 2X_{\{\{1, 3\}, \{2, 4, 5\}\}}$$

$$+ 6X_{\{\{1, 2\}, \{3, 4\}, \{5\}\}} + 2X_{\{\{1, 2, 4\}, \{3, 5\}\}} + 2X_{\{\{1, 2, 3, 4\}, \{5\}\}} + 6X_{\{\{1, 2\}, \{3\}, \{4, 5\}\}}$$

$$+ 6X_{\{\{2\}, \{3, 4\}, \{1, 5\}\}} + 6X_{\{\{1\}, \{2, 4\}, \{3, 5\}\}} + 2X_{\{\{1, 2, 3\}, \{4, 5\}\}} + 26X_{\{\{3\}, \{1\}, \{2, 4\}, \{5\}\}}$$

$$+ 6X_{\{\{2\}, \{1, 4\}, \{3, 5\}\}} + 2X_{\{\{1, 4\}, \{2, 3, 5\}\}} + 6X_{\{\{3\}, \{2\}, \{1, 4, 5\}\}} + X_{\{\{1, 2, 3, 4, 5\}\}}$$

$$+ 6X_{\{\{1, 3\}, \{2, 4\}, \{5\}\}} + 6X_{\{\{3\}, \{2, 4\}, \{1, 5\}\}} + 6X_{\{\{1\}, \{2, 3\}, \{4, 5\}\}} + 2X_{\{\{4\}, \{1, 2, 3, 5\}\}}$$

$$+ 26X_{\{\{1\}, \{2, 3\}, \{4\}, \{5\}\}} + 26X_{\{\{3\}, \{2\}, \{1, 4\}, \{5\}\}} + 6X_{\{\{2\}, \{1, 3\}, \{4, 5\}\}}$$

$$+ 26X_{\{\{3\}, \{1\}, \{4\}, \{2, 5\}\}} + 6X_{\{\{2, 3\}, \{1, 4\}, \{5\}\}} + 150X_{\{\{3\}, \{2\}, \{1\}, \{4\}, \{5\}\}}$$

$$+ 2X_{\{\{2, 3, 4\}, \{1, 5\}\}} + 6X_{\{\{1, 3\}, \{4\}, \{2, 5\}\}} + 2X_{\{\{2, 3\}, \{1, 4, 5\}\}} + 6X_{\{\{1\}, \{2, 3, 4\}, \{5\}\}}$$

$$+ 6X_{\{\{1\}, \{3, 4\}, \{2, 5\}\}} + 6X_{\{\{2\}, \{1, 3, 4\}, \{5\}\}} + 2X_{\{\{3\}, \{1, 2, 4, 5\}\}} + 26X_{\{\{1, 2\}, \{3\}, \{4\}, \{5\}\}}$$

$$+ 6X_{\{\{3\}, \{1, 4\}, \{2, 5\}\}} + 6X_{\{\{3\}, \{4\}, \{1, 2, 5\}\}} + 2X_{\{\{2\}, \{1, 3, 4, 5\}\}} + 6X_{\{\{2\}, \{1\}, \{3, 4, 5\}\}}$$

$$+ 6X_{\{\{1, 2, 3\}, \{4\}, \{5\}\}} + 6X_{\{\{1\}, \{4\}, \{2, 3, 5\}\}} + 2X_{\{\{1, 3, 4\}, \{2, 5\}\}} + 2X_{\{\{1\}, \{2, 3, 4, 5\}\}}$$

$$+ 2X_{\{\{1, 2\}, \{3, 4, 5\}\}}$$

$$\Omega(X_{\{\{1, 2, 3, 4, 5, 6\}\}}) = -6X_{\{\{1, 4\}, \{6\}, \{2, 3, 5\}\}} - 26X_{\{\{3\}, \{4\}, \{6\}, \{1, 2, 5\}\}}$$

$$- 26X_{\{\{3\}, \{1\}, \{4, 6\}, \{2, 5\}\}} - 2X_{\{\{2, 3\}, \{1, 4, 5, 6\}\}} - 6X_{\{\{4\}, \{1, 2, 3, 6\}, \{5\}\}}$$

$$- 6X_{\{\{2\}, \{3, 4, 6\}, \{1, 5\}\}} - 26X_{\{\{1, 2\}, \{3\}, \{6\}, \{4, 5\}\}} - 26X_{\{\{3\}, \{1, 4\}, \{6\}, \{2, 5\}\}}$$

$$\begin{aligned}
& -6X_{\{1, 2, 3, 4\}, \{6\}, \{5\}} - 26X_{\{3\}, \{1\}, \{4\}, \{2, 5, 6\}} - 2X_{\{1, 2, 3, 4\}, \{5, 6\}} \\
& - 150X_{\{3\}, \{2\}, \{4\}, \{1, 6\}, \{5\}} - 6X_{\{2, 3, 4\}, \{1, 6\}, \{5\}} - 26X_{\{1\}, \{2, 4\}, \{3, 6\}, \{5\}} \\
& - 6X_{\{2\}, \{1, 6\}, \{3, 4, 5\}} - 26X_{\{1, 3\}, \{2, 4\}, \{6\}, \{5\}} - 2X_{\{2, 3, 6\}, \{1, 4, 5\}} \\
& - 26X_{\{1\}, \{2, 3\}, \{4, 6\}, \{5\}} - 26X_{\{1, 2\}, \{4\}, \{6\}, \{3, 5\}} - 26X_{\{3\}, \{2, 4\}, \{1, 6\}, \{5\}} \\
& - 6X_{\{1\}, \{2, 3\}, \{4, 5, 6\}} - 2X_{\{1, 3, 4\}, \{2, 5, 6\}} - X_{\{1, 2, 3, 4, 5, 6\}} - 6X_{\{3\}, \{2, 6\}, \{1, 4, 5\}} \\
& - 6X_{\{1, 2\}, \{4, 6\}, \{3, 5\}} - 26X_{\{1\}, \{2, 3\}, \{6\}, \{4, 5\}} - 6X_{\{1\}, \{6\}, \{2, 3, 4, 5\}} \\
& - 6X_{\{2\}, \{1\}, \{3, 4, 5, 6\}} - 6X_{\{4\}, \{6\}, \{1, 2, 3, 5\}} - 2X_{\{3, 4\}, \{1, 2, 5, 6\}} \\
& - 2X_{\{1, 2, 3, 6\}, \{4, 5\}} - 26X_{\{3\}, \{1\}, \{2, 4\}, \{5, 6\}} - 6X_{\{2\}, \{4, 6\}, \{1, 3, 5\}} \\
& - 6X_{\{1, 4\}, \{2, 6\}, \{3, 5\}} - 6X_{\{2\}, \{1, 3, 6\}, \{4, 5\}} - 26X_{\{3\}, \{4\}, \{1, 2, 6\}, \{5\}} \\
& - 6X_{\{1, 3\}, \{2, 4\}, \{5, 6\}} - 6X_{\{1, 3\}, \{4\}, \{2, 5, 6\}} - 150X_{\{3\}, \{2\}, \{1, 4\}, \{6\}, \{5\}} \\
& - 6X_{\{1, 3\}, \{4, 6\}, \{2, 5\}} - 26X_{\{2\}, \{4\}, \{1, 6\}, \{3, 5\}} - 2X_{\{1, 2\}, \{3, 4, 5, 6\}} \\
& - 26X_{\{2\}, \{1\}, \{6\}, \{3, 4, 5\}} - 26X_{\{1, 2\}, \{3\}, \{4, 6\}, \{5\}} - 6X_{\{3\}, \{2, 4, 6\}, \{1, 5\}} \\
& - 150X_{\{3\}, \{1\}, \{4\}, \{6\}, \{2, 5\}} - 6X_{\{1, 2, 4\}, \{6\}, \{3, 5\}} - 150X_{\{3\}, \{1\}, \{4\}, \{2, 6\}, \{5\}} \\
& - 6X_{\{1\}, \{2, 4, 6\}, \{3, 5\}} - 2X_{\{1, 4\}, \{2, 3, 5, 6\}} - 6X_{\{1, 2, 4\}, \{3, 6\}, \{5\}} \\
& - 6X_{\{2, 3\}, \{6\}, \{1, 4, 5\}} - 6X_{\{3\}, \{1, 4\}, \{2, 5, 6\}} - 26X_{\{2\}, \{1\}, \{3, 4, 6\}, \{5\}} \\
& - 26X_{\{2\}, \{4\}, \{6\}, \{1, 3, 5\}} - 150X_{\{2\}, \{1, 3\}, \{4\}, \{6\}, \{5\}} - 26X_{\{2\}, \{1, 3\}, \{4, 6\}, \{5\}} \\
& - 26X_{\{3\}, \{2\}, \{4\}, \{1, 5, 6\}} - 26X_{\{3\}, \{2\}, \{1, 4\}, \{5, 6\}} - 6X_{\{3\}, \{1, 6\}, \{2, 4, 5\}} \\
& - 26X_{\{2, 3\}, \{4\}, \{6\}, \{1, 5\}} - 2X_{\{1\}, \{2, 3, 4, 5, 6\}} - 26X_{\{2, 3\}, \{4\}, \{1, 6\}, \{5\}} \\
& - 6X_{\{3, 4\}, \{2, 6\}, \{1, 5\}} - 6X_{\{3\}, \{1, 2, 4, 6\}, \{5\}} - 6X_{\{3\}, \{1, 2, 6\}, \{4, 5\}} \\
& - 6X_{\{2\}, \{1, 3\}, \{4, 5, 6\}} - 2X_{\{4\}, \{1, 2, 3, 5, 6\}} - 26X_{\{2\}, \{4\}, \{1, 3, 6\}, \{5\}} \\
& - 26X_{\{1, 3\}, \{4\}, \{2, 6\}, \{5\}} - 26X_{\{3\}, \{2\}, \{6\}, \{1, 4, 5\}} - 6X_{\{2, 3\}, \{1, 4\}, \{5, 6\}} \\
& - 6X_{\{1\}, \{3, 4, 6\}, \{2, 5\}} - 6X_{\{2, 4\}, \{1, 6\}, \{3, 5\}} - 6X_{\{1\}, \{2, 3, 4, 6\}, \{5\}} \\
& - 2X_{\{2, 3, 4, 6\}, \{1, 5\}} - 26X_{\{2, 3\}, \{1, 4\}, \{6\}, \{5\}} - 26X_{\{1\}, \{3, 4\}, \{2, 6\}, \{5\}} \\
& - 6X_{\{4\}, \{2, 3, 6\}, \{1, 5\}} - 26X_{\{2\}, \{1, 3\}, \{6\}, \{4, 5\}} - 6X_{\{1\}, \{2, 3, 4\}, \{5, 6\}} \\
& - 6X_{\{1, 2\}, \{3\}, \{4, 5, 6\}} - 26X_{\{3\}, \{2\}, \{4, 6\}, \{1, 5\}} - 6X_{\{1, 3\}, \{2, 4, 6\}, \{5\}} \\
& - 6X_{\{2\}, \{4\}, \{6\}, \{1, 3, 5, 6\}} - 2X_{\{1, 3, 6\}, \{2, 4, 5\}} - 26X_{\{3\}, \{1, 2, 4\}, \{6\}, \{5\}} \\
& - 6X_{\{3, 4\}, \{6\}, \{1, 2, 5\}} - 6X_{\{1\}, \{3, 4\}, \{2, 5, 6\}} - 2X_{\{1, 2, 4\}, \{3, 5, 6\}} \\
& - 6X_{\{2, 3, 4\}, \{6\}, \{1, 5\}} - 6X_{\{2, 3\}, \{4\}, \{1, 5, 6\}} - 26X_{\{1\}, \{2, 4\}, \{6\}, \{3, 5\}} \\
& - 26X_{\{3\}, \{1, 4\}, \{2, 6\}, \{5\}} - 6X_{\{3\}, \{2\}, \{1, 4, 5, 6\}} - 2X_{\{1, 6\}, \{2, 3, 4, 5\}} \\
& - 26X_{\{3\}, \{2\}, \{1, 6\}, \{4, 5\}} - 2X_{\{2, 4, 6\}, \{1, 3, 5\}} - 26X_{\{3\}, \{1\}, \{2, 6\}, \{4, 5\}} \\
& - 26X_{\{2\}, \{3, 4\}, \{6\}, \{1, 5\}} - 26X_{\{2\}, \{3, 4\}, \{1, 6\}, \{5\}} - 6X_{\{3\}, \{4, 6\}, \{1, 2, 5\}} \\
& - 6X_{\{3\}, \{1\}, \{2, 4, 5, 6\}} - 26X_{\{3\}, \{4\}, \{1, 6\}, \{2, 5\}} - 26X_{\{1, 3\}, \{4\}, \{6\}, \{2, 5\}} \\
& - 6X_{\{2, 4\}, \{6\}, \{1, 3, 5\}} - 26X_{\{1\}, \{4\}, \{6\}, \{2, 3, 5\}} - 6X_{\{2\}, \{1, 3, 4\}, \{5, 6\}} \\
& - 6X_{\{1, 3, 4\}, \{6\}, \{2, 5\}} - 6X_{\{1, 3, 4\}, \{2, 6\}, \{5\}} - 26X_{\{1\}, \{2, 3, 4\}, \{6\}, \{5\}} \\
& - 2X_{\{4, 6\}, \{1, 2, 3, 5\}} - 2X_{\{3\}, \{1, 2, 4, 5, 6\}} - 6X_{\{3, 4\}, \{1, 2, 6\}, \{5\}} \\
& - 6X_{\{2, 4\}, \{1, 3, 6\}, \{5\}} - 2X_{\{2, 3, 4\}, \{1, 5, 6\}} - 6X_{\{1\}, \{2, 4\}, \{3, 5, 6\}} \\
& - 6X_{\{3\}, \{4\}, \{1, 2, 5, 6\}} - 150X_{\{2\}, \{1\}, \{3, 4\}, \{6\}, \{5\}} - 2X_{\{3, 4, 6\}, \{1, 2, 5\}} \\
& - 6X_{\{2, 3\}, \{4, 6\}, \{1, 5\}} - 26X_{\{1\}, \{4\}, \{2, 6\}, \{3, 5\}} - 6X_{\{1, 2\}, \{6\}, \{3, 4, 5\}} \\
& - 2X_{\{2\}, \{1, 3, 4, 5, 6\}} - 150X_{\{1\}, \{2, 3\}, \{4\}, \{6\}, \{5\}} - 2X_{\{2, 6\}, \{1, 3, 4, 5\}} \\
& - 6X_{\{3, 4\}, \{1, 6\}, \{2, 5\}} - 6X_{\{2\}, \{3, 4\}, \{1, 5, 6\}} - 6X_{\{1, 2, 3\}, \{6\}, \{4, 5\}} \\
& - 26X_{\{2\}, \{1, 4\}, \{6\}, \{3, 5\}} - 26X_{\{1, 2\}, \{4\}, \{3, 6\}, \{5\}} - 6X_{\{2\}, \{6\}, \{1, 3, 4, 5\}} \\
& - 2X_{\{1, 2, 6\}, \{3, 4, 5\}} - 26X_{\{1\}, \{4\}, \{2, 3, 6\}, \{5\}} - 6X_{\{1, 3\}, \{6\}, \{2, 4, 5\}} \\
& - 2X_{\{2, 4\}, \{1, 3, 5, 6\}} - 26X_{\{1\}, \{3, 4\}, \{6\}, \{2, 5\}} - 150X_{\{3\}, \{2\}, \{1\}, \{4, 6\}, \{5\}} \\
& - 6X_{\{1, 2, 3\}, \{4, 6\}, \{5\}} - 26X_{\{3\}, \{2, 4\}, \{6\}, \{1, 5\}} - 26X_{\{3\}, \{4\}, \{2, 6\}, \{1, 5\}}
\end{aligned}$$

$$\begin{aligned}
& -26X_{\{2\}, \{1, 3, 4\}, \{6\}, \{5\}} - 26X_{\{2\}, \{1, 4\}, \{3, 6\}, \{5\}} - 6X_{\{2\}, \{1, 4\}, \{3, 5, 6\}} \\
& - 2X_{\{1, 2, 3\}, \{4, 5, 6\}} - 26X_{\{3\}, \{2\}, \{1, 4, 6\}, \{5\}} - 6X_{\{1, 2\}, \{3, 4, 6\}, \{5\}} \\
& - 150X_{\{3\}, \{2\}, \{1\}, \{6\}, \{4, 5\}} - 1082X_{\{3\}, \{2\}, \{1\}, \{4\}, \{6\}, \{5\}} - 26X_{\{1, 2\}, \{3, 4\}, \{6\}, \{5\}} \\
& - 6X_{\{1\}, \{2, 3, 6\}, \{4, 5\}} - 26X_{\{2\}, \{1\}, \{4, 6\}, \{3, 5\}} - 150X_{\{2\}, \{1\}, \{4\}, \{6\}, \{3, 5\}} \\
& - 2X_{\{6\}, \{1, 2, 3, 4, 5\}} - 26X_{\{2\}, \{1\}, \{4\}, \{3, 5, 6\}} - 6X_{\{3\}, \{6\}, \{1, 2, 4, 5\}} \\
& - 26X_{\{2\}, \{4\}, \{3, 6\}, \{1, 5\}} - 6X_{\{1\}, \{4, 6\}, \{2, 3, 5\}} - 6X_{\{4\}, \{2, 6\}, \{1, 3, 5\}} \\
& - 6X_{\{1, 4\}, \{2, 3, 6\}, \{5\}} - 6X_{\{4\}, \{1, 6\}, \{2, 3, 5\}} - 2X_{\{1, 3\}, \{2, 4, 5, 6\}} \\
& - 26X_{\{3\}, \{2\}, \{1\}, \{4, 5, 6\}} - 150X_{\{1, 2\}, \{3\}, \{4\}, \{6\}, \{5\}} - 6X_{\{1, 2\}, \{4\}, \{3, 5, 6\}} \\
& - 2X_{\{1, 2, 3, 4, 6\}, \{5\}} - 6X_{\{2, 3\}, \{1, 4, 6\}, \{5\}} - 6X_{\{3\}, \{1, 4, 6\}, \{2, 5\}} \\
& - 6X_{\{4\}, \{1, 2, 6\}, \{3, 5\}} - 2X_{\{1, 4, 6\}, \{2, 3, 5\}} - 6X_{\{1, 3\}, \{2, 6\}, \{4, 5\}} \\
& - 26X_{\{3\}, \{1\}, \{6\}, \{2, 4, 5\}} - 6X_{\{2\}, \{1, 3, 4, 6\}, \{5\}} - 6X_{\{1\}, \{4\}, \{2, 3, 5, 6\}} \\
& - 6X_{\{3\}, \{2, 4\}, \{1, 5, 6\}} - 26X_{\{2\}, \{1\}, \{3, 6\}, \{4, 5\}} - 150X_{\{2\}, \{1\}, \{4\}, \{3, 6\}, \{5\}} \\
& - 26X_{\{1, 2, 3\}, \{4\}, \{6\}, \{5\}} - 26X_{\{3\}, \{1\}, \{2, 4, 6\}, \{5\}} - 6X_{\{2, 4\}, \{3, 6\}, \{1, 5\}} \\
& - 6X_{\{4\}, \{3, 6\}, \{1, 2, 5\}} - 2X_{\{1, 3, 4, 6\}, \{2, 5\}} - 26X_{\{2\}, \{1\}, \{3, 4\}, \{5, 6\}} \\
& - 2X_{\{1, 2, 4, 6\}, \{3, 5\}} - 6X_{\{1, 2, 3\}, \{4\}, \{5, 6\}} - 6X_{\{1\}, \{2, 6\}, \{3, 4, 5\}} \\
& - 6X_{\{2\}, \{1, 4, 6\}, \{3, 5\}} - 26X_{\{1\}, \{4\}, \{3, 6\}, \{2, 5\}} - 6X_{\{1\}, \{3, 6\}, \{2, 4, 5\}} \\
& - 26X_{\{2\}, \{1, 3\}, \{4\}, \{5, 6\}} - 6X_{\{1, 2\}, \{3, 6\}, \{4, 5\}} - 6X_{\{1, 2\}, \{3, 4\}, \{5, 6\}} \\
& - 6X_{\{2\}, \{3, 6\}, \{1, 4, 5\}} - 150X_{\{3\}, \{2\}, \{1\}, \{4\}, \{5, 6\}} - 6X_{\{1, 4\}, \{3, 6\}, \{2, 5\}} \\
& - 6X_{\{2, 3\}, \{1, 6\}, \{4, 5\}} - 150X_{\{3\}, \{1\}, \{2, 4\}, \{6\}, \{5\}} - 26X_{\{1, 2\}, \{3\}, \{4\}, \{5, 6\}} \\
& - 6X_{\{3\}, \{1, 2, 4\}, \{5, 6\}} - 150X_{\{3\}, \{2\}, \{4\}, \{6\}, \{1, 5\}} - 26X_{\{1\}, \{2, 3\}, \{4\}, \{5, 6\}} \\
& - 2X_{\{3, 6\}, \{1, 2, 4, 5\}} - 6X_{\{4\}, \{1, 3, 6\}, \{2, 5\}}
\end{aligned}$$

```

> for i from 1 to 4 do
  for A in mylasp(i) do
    print(Omega(X[A]) = ToX(ToM(omega2(X[A]))));
  od;
od;


$$\Omega(X_{\{1\}}) = X_{\{1\}}$$


$$\Omega(X_{\{2\}, \{1\}}) = X_{\{2\}, \{1\}}$$


$$\Omega(X_{\{1, 2\}}) = -X_{\{1, 2\}} - 2X_{\{2\}, \{1\}}$$


$$\Omega(X_{\{3\}, \{2\}, \{1\}}) = X_{\{3\}, \{2\}, \{1\}}$$


$$\Omega(X_{\{2\}, \{1, 3\}}) = -X_{\{2\}, \{1, 3\}} - 2X_{\{3\}, \{2\}, \{1\}}$$


$$\Omega(X_{\{1\}, \{2, 3\}}) = -X_{\{1\}, \{2, 3\}} - 2X_{\{3\}, \{2\}, \{1\}}$$


$$\Omega(X_{\{1, 2\}, \{3\}}) = -X_{\{1, 2\}, \{3\}} - 2X_{\{3\}, \{2\}, \{1\}}$$


$$\Omega(X_{\{1, 2, 3\}}) = X_{\{1, 2, 3\}} + 2X_{\{2\}, \{1, 3\}} + 2X_{\{1\}, \{2, 3\}} + 2X_{\{1, 2\}, \{3\}} + 6X_{\{3\}, \{2\}, \{1\}}$$


$$\Omega(X_{\{3\}, \{2\}, \{1\}, \{4\}}) = X_{\{3\}, \{2\}, \{1\}, \{4\}}$$


$$\Omega(X_{\{3\}, \{2\}, \{1, 4\}}) = -2X_{\{3\}, \{2\}, \{1\}, \{4\}} - X_{\{3\}, \{2\}, \{1, 4\}}$$


$$\Omega(X_{\{3\}, \{1\}, \{2, 4\}}) = -2X_{\{3\}, \{2\}, \{1\}, \{4\}} - X_{\{3\}, \{1\}, \{2, 4\}}$$


$$\Omega(X_{\{2\}, \{1\}, \{3, 4\}}) = -X_{\{2\}, \{1\}, \{3, 4\}} - 2X_{\{3\}, \{2\}, \{1\}, \{4\}}$$


$$\Omega(X_{\{2\}, \{1, 3\}, \{4\}}) = -X_{\{2\}, \{1, 3\}, \{4\}} - 2X_{\{3\}, \{2\}, \{1\}, \{4\}}$$


```

$$\Omega(X_{\{1, 2, 3, 4\}}) = -2X_{\{3, 2, 1, 4\}} - X_{\{1, 2, 3, 4\}}$$

$$\Omega(X_{\{1, 2, \{3, 4\}\}}) = -2X_{\{3, 2, \{1, 4\}\}} - X_{\{1, 2, \{3, 4\}\}}$$

$$\Omega(X_{\{2, \{1, 3, 4\}\}}) = 2X_{\{2, \{1, 3, 4\}\}} + 2X_{\{2, \{1, \{3, 4\}\}\}} + 6X_{\{3, \{2, \{1, \{4\}\}\}\}}$$

$$+ 2X_{\{3, \{2, \{1, 4\}\}\}} + X_{\{2, \{1, 3, 4\}\}}$$

$$\Omega(X_{\{1, \{2, 3, 4\}\}}) = 2X_{\{2, \{1, \{3, 4\}\}\}} + 6X_{\{3, \{2, \{1, \{4\}\}\}\}} + 2X_{\{3, \{1, \{2, 4\}\}\}}$$

$$+ X_{\{1, \{2, 3, 4\}\}} + 2X_{\{1, \{2, 3\}, \{4\}\}}$$

$$\Omega(X_{\{2, 3, \{1, 4\}\}}) = 4X_{\{3, \{2, \{1, \{4\}\}\}\}} + 2X_{\{3, \{2, \{1, 4\}\}\}} + X_{\{2, 3, \{1, 4\}\}}$$

$$+ 2X_{\{1, \{2, 3\}, \{4\}\}}$$

$$\Omega(X_{\{1, 3, \{2, 4\}\}}) = 2X_{\{2, \{1, 3, 4\}\}} + 4X_{\{3, \{2, \{1, \{4\}\}\}\}} + 2X_{\{3, \{1, \{2, 4\}\}\}}$$

$$+ X_{\{1, 3, \{2, 4\}\}}$$

$$\Omega(X_{\{1, 2, \{3, 4\}\}}) = 2X_{\{2, \{1, \{3, 4\}\}\}} + 4X_{\{3, \{2, \{1, \{4\}\}\}\}} + X_{\{1, 2, \{3, 4\}\}}$$

$$+ 2X_{\{1, 2, \{3\}, \{4\}\}}$$

$$\Omega(X_{\{3, \{1, 2, 4\}\}}) = X_{\{3, \{1, 2, 4\}\}} + 6X_{\{3, \{2, \{1, \{4\}\}\}\}} + 2X_{\{3, \{1, \{2, 4\}\}\}}$$

$$+ 2X_{\{3, \{2, \{1, 4\}\}\}} + 2X_{\{1, 2, \{3, \{4\}\}\}}$$

$$\Omega(X_{\{1, 2, 3, \{4\}\}}) = 2X_{\{2, \{1, 3, \{4\}\}\}} + 6X_{\{3, \{2, \{1, \{4\}\}\}\}} + X_{\{1, 2, 3, \{4\}\}}$$

$$+ 2X_{\{1, \{2, 3\}, \{4\}\}} + 2X_{\{1, 2, \{3, \{4\}\}\}}$$

$$\Omega(X_{\{1, 2, 3, 4\}}) = -6X_{\{2, \{1, 3, \{4\}\}\}} - 2X_{\{3, \{1, 2, 4\}\}} - 6X_{\{2, \{1, \{3, 4\}\}\}}$$

$$- 26X_{\{3, \{2, \{1, \{4\}\}\}\}} - 2X_{\{1, 2, \{3, 4\}\}} - 6X_{\{3, \{1, \{2, 4\}\}\}} - 2X_{\{1, 3, \{2, 4\}\}}$$

$$- 2X_{\{1, 2, 3, \{4\}\}} - 6X_{\{3, \{2, \{1, 4\}\}\}} - 2X_{\{2, 3, \{1, 4\}\}} - 2X_{\{1, \{2, 3, 4\}\}} - 2X_{\{2, \{1, 3, 4\}\}}$$

$$- X_{\{1, 2, 3, 4\}} - 6X_{\{1, \{2, 3, \{4\}\}\}} - 6X_{\{1, 2, \{3, \{4\}\}\}}$$

&gt;

## = omega2 on the M-basis

```
> for i from 1 to 6 do
    print(Omega(M[{\seq(j,j=1..i)}])) =
ToM(omega2(M[{\seq(j,j=1..i)}]));
od;

$$\Omega(M_{\{1\}}) = M_{\{1\}}$$


$$\Omega(M_{\{1, 2\}}) = -M_{\{1, 2\}}$$


$$\Omega(M_{\{1, 2, 3\}}) = M_{\{1, 2, 3\}}$$


$$\Omega(M_{\{1, 2, 3, 4\}}) = -M_{\{1, 2, 3, 4\}}$$


$$\Omega(M_{\{1, 2, 3, 4, 5\}}) = M_{\{1, 2, 3, 4, 5\}}$$


$$\Omega(M_{\{1, 2, 3, 4, 5, 6\}}) = -M_{\{1, 2, 3, 4, 5, 6\}}$$

> for i from 1 to 5 do
    for A in mylasp(i) do
        print(Omega(M[A]) = ToM(omega2(M[A])));
    od;
od;
```

$$\Omega(M_{\{1\}}) = M_{\{1\}}$$

$$\Omega(M_{\{2\}, \{1\}}) = M_{\{2\}, \{1\}} + 2 M_{\{1, 2\}}$$

$$\Omega(M_{\{1, 2\}}) = -M_{\{1, 2\}}$$

$$\begin{aligned} \Omega(M_{\{3\}, \{2\}, \{1\}}) &= M_{\{3\}, \{2\}, \{1\}} + 2 M_{\{1, 2\}, \{3\}} + 6 M_{\{1, 2, 3\}} + 2 M_{\{1\}, \{2, 3\}} \\ &+ 2 M_{\{2\}, \{1, 3\}} \end{aligned}$$

$$\Omega(M_{\{2\}, \{1, 3\}}) = -2 M_{\{1, 2, 3\}} - M_{\{2\}, \{1, 3\}}$$

$$\Omega(M_{\{1\}, \{2, 3\}}) = -2 M_{\{1, 2, 3\}} - M_{\{1\}, \{2, 3\}}$$

$$\Omega(M_{\{1, 2\}, \{3\}}) = -M_{\{1, 2\}, \{3\}} - 2 M_{\{1, 2, 3\}}$$

$$\Omega(M_{\{1, 2, 3\}}) = M_{\{1, 2, 3\}}$$

$$\begin{aligned} \Omega(M_{\{3\}, \{2\}, \{1\}, \{4\}}) &= 2 M_{\{2\}, \{1, 3\}, \{4\}} + 6 M_{\{3\}, \{1, 2, 4\}} + 2 M_{\{1\}, \{2, 3\}, \{4\}} \\ &+ 2 M_{\{1, 2\}, \{3\}, \{4\}} + 6 M_{\{1, 2, 3\}, \{4\}} + M_{\{3\}, \{2\}, \{1\}, \{4\}} + 6 M_{\{2\}, \{1, 3, 4\}} \\ &+ 6 M_{\{1\}, \{2, 3, 4\}} + 4 M_{\{2, 3\}, \{1, 4\}} + 2 M_{\{3\}, \{2\}, \{1, 4\}} + 4 M_{\{1, 3\}, \{2, 4\}} \\ &+ 4 M_{\{1, 2\}, \{3, 4\}} + 24 M_{\{1, 2, 3, 4\}} + 2 M_{\{3\}, \{1\}, \{2, 4\}} + 2 M_{\{2\}, \{1\}, \{3, 4\}} \end{aligned}$$

$$\begin{aligned} \Omega(M_{\{3\}, \{2\}, \{1, 4\}}) &= -2 M_{\{3\}, \{1, 2, 4\}} - 6 M_{\{1, 2, 3, 4\}} - M_{\{3\}, \{2\}, \{1, 4\}} - 2 M_{\{2, 3\}, \{1, 4\}} \\ &- 2 M_{\{2\}, \{1, 3, 4\}} \end{aligned}$$

$$\begin{aligned} \Omega(M_{\{3\}, \{1\}, \{2, 4\}}) &= -2 M_{\{3\}, \{1, 2, 4\}} - 6 M_{\{1, 2, 3, 4\}} - M_{\{3\}, \{1\}, \{2, 4\}} - 2 M_{\{1, 3\}, \{2, 4\}} \\ &- 2 M_{\{1\}, \{2, 3, 4\}} \end{aligned}$$

$$\begin{aligned} \Omega(M_{\{2\}, \{1\}, \{3, 4\}}) &= -M_{\{2\}, \{1\}, \{3, 4\}} - 2 M_{\{1, 2\}, \{3, 4\}} - 6 M_{\{1, 2, 3, 4\}} - 2 M_{\{1\}, \{2, 3, 4\}} \\ &- 2 M_{\{2\}, \{1, 3, 4\}} \end{aligned}$$

$$\begin{aligned} \Omega(M_{\{2\}, \{1, 3\}, \{4\}}) &= -2 M_{\{1, 2, 3\}, \{4\}} - 6 M_{\{1, 2, 3, 4\}} - M_{\{2\}, \{1, 3\}, \{4\}} - 2 M_{\{1, 3\}, \{2, 4\}} \\ &- 2 M_{\{2\}, \{1, 3, 4\}} \end{aligned}$$

$$\begin{aligned} \Omega(M_{\{1\}, \{2, 3\}, \{4\}}) &= -2 M_{\{1, 2, 3\}, \{4\}} - 6 M_{\{1, 2, 3, 4\}} - M_{\{1\}, \{2, 3\}, \{4\}} - 2 M_{\{2, 3\}, \{1, 4\}} \\ &- 2 M_{\{1\}, \{2, 3, 4\}} \end{aligned}$$

$$\begin{aligned} \Omega(M_{\{1, 2\}, \{3\}, \{4\}}) &= -M_{\{1, 2\}, \{3\}, \{4\}} - 2 M_{\{1, 2, 3\}, \{4\}} - 2 M_{\{3\}, \{1, 2, 4\}} - 2 M_{\{1, 2\}, \{3, 4\}} \\ &- 6 M_{\{1, 2, 3, 4\}} \end{aligned}$$

$$\Omega(M_{\{2\}, \{1, 3, 4\}}) = 2 M_{\{1, 2, 3, 4\}} + M_{\{2\}, \{1, 3, 4\}}$$

$$\Omega(M_{\{1\}, \{2, 3, 4\}}) = 2 M_{\{1, 2, 3, 4\}} + M_{\{1\}, \{2, 3, 4\}}$$

$$\Omega(M_{\{2, 3\}, \{1, 4\}}) = 2 M_{\{1, 2, 3, 4\}} + M_{\{2, 3\}, \{1, 4\}}$$

$$\Omega(M_{\{1, 3\}, \{2, 4\}}) = 2 M_{\{1, 2, 3, 4\}} + M_{\{1, 3\}, \{2, 4\}}$$

$$\Omega(M_{\{1, 2\}, \{3, 4\}}) = M_{\{1, 2\}, \{3, 4\}} + 2 M_{\{1, 2, 3, 4\}}$$

$$\Omega(M_{\{3\}, \{1, 2, 4\}}) = M_{\{3\}, \{1, 2, 4\}} + 2 M_{\{1, 2, 3, 4\}}$$

$$\Omega(M_{\{1, 2, 3\}, \{4\}}) = M_{\{1, 2, 3\}, \{4\}} + 2 M_{\{1, 2, 3, 4\}}$$

$$\Omega(M_{\{1, 2, 3, 4\}}) = -M_{\{1, 2, 3, 4\}}$$

$$\Omega(M_{\{3\}, \{2\}, \{1\}, \{4\}, \{5\}}) = 4 M_{\{2, 3\}, \{1, 4\}, \{5\}} + 6 M_{\{3\}, \{2\}, \{1, 4, 5\}} + 4 M_{\{2\}, \{3, 4\}, \{1, 5\}}$$

$$\begin{aligned}
& + 4 M_{\{\{1, 3\}, \{2, 4\}, \{5\}\}} + 6 M_{\{\{1\}, \{2, 3, 4\}, \{5\}\}} + 4 M_{\{\{1\}, \{2, 3\}, \{4, 5\}\}} + 24 M_{\{\{1, 2, 3, 4\}, \{5\}\}} \\
& + 2 M_{\{\{3\}, \{2\}, \{1\}, \{4, 5\}\}} + 24 M_{\{\{3\}, \{1, 2, 4, 5\}\}} + 4 M_{\{\{2, 3\}, \{4\}, \{1, 5\}\}} \\
& + 4 M_{\{\{1\}, \{2, 4\}, \{3, 5\}\}} + 12 M_{\{\{2, 3, 4\}, \{1, 5\}\}} + 4 M_{\{\{1, 3\}, \{4\}, \{2, 5\}\}} + 24 M_{\{\{1\}, \{2, 3, 4, 5\}\}} \\
& + 12 M_{\{\{2, 4\}, \{1, 3, 5\}\}} + 6 M_{\{\{3\}, \{1, 2, 4\}, \{5\}\}} + 4 M_{\{\{1, 2\}, \{3, 4\}, \{5\}\}} \\
& + 2 M_{\{\{1\}, \{2, 3\}, \{4\}, \{5\}\}} + 2 M_{\{\{1, 2\}, \{3\}, \{4\}, \{5\}\}} + 12 M_{\{\{3, 4\}, \{1, 2, 5\}\}} \\
& + 4 M_{\{\{2\}, \{1, 3\}, \{4, 5\}\}} + 4 M_{\{\{3\}, \{2, 4\}, \{1, 5\}\}} + 6 M_{\{\{3\}, \{4\}, \{1, 2, 5\}\}} + 12 M_{\{\{1, 2, 3\}, \{4, 5\}\}} \\
& + 4 M_{\{\{1, 2\}, \{3\}, \{4, 5\}\}} + 6 M_{\{\{1, 2, 3\}, \{4\}, \{5\}\}} + 2 M_{\{\{2\}, \{1\}, \{3, 4\}, \{5\}\}} \\
& + 2 M_{\{\{2\}, \{1\}, \{4\}, \{3, 5\}\}} + 12 M_{\{\{1, 2, 4\}, \{3, 5\}\}} + 4 M_{\{\{1, 2\}, \{4\}, \{3, 5\}\}} \\
& + 4 M_{\{\{1\}, \{3, 4\}, \{2, 5\}\}} + 6 M_{\{\{2\}, \{4\}, \{1, 3, 5\}\}} + 6 M_{\{\{1\}, \{4\}, \{2, 3, 5\}\}} + 120 M_{\{\{1, 2, 3, 4, 5\}\}} \\
& + 2 M_{\{\{3\}, \{1\}, \{2, 4\}, \{5\}\}} + 6 M_{\{\{3\}, \{1\}, \{2, 4, 5\}\}} + 24 M_{\{\{2\}, \{1, 3, 4, 5\}\}} \\
& + 12 M_{\{\{1, 3\}, \{2, 4, 5\}\}} + 4 M_{\{\{2\}, \{1, 4\}, \{3, 5\}\}} + 2 M_{\{\{3\}, \{2\}, \{1, 4\}, \{5\}\}} \\
& + 2 M_{\{\{3\}, \{1\}, \{4\}, \{2, 5\}\}} + 2 M_{\{\{2\}, \{1, 3\}, \{4\}, \{5\}\}} + M_{\{\{3\}, \{2\}, \{1\}, \{4\}, \{5\}\}} \\
& + 12 M_{\{\{1, 2\}, \{3, 4, 5\}\}} + 12 M_{\{\{1, 4\}, \{2, 3, 5\}\}} + 2 M_{\{\{3\}, \{2\}, \{4\}, \{1, 5\}\}} + 24 M_{\{\{4\}, \{1, 2, 3, 5\}\}} \\
& + 6 M_{\{\{2\}, \{1, 3, 4\}, \{5\}\}} + 12 M_{\{\{2, 3\}, \{1, 4, 5\}\}} + 4 M_{\{\{3\}, \{1, 4\}, \{2, 5\}\}} + 6 M_{\{\{2\}, \{1\}, \{3, 4, 5\}\}} \\
& + 12 M_{\{\{1, 3, 4\}, \{2, 5\}\}}
\end{aligned}$$

$$\begin{aligned}
\Omega(M_{\{\{3\}, \{2\}, \{1\}, \{4, 5\}\}}) = & -2 M_{\{\{3\}, \{2\}, \{1, 4, 5\}\}} - 2 M_{\{\{1\}, \{2, 3\}, \{4, 5\}\}} - M_{\{\{3\}, \{2\}, \{1\}, \{4, 5\}\}} \\
& - 6 M_{\{\{3\}, \{1, 2, 4, 5\}\}} - 6 M_{\{\{1\}, \{2, 3, 4, 5\}\}} - 2 M_{\{\{2\}, \{1, 3\}, \{4, 5\}\}} - 6 M_{\{\{1, 2, 3\}, \{4, 5\}\}} \\
& - 2 M_{\{\{1, 2\}, \{3\}, \{4, 5\}\}} - 24 M_{\{\{1, 2, 3, 4, 5\}\}} - 2 M_{\{\{3\}, \{1\}, \{2, 4, 5\}\}} - 6 M_{\{\{2\}, \{1, 3, 4, 5\}\}} \\
& - 4 M_{\{\{1, 3\}, \{2, 4, 5\}\}} - 4 M_{\{\{1, 2\}, \{3, 4, 5\}\}} - 4 M_{\{\{2, 3\}, \{1, 4, 5\}\}} - 2 M_{\{\{2\}, \{1\}, \{3, 4, 5\}\}}
\end{aligned}$$

$$\begin{aligned}
\Omega(M_{\{\{3\}, \{2\}, \{4\}, \{1, 5\}\}}) = & -2 M_{\{\{3\}, \{2\}, \{1, 4, 5\}\}} - 2 M_{\{\{2\}, \{3, 4\}, \{1, 5\}\}} - 6 M_{\{\{3\}, \{1, 2, 4, 5\}\}} \\
& - 2 M_{\{\{2, 3\}, \{4\}, \{1, 5\}\}} - 6 M_{\{\{2, 3, 4\}, \{1, 5\}\}} - 4 M_{\{\{2, 4\}, \{1, 3, 5\}\}} - 4 M_{\{\{3, 4\}, \{1, 2, 5\}\}} \\
& - 2 M_{\{\{3\}, \{2, 4\}, \{1, 5\}\}} - 2 M_{\{\{3\}, \{4\}, \{1, 2, 5\}\}} - 2 M_{\{\{2\}, \{4\}, \{1, 3, 5\}\}} - 24 M_{\{\{1, 2, 3, 4, 5\}\}} \\
& - 6 M_{\{\{2\}, \{1, 3, 4, 5\}\}} - M_{\{\{3\}, \{2\}, \{4\}, \{1, 5\}\}} - 6 M_{\{\{4\}, \{1, 2, 3, 5\}\}} - 4 M_{\{\{2, 3\}, \{1, 4, 5\}\}}
\end{aligned}$$

$$\begin{aligned}
\Omega(M_{\{\{3\}, \{1\}, \{4\}, \{2, 5\}\}}) = & -6 M_{\{\{3\}, \{1, 2, 4, 5\}\}} - 2 M_{\{\{1, 3\}, \{4\}, \{2, 5\}\}} - 6 M_{\{\{1\}, \{2, 3, 4, 5\}\}} \\
& - 4 M_{\{\{3, 4\}, \{1, 2, 5\}\}} - 2 M_{\{\{3\}, \{4\}, \{1, 2, 5\}\}} - 2 M_{\{\{1\}, \{3, 4\}, \{2, 5\}\}} - 2 M_{\{\{1\}, \{4\}, \{2, 3, 5\}\}} \\
& - 24 M_{\{\{1, 2, 3, 4, 5\}\}} - 2 M_{\{\{3\}, \{1\}, \{2, 4, 5\}\}} - 4 M_{\{\{1, 3\}, \{2, 4, 5\}\}} - M_{\{\{3\}, \{1\}, \{4\}, \{2, 5\}\}} \\
& - 4 M_{\{\{1, 4\}, \{2, 3, 5\}\}} - 6 M_{\{\{4\}, \{1, 2, 3, 5\}\}} - 2 M_{\{\{3\}, \{1, 4\}, \{2, 5\}\}} - 6 M_{\{\{1, 3, 4\}, \{2, 5\}\}}
\end{aligned}$$

$$\begin{aligned}
\Omega(M_{\{\{2\}, \{1\}, \{4\}, \{3, 5\}\}}) = & -2 M_{\{\{1\}, \{2, 4\}, \{3, 5\}\}} - 6 M_{\{\{1\}, \{2, 3, 4, 5\}\}} - 4 M_{\{\{2, 4\}, \{1, 3, 5\}\}} \\
& - M_{\{\{2\}, \{1\}, \{4\}, \{3, 5\}\}} - 6 M_{\{\{1, 2, 4\}, \{3, 5\}\}} - 2 M_{\{\{1, 2\}, \{4\}, \{3, 5\}\}} - 2 M_{\{\{2\}, \{4\}, \{1, 3, 5\}\}} \\
& - 2 M_{\{\{1\}, \{4\}, \{2, 3, 5\}\}} - 24 M_{\{\{1, 2, 3, 4, 5\}\}} - 6 M_{\{\{2\}, \{1, 3, 4, 5\}\}} - 2 M_{\{\{2\}, \{1, 4\}, \{3, 5\}\}} \\
& - 4 M_{\{\{1, 2\}, \{3, 4, 5\}\}} - 4 M_{\{\{1, 4\}, \{2, 3, 5\}\}} - 6 M_{\{\{4\}, \{1, 2, 3, 5\}\}} - 2 M_{\{\{2\}, \{1\}, \{3, 4, 5\}\}}
\end{aligned}$$

$$\begin{aligned}
\Omega(M_{\{\{3\}, \{2\}, \{1, 4\}, \{5\}\}}) = & -2 M_{\{\{2, 3\}, \{1, 4\}, \{5\}\}} - 2 M_{\{\{3\}, \{2\}, \{1, 4, 5\}\}} - 6 M_{\{\{1, 2, 3, 4\}, \{5\}\}} \\
& - 6 M_{\{\{3\}, \{1, 2, 4, 5\}\}} - 2 M_{\{\{3\}, \{1, 2, 4\}, \{5\}\}} - 4 M_{\{\{1, 2, 4\}, \{3, 5\}\}} - 24 M_{\{\{1, 2, 3, 4, 5\}\}} \\
& - 6 M_{\{\{2\}, \{1, 3, 4, 5\}\}} - 2 M_{\{\{2\}, \{1, 4\}, \{3, 5\}\}} - M_{\{\{3\}, \{2\}, \{1, 4\}, \{5\}\}} - 6 M_{\{\{1, 4\}, \{2, 3, 5\}\}} \\
& - 2 M_{\{\{2\}, \{1, 3, 4\}, \{5\}\}} - 4 M_{\{\{2, 3\}, \{1, 4, 5\}\}} - 2 M_{\{\{3\}, \{1, 4\}, \{2, 5\}\}} - 4 M_{\{\{1, 3, 4\}, \{2, 5\}\}}
\end{aligned}$$

$$\begin{aligned}
\Omega(M_{\{\{3\}, \{1\}, \{2, 4\}, \{5\}\}}) = & -2 M_{\{\{1, 3\}, \{2, 4\}, \{5\}\}} - 2 M_{\{\{1\}, \{2, 3, 4\}, \{5\}\}} - 6 M_{\{\{1, 2, 3, 4\}, \{5\}\}} \\
& - 6 M_{\{\{3\}, \{1, 2, 4, 5\}\}} - 2 M_{\{\{1\}, \{2, 4\}, \{3, 5\}\}} - 4 M_{\{\{2, 3, 4\}, \{1, 5\}\}} - 6 M_{\{\{1\}, \{2, 3, 4, 5\}\}} \\
& - 6 M_{\{\{2, 4\}, \{1, 3, 5\}\}} - 2 M_{\{\{3\}, \{1, 2, 4\}, \{5\}\}} - 2 M_{\{\{3\}, \{2, 4\}, \{1, 5\}\}} - 4 M_{\{\{1, 2, 4\}, \{3, 5\}\}} \\
& - 24 M_{\{\{1, 2, 3, 4, 5\}\}} - M_{\{\{3\}, \{1\}, \{2, 4\}, \{5\}\}} - 2 M_{\{\{3\}, \{1\}, \{2, 4, 5\}\}} - 4 M_{\{\{1, 3\}, \{2, 4, 5\}\}}
\end{aligned}$$

$$\begin{aligned}
\Omega(M_{\{\{2\}, \{1\}, \{3, 4\}, \{5\}\}}) = & -2 M_{\{\{2\}, \{3, 4\}, \{1, 5\}\}} - 2 M_{\{\{1\}, \{2, 3, 4\}, \{5\}\}} - 6 M_{\{\{1, 2, 3, 4\}, \{5\}\}} \\
& - 4 M_{\{\{2, 3, 4\}, \{1, 5\}\}} - 6 M_{\{\{1\}, \{2, 3, 4, 5\}\}} - 2 M_{\{\{1, 2\}, \{3, 4\}, \{5\}\}} - 6 M_{\{\{3, 4\}, \{1, 2, 5\}\}}
\end{aligned}$$

$$- M_{\{\{2\}, \{1\}, \{3, 4\}, \{5\}\}} - 2 M_{\{\{1\}, \{3, 4\}, \{2, 5\}\}} - 24 M_{\{\{1, 2, 3, 4, 5\}\}} - 6 M_{\{\{2\}, \{1, 3, 4, 5\}\}} \\ - 4 M_{\{\{1, 2\}, \{3, 4, 5\}\}} - 2 M_{\{\{2\}, \{1, 3, 4\}, \{5\}\}} - 2 M_{\{\{2\}, \{1\}, \{3, 4, 5\}\}} - 4 M_{\{\{1, 3, 4\}, \{2, 5\}\}}$$

$$\Omega(M_{\{\{2\}, \{1, 3\}, \{4\}, \{5\}\}}) = -2 M_{\{\{1, 3\}, \{2, 4\}, \{5\}\}} - 6 M_{\{\{1, 2, 3, 4\}, \{5\}\}} - 2 M_{\{\{1, 3\}, \{4\}, \{2, 5\}\}} \\ - 4 M_{\{\{2, 4\}, \{1, 3, 5\}\}} - 2 M_{\{\{2\}, \{1, 3\}, \{4, 5\}\}} - 4 M_{\{\{1, 2, 3\}, \{4, 5\}\}} - 2 M_{\{\{1, 2, 3\}, \{4\}, \{5\}\}} \\ - 2 M_{\{\{2\}, \{4\}, \{1, 3, 5\}\}} - 24 M_{\{\{1, 2, 3, 4, 5\}\}} - 6 M_{\{\{2\}, \{1, 3, 4, 5\}\}} - 6 M_{\{\{1, 3\}, \{2, 4, 5\}\}} \\ - M_{\{\{2\}, \{1, 3\}, \{4\}, \{5\}\}} - 6 M_{\{\{4\}, \{1, 2, 3, 5\}\}} - 2 M_{\{\{2\}, \{1, 3, 4\}, \{5\}\}} - 4 M_{\{\{1, 3, 4\}, \{2, 5\}\}}$$

$$\Omega(M_{\{\{1\}, \{2, 3\}, \{4\}, \{5\}\}}) = -2 M_{\{\{2, 3\}, \{1, 4\}, \{5\}\}} - 2 M_{\{\{1\}, \{2, 3, 4\}, \{5\}\}} - 2 M_{\{\{1\}, \{2, 3\}, \{4, 5\}\}} \\ - 6 M_{\{\{1, 2, 3, 4\}, \{5\}\}} - 2 M_{\{\{2, 3\}, \{4\}, \{1, 5\}\}} - 4 M_{\{\{2, 3, 4\}, \{1, 5\}\}} - 6 M_{\{\{1\}, \{2, 3, 4, 5\}\}} \\ - M_{\{\{1\}, \{2, 3\}, \{4\}, \{5\}\}} - 4 M_{\{\{1, 2, 3\}, \{4, 5\}\}} - 2 M_{\{\{1, 2, 3\}, \{4\}, \{5\}\}} - 2 M_{\{\{1\}, \{4\}, \{2, 3, 5\}\}} \\ - 24 M_{\{\{1, 2, 3, 4, 5\}\}} - 4 M_{\{\{1, 4\}, \{2, 3, 5\}\}} - 6 M_{\{\{4\}, \{1, 2, 3, 5\}\}} - 6 M_{\{\{2, 3\}, \{1, 4, 5\}\}}$$

$$\Omega(M_{\{\{1, 2\}, \{3\}, \{4\}, \{5\}\}}) = -6 M_{\{\{1, 2, 3, 4\}, \{5\}\}} - 6 M_{\{\{3\}, \{1, 2, 4, 5\}\}} - 2 M_{\{\{3\}, \{1, 2, 4\}, \{5\}\}} \\ - 2 M_{\{\{1, 2\}, \{3, 4\}, \{5\}\}} - M_{\{\{1, 2\}, \{3\}, \{4\}, \{5\}\}} - 4 M_{\{\{3, 4\}, \{1, 2, 5\}\}} - 2 M_{\{\{3\}, \{4\}, \{1, 2, 5\}\}} \\ - 4 M_{\{\{1, 2, 3\}, \{4, 5\}\}} - 2 M_{\{\{1, 2\}, \{3\}, \{4, 5\}\}} - 2 M_{\{\{1, 2, 3\}, \{4\}, \{5\}\}} - 4 M_{\{\{1, 2, 4\}, \{3, 5\}\}} \\ - 2 M_{\{\{1, 2\}, \{4\}, \{3, 5\}\}} - 24 M_{\{\{1, 2, 3, 4, 5\}\}} - 6 M_{\{\{1, 2\}, \{3, 4, 5\}\}} - 6 M_{\{\{4\}, \{1, 2, 3, 5\}\}}$$

$$\Omega(M_{\{\{3\}, \{2\}, \{1, 4, 5\}\}}) = 6 M_{\{\{1, 2, 3, 4, 5\}\}} + 2 M_{\{\{3\}, \{1, 2, 4, 5\}\}} + M_{\{\{3\}, \{2\}, \{1, 4, 5\}\}} \\ + 2 M_{\{\{2, 3\}, \{1, 4, 5\}\}} + 2 M_{\{\{2\}, \{1, 3, 4, 5\}\}}$$

$$\Omega(M_{\{\{3\}, \{1\}, \{2, 4, 5\}\}}) = 6 M_{\{\{1, 2, 3, 4, 5\}\}} + 2 M_{\{\{3\}, \{1, 2, 4, 5\}\}} + M_{\{\{3\}, \{1\}, \{2, 4, 5\}\}} \\ + 2 M_{\{\{1, 3\}, \{2, 4, 5\}\}} + 2 M_{\{\{1\}, \{2, 3, 4, 5\}\}}$$

$$\Omega(M_{\{\{2\}, \{1\}, \{3, 4, 5\}\}}) = 6 M_{\{\{1, 2, 3, 4, 5\}\}} + M_{\{\{2\}, \{1\}, \{3, 4, 5\}\}} + 2 M_{\{\{1, 2\}, \{3, 4, 5\}\}} \\ + 2 M_{\{\{1\}, \{2, 3, 4, 5\}\}} + 2 M_{\{\{2\}, \{1, 3, 4, 5\}\}}$$

$$\Omega(M_{\{\{2\}, \{1, 3\}, \{4, 5\}\}}) = 2 M_{\{\{1, 2, 3\}, \{4, 5\}\}} + 6 M_{\{\{1, 2, 3, 4, 5\}\}} + M_{\{\{2\}, \{1, 3\}, \{4, 5\}\}} \\ + 2 M_{\{\{1, 3\}, \{2, 4, 5\}\}} + 2 M_{\{\{2\}, \{1, 3, 4, 5\}\}}$$

$$\Omega(M_{\{\{1\}, \{2, 3\}, \{4, 5\}\}}) = 2 M_{\{\{1, 2, 3\}, \{4, 5\}\}} + 6 M_{\{\{1, 2, 3, 4, 5\}\}} + M_{\{\{1\}, \{2, 3\}, \{4, 5\}\}} \\ + 2 M_{\{\{2, 3\}, \{1, 4, 5\}\}} + 2 M_{\{\{1\}, \{2, 3, 4, 5\}\}}$$

$$\Omega(M_{\{\{1, 2\}, \{3\}, \{4, 5\}\}}) = M_{\{\{1, 2\}, \{3\}, \{4, 5\}\}} + 2 M_{\{\{1, 2, 3\}, \{4, 5\}\}} + 6 M_{\{\{1, 2, 3, 4, 5\}\}} \\ + 2 M_{\{\{3\}, \{1, 2, 4, 5\}\}} + 2 M_{\{\{1, 2\}, \{3, 4, 5\}\}}$$

$$\Omega(M_{\{\{2\}, \{4\}, \{1, 3, 5\}\}}) = 2 M_{\{\{4\}, \{1, 2, 3, 5\}\}} + 6 M_{\{\{1, 2, 3, 4, 5\}\}} + M_{\{\{2\}, \{4\}, \{1, 3, 5\}\}} \\ + 2 M_{\{\{2, 4\}, \{1, 3, 5\}\}} + 2 M_{\{\{2\}, \{1, 3, 4, 5\}\}}$$

$$\Omega(M_{\{\{1\}, \{4\}, \{2, 3, 5\}\}}) = 2 M_{\{\{4\}, \{1, 2, 3, 5\}\}} + 6 M_{\{\{1, 2, 3, 4, 5\}\}} + M_{\{\{1\}, \{4\}, \{2, 3, 5\}\}} \\ + 2 M_{\{\{1, 4\}, \{2, 3, 5\}\}} + 2 M_{\{\{1\}, \{2, 3, 4, 5\}\}}$$

$$\Omega(M_{\{\{3\}, \{2, 4\}, \{1, 5\}\}}) = M_{\{\{3\}, \{2, 4\}, \{1, 5\}\}} + 2 M_{\{\{2, 3, 4\}, \{1, 5\}\}} + 2 M_{\{\{2, 4\}, \{1, 3, 5\}\}} \\ + 6 M_{\{\{1, 2, 3, 4, 5\}\}} + 2 M_{\{\{3\}, \{1, 2, 4, 5\}\}}$$

$$\Omega(M_{\{\{2\}, \{3, 4\}, \{1, 5\}\}}) = 2 M_{\{\{3, 4\}, \{1, 2, 5\}\}} + 6 M_{\{\{1, 2, 3, 4, 5\}\}} + M_{\{\{2\}, \{3, 4\}, \{1, 5\}\}} \\ + 2 M_{\{\{2, 3, 4\}, \{1, 5\}\}} + 2 M_{\{\{2\}, \{1, 3, 4, 5\}\}}$$

$$\Omega(M_{\{\{2, 3\}, \{4\}, \{1, 5\}\}}) = 2 M_{\{\{4\}, \{1, 2, 3, 5\}\}} + 6 M_{\{\{1, 2, 3, 4, 5\}\}} + M_{\{\{2, 3\}, \{4\}, \{1, 5\}\}} \\ + 2 M_{\{\{2, 3, 4\}, \{1, 5\}\}} + 2 M_{\{\{2, 3\}, \{1, 4, 5\}\}}$$

$$\Omega(M_{\{\{3\}, \{1, 4\}, \{2, 5\}\}}) = M_{\{\{3\}, \{1, 4\}, \{2, 5\}\}} + 2 M_{\{\{1, 3, 4\}, \{2, 5\}\}} + 2 M_{\{\{1, 4\}, \{2, 3, 5\}\}} \\ + 6 M_{\{\{1, 2, 3, 4, 5\}\}} + 2 M_{\{\{3\}, \{1, 2, 4, 5\}\}}$$

$$\begin{aligned}
\Omega(M_{\{1, \{3, 4\}, \{2, 5\}}}) &= 2M_{\{3, 4\}, \{1, 2, 5\}} + 6M_{\{1, 2, 3, 4, 5\}} + M_{\{1\}, \{3, 4\}, \{2, 5\}} \\
&\quad + 2M_{\{1, 3, 4\}, \{2, 5\}} + 2M_{\{1\}, \{2, 3, 4, 5\}} \\
\Omega(M_{\{1, \{3\}, \{4\}, \{2, 5\}}}) &= 2M_{\{4\}, \{1, 2, 3, 5\}} + 6M_{\{1, 2, 3, 4, 5\}} + M_{\{1, 3\}, \{4\}, \{2, 5\}} \\
&\quad + 2M_{\{1, 3, 4\}, \{2, 5\}} + 2M_{\{1, 3\}, \{2, 4, 5\}} \\
\Omega(M_{\{2\}, \{1, 4\}, \{3, 5\}}) &= 2M_{\{1, 2, 4\}, \{3, 5\}} + 6M_{\{1, 2, 3, 4, 5\}} + M_{\{2\}, \{1, 4\}, \{3, 5\}} \\
&\quad + 2M_{\{1, 4\}, \{2, 3, 5\}} + 2M_{\{2\}, \{1, 3, 4, 5\}} \\
\Omega(M_{\{1\}, \{2, 4\}, \{3, 5\}}) &= 2M_{\{1, 2, 4\}, \{3, 5\}} + 6M_{\{1, 2, 3, 4, 5\}} + M_{\{1\}, \{2, 4\}, \{3, 5\}} \\
&\quad + 2M_{\{2, 4\}, \{1, 3, 5\}} + 2M_{\{1\}, \{2, 3, 4, 5\}} \\
\Omega(M_{\{1, 2\}, \{4\}, \{3, 5\}}) &= M_{\{1, 2\}, \{4\}, \{3, 5\}} + 2M_{\{1, 2, 4\}, \{3, 5\}} + 2M_{\{4\}, \{1, 2, 3, 5\}} \\
&\quad + 6M_{\{1, 2, 3, 4, 5\}} + 2M_{\{1, 2\}, \{3, 4, 5\}} \\
\Omega(M_{\{3\}, \{4\}, \{1, 2, 5\}}) &= M_{\{3\}, \{4\}, \{1, 2, 5\}} + 2M_{\{3, 4\}, \{1, 2, 5\}} + 2M_{\{4\}, \{1, 2, 3, 5\}} \\
&\quad + 6M_{\{1, 2, 3, 4, 5\}} + 2M_{\{3\}, \{1, 2, 4, 5\}} \\
\Omega(M_{\{2\}, \{1, 3, 4\}, \{5\}}) &= 2M_{\{1, 2, 3, 4\}, \{5\}} + 6M_{\{1, 2, 3, 4, 5\}} + M_{\{2\}, \{1, 3, 4\}, \{5\}} \\
&\quad + 2M_{\{1, 3, 4\}, \{2, 5\}} + 2M_{\{2\}, \{1, 3, 4, 5\}} \\
\Omega(M_{\{1\}, \{2, 3, 4\}, \{5\}}) &= 2M_{\{1, 2, 3, 4\}, \{5\}} + 6M_{\{1, 2, 3, 4, 5\}} + M_{\{1\}, \{2, 3, 4\}, \{5\}} \\
&\quad + 2M_{\{2, 3, 4\}, \{1, 5\}} + 2M_{\{1\}, \{2, 3, 4, 5\}} \\
\Omega(M_{\{2, 3\}, \{1, 4\}, \{5\}}) &= 2M_{\{1, 2, 3, 4\}, \{5\}} + 6M_{\{1, 2, 3, 4, 5\}} + M_{\{2, 3\}, \{1, 4\}, \{5\}} \\
&\quad + 2M_{\{1, 4\}, \{2, 3, 5\}} + 2M_{\{2, 3\}, \{1, 4, 5\}} \\
\Omega(M_{\{1, 3\}, \{2, 4\}, \{5\}}) &= 2M_{\{1, 2, 3, 4\}, \{5\}} + 6M_{\{1, 2, 3, 4, 5\}} + M_{\{1, 3\}, \{2, 4\}, \{5\}} \\
&\quad + 2M_{\{2, 4\}, \{1, 3, 5\}} + 2M_{\{1, 3\}, \{2, 4, 5\}} \\
\Omega(M_{\{1, 2\}, \{3, 4\}, \{5\}}) &= M_{\{1, 2\}, \{3, 4\}, \{5\}} + 2M_{\{1, 2, 3, 4\}, \{5\}} + 2M_{\{3, 4\}, \{1, 2, 5\}} \\
&\quad + 6M_{\{1, 2, 3, 4, 5\}} + 2M_{\{1, 2\}, \{3, 4, 5\}} \\
\Omega(M_{\{3\}, \{1, 2, 4\}, \{5\}}) &= M_{\{3\}, \{1, 2, 4\}, \{5\}} + 2M_{\{1, 2, 3, 4\}, \{5\}} + 2M_{\{1, 2, 4\}, \{3, 5\}} \\
&\quad + 6M_{\{1, 2, 3, 4, 5\}} + 2M_{\{3\}, \{1, 2, 4, 5\}} \\
\Omega(M_{\{1, 2, 3\}, \{4\}, \{5\}}) &= M_{\{1, 2, 3\}, \{4\}, \{5\}} + 2M_{\{1, 2, 3, 4\}, \{5\}} + 2M_{\{4\}, \{1, 2, 3, 5\}} \\
&\quad + 2M_{\{1, 2, 3\}, \{4, 5\}} + 6M_{\{1, 2, 3, 4, 5\}}
\end{aligned}$$

$$\begin{aligned}
\Omega(M_{\{2\}, \{1, 3, 4, 5\}}) &= -2M_{\{1, 2, 3, 4, 5\}} - M_{\{2\}, \{1, 3, 4, 5\}} \\
\Omega(M_{\{1\}, \{2, 3, 4, 5\}}) &= -2M_{\{1, 2, 3, 4, 5\}} - M_{\{1\}, \{2, 3, 4, 5\}} \\
\Omega(M_{\{2, 3\}, \{1, 4, 5\}}) &= -2M_{\{1, 2, 3, 4, 5\}} - M_{\{2, 3\}, \{1, 4, 5\}} \\
\Omega(M_{\{1, 3\}, \{2, 4, 5\}}) &= -2M_{\{1, 2, 3, 4, 5\}} - M_{\{1, 3\}, \{2, 4, 5\}} \\
\Omega(M_{\{1, 2\}, \{3, 4, 5\}}) &= -2M_{\{1, 2, 3, 4, 5\}} - M_{\{1, 2\}, \{3, 4, 5\}} \\
\Omega(M_{\{3\}, \{1, 2, 4, 5\}}) &= -2M_{\{1, 2, 3, 4, 5\}} - M_{\{3\}, \{1, 2, 4, 5\}} \\
\Omega(M_{\{1, 2, 3\}, \{4, 5\}}) &= -M_{\{1, 2, 3\}, \{4, 5\}} - 2M_{\{1, 2, 3, 4, 5\}} \\
\Omega(M_{\{2, 4\}, \{1, 3, 5\}}) &= -M_{\{2, 4\}, \{1, 3, 5\}} - 2M_{\{1, 2, 3, 4, 5\}} \\
\Omega(M_{\{1, 4\}, \{2, 3, 5\}}) &= -M_{\{1, 4\}, \{2, 3, 5\}} - 2M_{\{1, 2, 3, 4, 5\}}
\end{aligned}$$

$$\begin{aligned}
\Omega(M_{\{2, 3, 4\}, \{1, 5\}}) &= -M_{\{2, 3, 4\}, \{1, 5\}} - 2 M_{\{1, 2, 3, 4, 5\}} \\
\Omega(M_{\{1, 3, 4\}, \{2, 5\}}) &= -M_{\{1, 3, 4\}, \{2, 5\}} - 2 M_{\{1, 2, 3, 4, 5\}} \\
\Omega(M_{\{4\}, \{1, 2, 3, 5\}}) &= -M_{\{4\}, \{1, 2, 3, 5\}} - 2 M_{\{1, 2, 3, 4, 5\}} \\
\Omega(M_{\{1, 2, 4\}, \{3, 5\}}) &= -M_{\{1, 2, 4\}, \{3, 5\}} - 2 M_{\{1, 2, 3, 4, 5\}} \\
\Omega(M_{\{3, 4\}, \{1, 2, 5\}}) &= -M_{\{3, 4\}, \{1, 2, 5\}} - 2 M_{\{1, 2, 3, 4, 5\}} \\
\Omega(M_{\{1, 2, 3, 4\}, \{5\}}) &= -M_{\{1, 2, 3, 4\}, \{5\}} - 2 M_{\{1, 2, 3, 4, 5\}} \\
\Omega(M_{\{1, 2, 3, 4, 5\}}) &= M_{\{1, 2, 3, 4, 5\}}
\end{aligned}$$

&gt;

## = omega2(antipode(\*)) on X-basis

```

> for i from 1 to 5 do
  print(Omega(S[{{seq(j,j=1..i)}}])) =
  ToX(ToM(omega2(antipode(X[{{seq(j,j=1..i)}}]))));
od;

```

$$\begin{aligned}
\Omega(S(X_{\{1\}})) &= -X_{\{1\}} \\
\Omega(S(X_{\{1, 2\}})) &= X_{\{1, 2\}} \\
\Omega(S(X_{\{1, 2, 3\}})) &= -X_{\{1, 2, 3\}} - 2 X_{\{2, \{1, 3\}\}} + X_{\{1, \{2, 3\}\}} + X_{\{1, 2, \{3\}\}} \\
\Omega(S(X_{\{1, 2, 3, 4\}})) &= -6 X_{\{2, \{1, 3\}, \{4\}\}} + 2 X_{\{3, \{1, 2, 4\}\}} - 4 X_{\{1, 2, \{3, 4\}\}} - 6 X_{\{\{3\}, \{1\}, \{2, 4\}\}} \\
&\quad + 2 X_{\{\{1, 3\}, \{2, 4\}\}} - 2 X_{\{\{1, 2, 3\}, \{4\}\}} + 6 X_{\{\{3\}, \{2\}, \{1, 4\}\}} + 2 X_{\{\{2, 3\}, \{1, 4\}\}} \\
&\quad - 2 X_{\{\{1\}, \{2, 3, 4\}\}} + 2 X_{\{\{2\}, \{1, 3, 4\}\}} + X_{\{\{1, 2, 3, 4\}\}} + 6 X_{\{\{1\}, \{2, 3\}, \{4\}\}} \\
\Omega(S(X_{\{1, 2, 3, 4, 5\}})) &= -6 X_{\{\{2\}, \{4\}, \{1, 3, 5\}\}} - 11 X_{\{\{2\}, \{1, 3\}, \{4\}, \{5\}\}} - 11 X_{\{\{2\}, \{1\}, \{4\}, \{3, 5\}\}} \\
&\quad + 9 X_{\{\{3\}, \{1\}, \{2, 4, 5\}\}} - 2 X_{\{\{2, 4\}, \{1, 3, 5\}\}} + 9 X_{\{\{3\}, \{1, 2, 4\}, \{5\}\}} - 2 X_{\{\{3, 4\}, \{1, 2, 5\}\}} \\
&\quad - 6 X_{\{\{2, 3\}, \{4\}, \{1, 5\}\}} + 14 X_{\{\{2\}, \{1\}, \{3, 4\}, \{5\}\}} + 24 X_{\{\{1, 2\}, \{4\}, \{3, 5\}\}} \\
&\quad - X_{\{\{3\}, \{2\}, \{1\}, \{4, 5\}\}} - 26 X_{\{\{3\}, \{2\}, \{4\}, \{1, 5\}\}} - 2 X_{\{\{1, 3\}, \{2, 4, 5\}\}} - 21 X_{\{\{1, 2\}, \{3, 4\}, \{5\}\}} \\
&\quad - 2 X_{\{\{1, 2, 4\}, \{3, 5\}\}} + 3 X_{\{\{1, 2, 3, 4\}, \{5\}\}} - 6 X_{\{\{1, 2\}, \{3\}, \{4, 5\}\}} - 6 X_{\{\{2\}, \{3, 4\}, \{1, 5\}\}} \\
&\quad + 9 X_{\{\{1\}, \{2, 4\}, \{3, 5\}\}} + 8 X_{\{\{1, 2, 3\}, \{4, 5\}\}} - 56 X_{\{\{3\}, \{1\}, \{2, 4\}, \{5\}\}} - 6 X_{\{\{2\}, \{1, 4\}, \{3, 5\}\}} \\
&\quad - 2 X_{\{\{1, 4\}, \{2, 3, 5\}\}} - 6 X_{\{\{3\}, \{2\}, \{1, 4, 5\}\}} - X_{\{\{1, 2, 3, 4, 5\}\}} + 9 X_{\{\{1, 3\}, \{2, 4\}, \{5\}\}} \\
&\quad - 6 X_{\{\{3\}, \{2, 4\}, \{1, 5\}\}} - 21 X_{\{\{1\}, \{2, 3\}, \{4, 5\}\}} - 2 X_{\{\{4\}, \{1, 2, 3, 5\}\}} + 14 X_{\{\{1\}, \{2, 3\}, \{4\}, \{5\}\}} \\
&\quad + 39 X_{\{\{3\}, \{2\}, \{1, 4\}, \{5\}\}} + 24 X_{\{\{2\}, \{1, 3\}, \{4, 5\}\}} + 39 X_{\{\{3\}, \{1\}, \{4\}, \{2, 5\}\}} \\
&\quad + 9 X_{\{\{2, 3\}, \{1, 4\}, \{5\}\}} - 2 X_{\{\{2, 3, 4\}, \{1, 5\}\}} - 6 X_{\{\{1, 3\}, \{4\}, \{2, 5\}\}} - 2 X_{\{\{2, 3\}, \{1, 4, 5\}\}} \\
&\quad - 16 X_{\{\{1\}, \{2, 3, 4\}, \{5\}\}} + 9 X_{\{\{1\}, \{3, 4\}, \{2, 5\}\}} + 9 X_{\{\{2\}, \{1, 3, 4\}, \{5\}\}} - 2 X_{\{\{3\}, \{1, 2, 4, 5\}\}} \\
&\quad - X_{\{\{1, 2\}, \{3\}, \{4\}, \{5\}\}} - 6 X_{\{\{3\}, \{1, 4\}, \{2, 5\}\}} - 6 X_{\{\{3\}, \{4\}, \{1, 2, 5\}\}} - 2 X_{\{\{2\}, \{1, 3, 4, 5\}\}} \\
&\quad - X_{\{\{2\}, \{1\}, \{3, 4, 5\}\}} - X_{\{\{1, 2, 3\}, \{4\}, \{5\}\}} + 9 X_{\{\{1\}, \{4\}, \{2, 3, 5\}\}} - 2 X_{\{\{1, 3, 4\}, \{2, 5\}\}} \\
&\quad + 3 X_{\{\{1\}, \{2, 3, 4, 5\}\}} + 8 X_{\{\{1, 2\}, \{3, 4, 5\}\}}
\end{aligned}$$

```

> for i from 1 to 4 do
  for A in mylasp(i) do
    print(S(Omega(X[A]))) = ToX(ToM(antipode(omega2(X[A]))));
  od;
od;

```

$$S(\Omega(X_{\{1\}})) = -X_{\{1\}}$$

$$S(\Omega(X_{\{2\}, \{1\}})) = X_{\{2\}, \{1\}}$$

$$S(\Omega(X_{\{1, 2\}})) = X_{\{1, 2\}}$$

$$S(\Omega(X_{\{3\}, \{2\}, \{1\}})) = -X_{\{3\}, \{2\}, \{1\}}$$

$$S(\Omega(X_{\{2\}, \{1, 3\}})) = X_{\{2\}, \{1, 3\}} - X_{\{1\}, \{2, 3\}} - X_{\{1, 2\}, \{3\}}$$

$$S(\Omega(X_{\{1\}, \{2, 3\}})) = -X_{\{1, 2\}, \{3\}}$$

$$S(\Omega(X_{\{1, 2\}, \{3\}})) = -X_{\{1\}, \{2, 3\}}$$

$$S(\Omega(X_{\{1, 2, 3\}})) = -X_{\{1, 2, 3\}} - 2X_{\{2\}, \{1, 3\}} + X_{\{1\}, \{2, 3\}} + X_{\{1, 2\}, \{3\}}$$

$$S(\Omega(X_{\{3\}, \{2\}, \{1\}, \{4\}})) = X_{\{3\}, \{2\}, \{1\}, \{4\}}$$

$$\begin{aligned} S(\Omega(X_{\{3\}, \{2\}, \{1, 4\}})) &= -2X_{\{2\}, \{1, 3\}, \{4\}} + X_{\{2\}, \{1\}, \{3, 4\}} - 2X_{\{3\}, \{1\}, \{2, 4\}} \\ &\quad + X_{\{3\}, \{2\}, \{1, 4\}} + 2X_{\{1\}, \{2, 3\}, \{4\}} + X_{\{1, 2\}, \{3\}, \{4\}} \end{aligned}$$

$$S(\Omega(X_{\{3\}, \{1\}, \{2, 4\}})) = -X_{\{2\}, \{1, 3\}, \{4\}} + X_{\{1\}, \{2, 3\}, \{4\}} + X_{\{1, 2\}, \{3\}, \{4\}}$$

$$S(\Omega(X_{\{2\}, \{1\}, \{3, 4\}})) = X_{\{1, 2\}, \{3\}, \{4\}}$$

$$S(\Omega(X_{\{2\}, \{1, 3\}, \{4\}})) = X_{\{2\}, \{1\}, \{3, 4\}} - X_{\{3\}, \{1\}, \{2, 4\}} + X_{\{1\}, \{2, 3\}, \{4\}}$$

$$S(\Omega(X_{\{1\}, \{2, 3\}, \{4\}})) = X_{\{1\}, \{2, 3\}, \{4\}}$$

$$S(\Omega(X_{\{1, 2\}, \{3\}, \{4\}})) = X_{\{2\}, \{1\}, \{3, 4\}}$$

$$\begin{aligned} S(\Omega(X_{\{2\}, \{1, 3, 4\}})) &= 2X_{\{2\}, \{1, 3\}, \{4\}} - 2X_{\{2\}, \{1\}, \{3, 4\}} + 4X_{\{3\}, \{1\}, \{2, 4\}} \\ &\quad + X_{\{1, 2, 3\}, \{4\}} - 2X_{\{3\}, \{2\}, \{1, 4\}} + X_{\{1\}, \{2, 3, 4\}} - X_{\{2\}, \{1, 3, 4\}} - X_{\{1\}, \{2, 3\}, \{4\}} \\ &\quad - X_{\{1, 2\}, \{3\}, \{4\}} \end{aligned}$$

$$S(\Omega(X_{\{1\}, \{2, 3, 4\}})) = 2X_{\{2\}, \{1, 3\}, \{4\}} + X_{\{1, 2, 3\}, \{4\}} - X_{\{1\}, \{2, 3\}, \{4\}} - X_{\{1, 2\}, \{3\}, \{4\}}$$

$$\begin{aligned} S(\Omega(X_{\{2, 3\}, \{1, 4\}})) &= 2X_{\{2\}, \{1, 3\}, \{4\}} + X_{\{2\}, \{1\}, \{3, 4\}} + 2X_{\{1, 2\}, \{3, 4\}} \\ &\quad + 2X_{\{3\}, \{1\}, \{2, 4\}} - 2X_{\{3\}, \{2\}, \{1, 4\}} - X_{\{2, 3\}, \{1, 4\}} - 4X_{\{1\}, \{2, 3\}, \{4\}} \\ &\quad + X_{\{1, 2\}, \{3\}, \{4\}} \end{aligned}$$

$$\begin{aligned} S(\Omega(X_{\{1, 3\}, \{2, 4\}})) &= X_{\{2\}, \{1\}, \{3, 4\}} + 2X_{\{1, 2\}, \{3, 4\}} - X_{\{1, 3\}, \{2, 4\}} - 2X_{\{1\}, \{2, 3\}, \{4\}} \\ &\quad + X_{\{1, 2\}, \{3\}, \{4\}} \end{aligned}$$

$$S(\Omega(X_{\{1, 2\}, \{3, 4\}})) = X_{\{1, 2\}, \{3, 4\}}$$

$$\begin{aligned} S(\Omega(X_{\{3\}, \{1, 2, 4\}})) &= 4X_{\{2\}, \{1, 3\}, \{4\}} - X_{\{3\}, \{1, 2, 4\}} - X_{\{2\}, \{1\}, \{3, 4\}} + 2X_{\{3\}, \{1\}, \{2, 4\}} \\ &\quad + X_{\{1, 2, 3\}, \{4\}} - 2X_{\{3\}, \{2\}, \{1, 4\}} + X_{\{1\}, \{2, 3, 4\}} - X_{\{1\}, \{2, 3\}, \{4\}} - 2X_{\{1, 2\}, \{3\}, \{4\}} \end{aligned}$$

$$S(\Omega(X_{\{1, 2, 3\}, \{4\}})) = -X_{\{2\}, \{1\}, \{3, 4\}} + 2X_{\{3\}, \{1\}, \{2, 4\}} + X_{\{1\}, \{2, 3, 4\}} - X_{\{1\}, \{2, 3\}, \{4\}}$$

$$\begin{aligned} S(\Omega(X_{\{1, 2, 3, 4\}})) &= -6X_{\{2\}, \{1, 3\}, \{4\}} + 2X_{\{3\}, \{1, 2, 4\}} - 4X_{\{1, 2\}, \{3, 4\}} - 6X_{\{3\}, \{1\}, \{2, 4\}} \\ &\quad + 2X_{\{1, 3\}, \{2, 4\}} - 2X_{\{1, 2, 3\}, \{4\}} + 6X_{\{3\}, \{2\}, \{1, 4\}} + 2X_{\{2, 3\}, \{1, 4\}} \\ &\quad - 2X_{\{1\}, \{2, 3, 4\}} + 2X_{\{2\}, \{1, 3, 4\}} + X_{\{1, 2, 3, 4\}} + 6X_{\{1\}, \{2, 3\}, \{4\}} \end{aligned}$$

[>

## = omega2(antipode(\*)) on the M-basis

```

> for i from 1 to 5 do
  print(Omega(S(M[{{seq(j,j=1..i)}}])) =
ToM(omega2(antipode(M[{{seq(j,j=1..i)}}]))));
od;

$$\Omega(S(M_{\{\{1\}\}})) = -M_{\{\{1\}\}}$$


$$\Omega(S(M_{\{\{1, 2\}\}})) = M_{\{\{1, 2\}\}}$$


$$\Omega(S(M_{\{\{1, 2, 3\}\}})) = -M_{\{\{1, 2, 3\}\}}$$


$$\Omega(S(M_{\{\{1, 2, 3, 4\}\}})) = M_{\{\{1, 2, 3, 4\}\}}$$


$$\Omega(S(M_{\{\{1, 2, 3, 4, 5\}\}})) = -M_{\{\{1, 2, 3, 4, 5\}\}}$$


> for i from 1 to 4 do
  for A in mylasp(i) do
    print(S(Omega(M[A])) = ToM(antipode(omega2(M[A]))));
  od;
od;

$$S(\Omega(M_{\{\{1\}\}})) = -M_{\{\{1\}\}}$$


$$S(\Omega(M_{\{\{2\}, \{1\}\}})) = M_{\{\{2\}, \{1\}\}}$$


$$S(\Omega(M_{\{\{1, 2\}\}})) = M_{\{\{1, 2\}\}}$$


$$S(\Omega(M_{\{\{3, 2, 1\}\}})) = -M_{\{\{3, 2, 1\}\}} + M_{\{\{1, 2, 3\}\}} + M_{\{\{1, 2, 3\}\}} - 2M_{\{\{2, 1, 3\}\}}$$


$$S(\Omega(M_{\{\{2, 1, 3\}\}})) = M_{\{\{2, 1, 3\}\}} - M_{\{\{1, 2, 3\}\}} - M_{\{\{1, 2, 3\}\}}$$


$$S(\Omega(M_{\{\{1, 2, 3\}\}})) = -M_{\{\{1, 2, 3\}\}}$$


$$S(\Omega(M_{\{\{1, 2, 3\}\}})) = -M_{\{\{1, 2, 3\}\}}$$


$$S(\Omega(M_{\{\{1, 2, 3\}\}})) = -M_{\{\{1, 2, 3\}\}}$$


$$S(\Omega(M_{\{\{3, 2, 1, 4\}\}})) = 4M_{\{\{2, 1, 3, 4\}\}} - 2M_{\{\{3, 1, 2, 4\}\}} - 4M_{\{\{1, 2, 3, 4\}\}}$$


$$- 2M_{\{\{1, 2, 3, 4\}\}} + 2M_{\{\{1, 2, 3, 4\}\}} + M_{\{\{3, 2, 1, 4\}\}} - 2M_{\{\{2, 1, 3, 4\}\}}$$


$$+ 2M_{\{\{1, 2, 3, 4\}\}} - 6M_{\{\{2, 3, 1, 4\}\}} + 6M_{\{\{1, 3, 2, 4\}\}} + 4M_{\{\{3, 1, 2, 4\}\}}$$


$$- 2M_{\{\{2, 1, 3, 4\}\}}$$


$$S(\Omega(M_{\{\{3, 2, 1, 4\}\}})) = -2M_{\{\{2, 1, 3, 4\}\}} + M_{\{\{3, 1, 2, 4\}\}} + 2M_{\{\{1, 2, 3, 4\}\}}$$


$$+ M_{\{\{1, 2, 3, 4\}\}} - M_{\{\{1, 2, 3, 4\}\}} + M_{\{\{2, 1, 3, 4\}\}} - M_{\{\{1, 2, 3, 4\}\}} + 4M_{\{\{2, 3, 1, 4\}\}}$$


$$+ M_{\{\{3, 2, 1, 4\}\}} - 4M_{\{\{1, 3, 2, 4\}\}} - 2M_{\{\{3, 1, 2, 4\}\}} + M_{\{\{2, 1, 3, 4\}\}}$$


$$S(\Omega(M_{\{\{3, 1, 2, 4\}\}})) = 2M_{\{\{3, 1, 2, 4\}\}} - M_{\{\{1, 2, 3, 4\}\}} - M_{\{\{1, 2, 3, 4\}\}} - M_{\{\{2, 1, 3, 4\}\}}$$


$$- M_{\{\{2, 1, 3, 4\}\}} + M_{\{\{1, 2, 3, 4\}\}} + M_{\{\{1, 2, 3, 4\}\}} + M_{\{\{2, 3, 1, 4\}\}}$$


$$S(\Omega(M_{\{\{2, 1, 3, 4\}\}})) = -M_{\{\{1, 2, 3, 4\}\}} + M_{\{\{2, 1, 3, 4\}\}} + M_{\{\{1, 2, 3, 4\}\}} - M_{\{\{1, 2, 3, 4\}\}}$$


$$+ M_{\{\{3, 2, 1, 4\}\}}$$


$$S(\Omega(M_{\{\{2, 1, 3, 4\}\}})) = -M_{\{\{1, 2, 3, 4\}\}} - M_{\{\{1, 2, 3, 4\}\}} - M_{\{\{3, 1, 2, 4\}\}} - M_{\{\{3, 1, 2, 4\}\}}$$


$$+ M_{\{\{2, 1, 3, 4\}\}} + M_{\{\{1, 2, 3, 4\}\}} + M_{\{\{2, 3, 1, 4\}\}} + 2M_{\{\{2, 1, 3, 4\}\}}$$


```

$$S(\Omega(M_{\{1\}, \{2, 3\}, \{4\}})) = -2 M_{\{1, 2\}, \{3, 4\}} + M_{\{1\}, \{2, 3\}, \{4\}} + 2 M_{\{2, 3\}, \{1, 4\}}$$

$$S(\Omega(M_{\{1, 2\}, \{3\}, \{4\}})) = M_{\{2\}, \{1\}, \{3, 4\}} + M_{\{2\}, \{1, 3, 4\}} - M_{\{1\}, \{2, 3, 4\}} + M_{\{3\}, \{1, 2, 4\}} \\ - M_{\{1, 2, 3\}, \{4\}}$$

$$S(\Omega(M_{\{2\}, \{1, 3, 4\}})) = -M_{\{2\}, \{1, 3, 4\}} + M_{\{1\}, \{2, 3, 4\}} + M_{\{1, 2, 3\}, \{4\}}$$

$$S(\Omega(M_{\{1\}, \{2, 3, 4\}})) = M_{\{1, 2, 3\}, \{4\}}$$

$$S(\Omega(M_{\{2, 3\}, \{1, 4\}})) = -M_{\{2, 3\}, \{1, 4\}} + 2 M_{\{1, 2\}, \{3, 4\}}$$

$$S(\Omega(M_{\{1, 3\}, \{2, 4\}})) = -M_{\{1, 3\}, \{2, 4\}} + 2 M_{\{1, 2\}, \{3, 4\}}$$

$$S(\Omega(M_{\{1, 2\}, \{3, 4\}})) = M_{\{1, 2\}, \{3, 4\}}$$

$$S(\Omega(M_{\{3\}, \{1, 2, 4\}})) = -M_{\{3\}, \{1, 2, 4\}} + M_{\{1\}, \{2, 3, 4\}} + M_{\{1, 2, 3\}, \{4\}}$$

$$S(\Omega(M_{\{1, 2, 3\}, \{4\}})) = M_{\{1\}, \{2, 3, 4\}}$$

$$S(\Omega(M_{\{1, 2, 3, 4\}})) = M_{\{1, 2, 3, 4\}}$$

>