BASICS OF THE SYMMETRIC GROUP, PARTITIONS, AND
TABLEAUX

(1) Find the inverse of every element in $S_3$.

(2) Compute the conjugacy class in $S_3$ of $\sigma = (1\ 3\ 2)$. (hint: use previous problem).

(3) Without computation, list all permutations in the conjugacy class of $(1\ 2)(3\ 4)$.

(4) * Prove that the number of self-conjugate partitions equals the number of partitions with distinct odd parts. Hint: find a bijection between the sets and consider the following diagram:

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(5) * Characterize the conjugate of a partition without using Ferrers diagrams.

(6) How many standard tableaux of shape (2,2,1) are there? (Do not list the standard tableaux).

(7) List the standard tableaux of shape (2,2,1). Make sure you have them all by comparing to your answer in the previous question.

(8) Explain why

$$f(\lambda_1, \lambda_2, \ldots, \lambda_\ell) = \sum_{i=1}^k f(\lambda_1, \lambda_1, \ldots, \lambda_i - 1, \ldots, \lambda_\ell),$$

where $f(\lambda_1, \lambda_2, \ldots, \lambda_i - 1, \ldots, \lambda_\ell) = 0$ when $\lambda_i = \lambda_{i+1}$.