Integration by Parts

Math 1310N

2 February 2010
Integration by Parts

*Integration by parts* is an integration technique that transforms the integral of products of functions into other (ideally simpler) integrals.
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Motivation: Product rule for differentiation.
Integration by Parts

Product rule:
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\frac{d}{dx} \left( f(x)g(x) \right) = f'(x)g(x) + f(x)g'(x).
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Integration by Parts

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Interpreted as an integration rule:

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f(x)g(x) + C = \int f'(x)g(x) \, dx + \int f(x)g'(x) \, dx
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Integration by Parts

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Proposition 4.4.1. If \( f \) and \( g \) are cont. diff., then
\[
\int f'(x)g(x) \, dx = f(x)g(x) - \int f(x)g'(x) \, dx
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**Strategy:** to evaluate \( \int H(x) \, dx \), write \( H(x) = f'(x)g(x) \).
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**Strategy:** to evaluate \( \int H(x) \, dx \), write \( H(x) = f'(x)g(x) \).

**Note.** In the transition from the left-hand side to the right, \( g \) is differentiated and \( f' \) is integrated. It is useful to choose:
- \( g \) as a function that simplifies when differentiated, and
- \( f' \) as a function that simplifies when integrated.
\[
\int f'(x)g(x) \, dx = f(x)g(x) - \int f(x)g'(x) \, dx
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**LIATE heuristic:** pick \( g \) as whichever comes first from the left; pick \( f' \) as whichever comes first from the right:

<table>
<thead>
<tr>
<th>Log</th>
<th>Inv. trig</th>
<th>Algebraic</th>
<th>Trig</th>
<th>Exp</th>
</tr>
</thead>
<tbody>
<tr>
<td>\ln, \log_2</td>
<td>arcsin, arctan</td>
<td>( x^2, 3x^{50} )</td>
<td>sin, tan</td>
<td>( e^x, 17^x )</td>
</tr>
</tbody>
</table>
Summary

- You should understand and be able to use integration by parts to evaluate indefinite and definite integrals.

- You should be able to derive recursion formulas and apply these formulas to evaluate specific integrals.