

**1 Note.**

For best results, set aside two hours and sit down to do this practice midterm without any outside help, just as you will with the actual midterm. When time is up, check your answers and figure out the ones you've missed.

I've put all the problems here on a single page in the hopes of conserving paper. . . The actual midterm will have more space for working on the test itself.

**Problem 1.** Evaluate the following limits.

- $\lim_{x \rightarrow 4} \frac{x^3 - 5x^2 + 2x + 8}{x^3 - 5x^2 + 4x}$
- $\lim_{x \rightarrow 0} \frac{\sin(5x)}{7x}$
- $\lim_{x \rightarrow 0} x \sin\left(\frac{1}{x}\right)$
- $\lim_{x \rightarrow \infty} \frac{x^3 + 5}{5x^3 - 2x^2 + 4}$

**Problem 2.** True or False!

- If a function has a vertical tangent line at  $x = x_0$ , then the derivative at  $x_0$  is 0.
- The product of two discontinuous functions is discontinuous.
- If  $f(1) = 25$  and  $f(2) = 36$ , then there is a number  $a$  such that  $f(a) = 27.5$ .
- If a function is continuous, then it is differentiable.
- If a function is differentiable, then it is continuous.

**Problem 3.** We've seen derivatives which use a limit as  $h \rightarrow 0$  and as  $t \rightarrow x_0$ . In this problem, we'll develop a definition of the derivative as  $q \rightarrow 1$ .

When  $x_0 \neq 0$ , we can consider the number  $qx_0$  obtained by multiplying  $x_0$  by  $q$ .

- What value of  $c$  gives  $\lim_{q \rightarrow c} qx_0 = x_0$ ?
- For an arbitrary function  $f(x)$ , sketch the secant line connecting  $f(x_0)$  to  $f(qx_0)$ .
- Write a definition of the derivative using the  $\lim_{q \rightarrow c}$ .
- Use this  $q$  definition of the derivative to find the derivative of  $f(x) = x^3$  at  $x_0 = 1$ .

**Problem 4.** Find the derivative of the following functions.

- $f(x) = \sin(\ln(x))^5$
- $g(x) = (x^2 + 2x + 10)^{(3x^3+1)}$
- $h(x) = \frac{\tan^3(x) \sin(x)}{5x^2 \ln(x)}$

**Problem 5.** Let  $f(x) = 3x - \lfloor 3x \rfloor$ . Graph  $f$  and find  $f'(x)$  where the derivative exists.

**Problem 6.** Consider the hyperbola given by the equation  $x^2 - 5y^2 = 6$ .

- When  $x = 3$ , find all possible values of  $y$ .
- Find the equation of the tangent line to the hyperbola through the point  $(6, -\sqrt{6})$ .

**Problem 7.** In four days I need to do an experiment with some culture of bacteria; for the experiment to be effective, I need at least  $K$  bacteria in my culture.

- If the size of the population of the bacteria on day  $t$  is given by the equation  $P(t) = P_0 e^{3t}$ , where  $P_0$  is the initial population, how large should my starting population be in order to have  $K$  bacteria on day 4?
- What is the rate of change of the population on day 4?
- Unfortunately, my clock is only accurate to within 2%. Estimate the error propagation in the population size based on my reading of my faulty clock on day 4.

**Problem 8.** Write the following system of equations as an augmented matrix and then use Gaussian elimination to solve the system.

$$\begin{array}{rcccc} & 2y & & -w & = & 1 \\ & y & & & = & 2 \\ x & +y & & & = & 3 \\ & y & +z & & 0 & = & 4 \end{array}$$