

### How to Play Craps:

Craps is a dice game that is played at most casinos. We will describe here the most common rules of the game with the intention of understanding the game well enough to analyze the probability of winning each of the different bets.

There is one player who is called the “shooter” who rolls the dice. He bets against the “house” that he will win the outcome of a round of craps. There are other ‘customers’ who are allowed to place bets on the outcome of rolls of the dice and of the round of the game.

To play one round of craps, the shooter casts a pair of dice.

- a) If the roll is a 7 or 11, the shooter wins the round.
- b) If the roll is 2,3 or 12, the shooter loses the round.
- c) If the first cast is a 4,5,6,8,9 or 10 the the shooter keeps casting the dice until:
  - 1) a 7 occurs
  - 2) the number from the first cast occurs again

In the former case the shooter loses, in the latter case the shooter wins. If the shooter had bet \$1 on the outcome of the round, the house takes the dollar if the shooter loses, and gives a dollar if the shooter wins.

We may now make our description a little more mathematical and construct an experiment based on the activity of the customer during one round of the game. Note that when the shooter plays a single round of craps, he often produces a large quantity of random numbers, the most relevant ones are the three quantities:

$U$  = the result of the first cast

$V$  = the result of the cast that decides the round

$$X = \begin{cases} W & \text{if the shooter wins} \\ L & \text{if the shooter loses} \end{cases}$$

We can see that if  $U = 2, 3, 7, 11$ , or  $12$  then the round is decided on the first cast,  $U = V$ , and

$$X = \begin{cases} W & \text{if } U = 7, 11 \\ L & \text{if } U = 2, 3, 12 \end{cases} .$$

If  $U = 4, 5, 6, 8, 9, 10$  then the game proceeds according to the following flow chart.

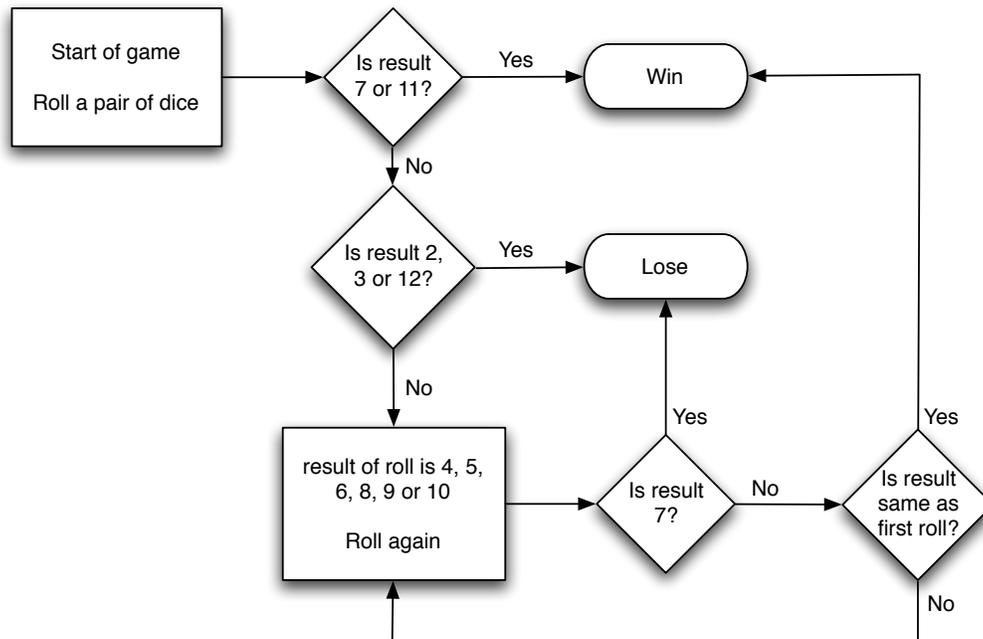


Figure 1: A flow chart to decide what happens after the first roll.

The goal here will be to create a fortune wheel such that spinning it once represents playing one round of craps. Since there are 17 possible outcomes of this game, we will represent one round of craps by a wheel with 17 regions on it. We shall build this wheel in steps.

Our first step will be to produce a wheel that simulates  $U$ , the first cast of the dice. It is clear that casting a pair of dice and recording the sum is equivalent to operating the following random device.

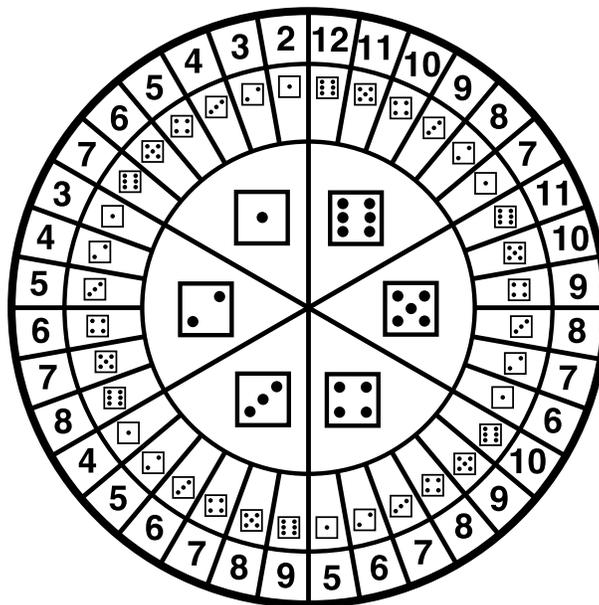


Figure 2: A wheel representing the sum of two dice

To simulate  $U$ , we can just spin the wheel just produced and record the number that stops in front of the arrow.

It is clear that we can simulate  $U$  just as well by any wheel that can be obtained by rearranging the labels on the outer edge in any order we wish. In particular, we can lump together in successive bunches all of the regions with the same type of tape. Indeed it is clear that what matters is not how each kind of tape is cut or in what order these various pieces are applied, but how much tape of each particular kind is used in building the wheel.

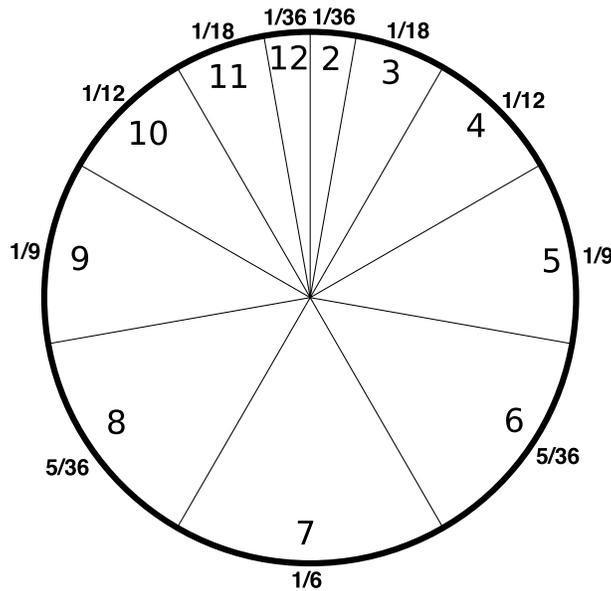


Figure 3: A wheel representing the sum of two dice

Our next task is to modify the wheel shown in figure 2 so that it not only produces  $U$ , but also  $V$  and  $X$  as well. We note that when  $U = 2, 3, 7, 11$ , or  $12$  that the values of  $V$  and  $X$  are completely determined already.

If  $U$  is one of the remaining possibilities then there are two choices for the value of the pair  $(V, X)$ . Consider the case when  $U = 4$ , then  $V$  is determined by spinning the wheel in figure 2 until either a 4 or a 7 comes up. The arcs labeled by 4 and 7 are ‘live’ and all of the remaining are ‘dead.’ One third of the ‘live’ arc is labeled by a 4 and two thirds of the ‘live’ arc is labeled by a 7 because

$$P[U = 4|U = 4 \text{ or } 7] = \frac{P[U = 4 \ \& \ U = 4 \ \text{or } 7]}{P[U = 4 \ \text{or } 7]} = \frac{1/12}{1/12 + 1/6} = 1/3$$

and similarly

$$P[U = 7|U = 4 \ \text{or } 7] = \frac{P[U = 7 \ \& \ U = 4 \ \text{or } 7]}{P[U = 4 \ \text{or } 7]} = \frac{1/6}{1/12 + 1/6} = 2/3.$$

In other words, when  $U = 4$ ,  $V$  can just as well be determined by spinning the ‘healthy’ wheel shown in figure 3 below.

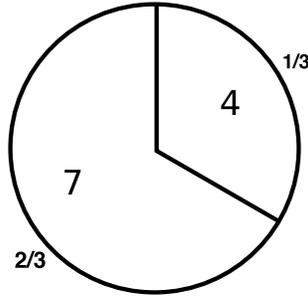


Figure 3: A wheel representing the remainder of a round once it is determined that  $U = 4$ .

Similarly, when  $U = 5$ ,  $V$  is either 5 or 7. Crippling the wheel so that only these two numbers come up means that  $V$  can be determined by spinning the healthy wheel shown in figure 4 below.

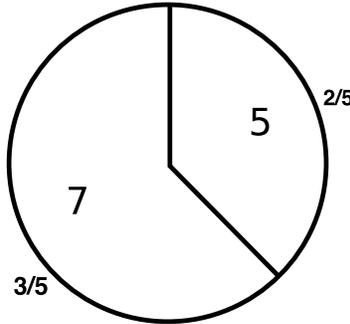


Figure 4: A wheel representing the remainder of a round once it is determined that  $U = 5$ .

The lengths of the arcs on the healthy wheels are computed from the conditional probabilities  $P[U = 5|U = 5 \text{ or } 7]$  and  $P[U = 7|U = 5 \text{ or } 7]$  which are easily calculated as  $2/5$  and  $3/5$  respectively.

And when  $U = 6$  we can get  $V$  by spinning the wheel below where the lengths of the arcs labeled 6 and 7 are determined from  $P[U = 6|U = 6 \text{ or } 7]$  and  $P[U = 7|U = 6 \text{ or } 7]$ .

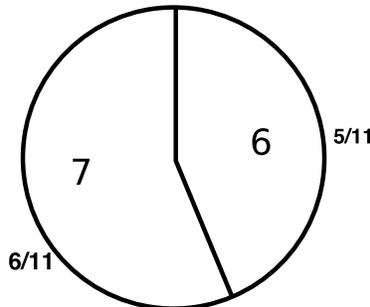


Figure 5: A wheel representing the remainder of a round once it is determined that  $U = 6$ .



game is  $P_{LOSE}$  then the expected value of the game is

$$A \cdot P_{WIN} - B \cdot P_{LOSE}$$

A \$1 bet on the come line of the table will pay off \$1 if the shooter wins. The expected value of the outcome this bet will be simply  $1 \cdot .4929 - 1 \cdot .5071 = -.0142$ . That means that on average the player will lose 1.4 cents per dollar that is bet.

**More betting:**

At the end of this section we described a number of other bets that are made at the craps table. The probability of winning those games may also be read off of this wheel with little difficulty.

A customer may place an ‘any seven’ bet that the next roll will be a 7. For this bet we look at the inner part of the wheel only and see that the probability that the next roll is a seven is  $1/6$ . The customer will win this bet only one in six times.

The payoff for an ‘any seven’ bet is four times the bet made, so the payoff for a \$1 dollar bet will be \$4. The expected value of this bet is  $4 \cdot \frac{1}{6} - 1 \cdot \frac{5}{6} = -.1667$ . Which means that on average the bettor will lose 16 cents for every dollar bet. This bet is much more advantageous to the house than the come line bet where the expected value was still in favor of the house but value is less than one-tenth of the any seven bet.

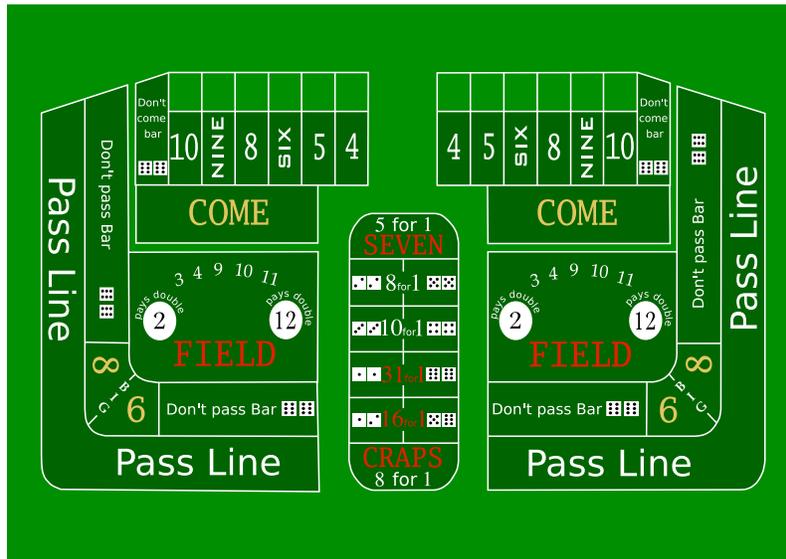


Figure 7: a craps betting table

**Betting Terminology:**

**any craps**

a bet that the next roll will be 2, 3, or 12. This bet pays 7:1 and has a house edge of 11.1%.

**3-way craps**

a bet made in units of 3 with one unit on 2, one unit on 3, and one unit on 12. This is a horn bet without the bet on 11.

**any seven**

a bet that the next roll will be 7. This bet pays 4:1 and has a house edge of 16.7%.

**big 6**

a bet that a 6 will be rolled before a 7 comes up. This bet pays even money, and has a house edge of 9.1%. A place bet on 6 pays 7:6 but is identical otherwise. The place bet is preferred, having a house edge of 1.5%

**big 8**

a bet that an 8 will be rolled before a 7 comes up. This bet pays even money, and has a house edge of 9.1%. A place bet on 8 pays 7:6 but is identical otherwise. The place bet is preferred, having a house edge of 1.5%

**buy bet**

giving the house a 5% commission in order to be paid correct odds for a place bet. The buy bets on 4 and 10 allow the player to reduce the house edge from 6.67% to 4% on these bets. Some casinos collect the commission only on winning bets, while others collect it at the time the bet is made.

**come bet**

A 'virtual pass line bet;' a bet made after the come out roll but in other respects exactly like a pass line bet.

**come out roll**

the first roll of the dice in a betting round is called the 'come out' roll. Pass bets win when the come out roll is 7 or 11, while pass bets lose when the come out roll is 2, 3, or 12. Don't bets lose when the come out roll is 7 or 11, and don't bets win when the come out roll is 2 or 3. Don't bets tie when the come out roll is 12 (2 in some casinos; the 'bar' roll on the layout indicates which roll is treated as a tie).

**dice pass**

The dice are said to 'pass' when the shooter rolls a 7 or 11 on the come-out roll. The dice 'don't pass' when the shooter rolls a 2, 3, or 12 on the come-out. If the come-out roll is a 4, 5, 6, 8, 9, or 10, this roll sets the 'point,' and the shooter continues to roll until the point is rolled again or a 7 is rolled (see 'seven out'). If the shooter rolls the point before rolling a seven, the dice pass. If the shooter sevens out, the dice don't pass and the shooter loses control of the dice. NOTE: in this context, 'pass' does NOT mean that the dice to given to the next player. Control of the dice

is transferred only when the shooter ‘sevens out’ or when the shooter has completed a game and no longer wishes to roll the dice.

**don’t come bet**

A ‘virtual don’t pass bet’; a bet made after the come out roll but in other respects exactly like a don’t pass bet.

**don’t pass bet**

a bet that the dice will not pass. This bet can be placed only immediately before a ‘come out’ roll. One result (either the 2 or the 12, depending on the casino) will result in a push. House edge on these bets is 1.40%. A don’t pass bet can be taken down, but not increased, after the come-out roll.

**double odds**

an odds bet that is about twice as large as the original pass/come bet. Some casinos offer higher odds, such as 5X or even 10X odds.

**field bet**

a bet that the next roll will be 2, 3, 4, 9, 10, 11, or 12. This bet pays even money for 3, 4, 9, 10, and 11, and usually pays 2:1 for 2 or 12. Some casinos pay 3:1 for either the 2 or 12 (but not both), and some casinos may make the 5 instead of the 9 a field roll.

**hard way**

a bet on 4, 6, 8, or 10 that wins only if the dice show the same face; e.g., ‘hard 8’ occurs when each die shows a four.

**hop bet**

a bet that the next roll will result in one particular combination of the dice, such as 2-2 (called a ‘hopping hardway’) or 3-5. 2-2, 3-3, 4-4, and 5-5 are paid the same as a one-roll 2; other hop bets are paid the same as a one-roll 11.

**horn bet**

a bet that the next roll will be 2, 3, 11, or 12, made in multiples of 4, with one unit on each of the numbers.

**horn high bet**

a bet made in multiples of 5 with one unit on 3 of the horn numbers, and two units on the ‘high’ number; e.g., ‘\$5 horn high eleven:’ \$1 each on 2, 3, 12, and \$2 on the 11.

**lay bet**

a bet that a particular number (4,5,6,8,9, or 10) will NOT be rolled before a 7 comes up. The casino takes 5% of the winnings on these bets. The 5% commission is usually taken up front, but some casinos take the commission after the bet wins.

**lay odds**

after a point has been established, the don't pass bettor can place an additional odds bet that will win if the original don't pass bet wins. The odds bet is paid at the correct odds for the point, and is a fair bet with no house edge. This also applies to a don't come bet. Making this bets is referred to 'laying the odds' for your don't bet.

**line bet**

a bet on the 'pass line' or the 'don't pass line' is called a 'line' bet. These bets are placed at the beginning of the game, before the 'come out' roll. The shooter is required to make a line bet in order to shoot the dice.

**odds off**

odds bets that are 'not working.' Odds bets can be called 'off' by the player at any time, but are left on the felt until the bet is resolved. Also, come odds bets are usually 'off' during the come out roll, unless the bettor asks to have the odds bets 'working.' Come odd bets that are 'off' will be returned to the player if the line bet loses on the come out roll. Don't come odds generally work on the come-out roll.

**pass bet**

a bet that the dice will pass, also known as a 'pass line' bet. This bet is generally placed immediately before a 'come out' roll, although you can make or increase this bet at any time. House edge on this bets is 1.41%.

**place bet (to win)**

a bet that a particular number (4, 5, 6, 8, 9, or 10) will be rolled before a 7 comes up. These bets are paid at slightly less than correct odds, giving the house an edge of 1.52% on 6/8, 4% on 5/9, and 6.67% on 4/10.

**place bet (to lose)**

a bet that a 7 will be rolled before the number you are placing (4,5,6,8,9, or 10) comes up. The casino requires you to lay slightly more than the correct odds, giving the house an edge of 3.03% on 4/10, 2.5% on 5/9, and 1.82% on 6/8.

**point**

if a 4, 5, 6, 8, 9, or 10 is rolled on the come out roll, then this number becomes the 'point.' The shooter must roll the point again, before rolling a seven, in order for the dice to 'pass.' A 'come

point' is just the number that is serving as a point for a come bet.

### **put bet**

1. A bet made on the pass line after the come out roll. This is allowed in Las Vegas and at Turning Stone, but not in Atlantic City and not at Foxwoods. This is not recommended, as 45% of your pass line wins are made on the come-out roll. 2. A bet made directly onto a come point number. E.g., 'Put \$5 and \$10 odds on the six.' Not recommended for the same reasons given in 1.

### **right bettor**

a player who bets that the dice will pass.

### **seven out**

when the shooter rolls seven after a point has been established. Control of the dice is transferred to the next shooter. Another term for this is 'miss out.' You will sometimes hear players call this something else, but we can't print those things here. This is often incorrectly called 'crap out.'

### **shooter**

the player who is rolling the dice. The shooter must place a 'line' bet ('pass' or 'don't pass') in order to be eligible to roll the dice. Of course, the shooter can place other bets in addition to the required 'line' bet. Most shooters (and players) tend to play the 'pass' line. Note that shooters who make 'don't pass' bets are not betting against themselves, they are simply betting that the dice will not 'pass.'

### **single odds**

an odds bet that is about as large as the original pass/come bet. Some casinos allow "double odds," or even larger odds bets.

### **take odds**

after a point has been established, the pass/come bettor can place an additional odds bet that will win if the original pass/come bet wins. The odds bet is paid at the correct odds for the point, and is a fair bet with no house edge.

### **two ways**

a phrase appended to a hardway or proposition bet to indicate that the player is betting one chip for the dealers along with his own bet. A \$2 bet two ways is \$1 for the player and \$1 for the dealers; a \$6 bet two ways is \$5 for the player and \$1 for the dealers; a \$10 bet two ways is \$5 for the player and \$5 for the dealers. E.g., 'Hard 6, two ways' or 'Two-way hard 6.'

### **working**

bets that are 'live' (i.e., can be resolved with the next roll) are said to be working. Generally, place bets, buy/lay bets, and come odds bets do not work on the come-out unless you tell the dealers to 'make them work.' All other bets (e.g., hardways) work unless you call them 'off' (i.e., tell the dealers you do not want them to 'work').

**world bet**

a bet that the next roll will be 2, 3, 7, 11, or 12, made in multiples of 5, with one unit on each of the numbers.

**wrong bettor**

a player who bets that the dice will not pass.

**EXERCISE:**

The table shown on the next page lists a number of common craps bets and their description. For each bet, calculate the probability of winning, the probability of losing, and the house advantage (= the negative of the expected value of the bet expressed as a percentage of the original bet).

Name of bet	Description	Payoff odds	P(Win)	P(Lose)	House advantage
Pass Bet	2,3,12 - lose 7,11 - win 4,5,6,8,9,10 this is the point shooter rolls again until point or 7 comes up, if point is first, then win. if 7 is first, then lose	1:1	.492929	.50707	1.4%
Don't Pass Bet	2,3 - win, 12 - roll again 7,11 - lose 4,5,6,8,9,10 this is the point shooter rolls again until point or 7 comes up, if point is first, then lose. if 7 is first, then win	1:1			
Field Bet	next roll is 2,3,4,9,10,11,12 - win 5,6,7,8 - lose	2:1 for 2 or 12 1:1 otherwise			
Field Bet (some casinos)	next roll is 2,3,4,9,10,11,12 - win 5,6,7,8 - lose	2:1 for 2 3:1 for 12 1:1 otherwise			
Any craps	next roll is 2, 3, 12- win 4,5,6,7,8,9,10,11 - lose	7:1			
Any 7	next roll is 7- win 2,3,4,5,6,8,9,10,11,12 - lose	4:1			
Big 6	if a 6 is rolled before a 7- win if a 7 is rolled before a 6 - lose	1:1			
Big 8	if a 8 is rolled before a 7- win if a 7 is rolled before a 8 - lose	1:1			
4 Hardway	if  is rolled before a 7 or a  , then win; otherwise lose	7:1			
10 Hardway	if  is rolled before a 7 or a  - win otherwise lose	7:1			
6 Hardway	if  is rolled before a 7 or a  or  , then win; otherwise lose	9:1			
8 Hardway	if  is rolled before a 7 or a  or  , then win; otherwise lose	9:1			

Name of bet	Description	Payoff odds	P(Win)	P(Lose)	House advantage
	next roll is 2 win otherwise lose	30:1			
	next roll is 3 win otherwise lose	15:1			
	next roll is 12 win otherwise lose	30:1			
	next roll is 11 win otherwise lose	15:1			
world bet	next roll is 2, 3, 7, 11, 12 otherwise lose; equiv to \$1 bet on each of 2, 12, 3, 11, any 7	26:5 for 2 or 12 11:5 for 3 or 11 0:5 for 7			
horn bet	next roll is 2, 3, 11, 12 otherwise lose; equivalent to \$1 bet on each of 2, 3, 11, 12	27:4 for 2 or 12 12:4 for 3 or 11			
take odds when point is 6 or 8	the point is rolled before 7 win 7 before the point loses (on existing pass bet only)	6:5			
take odds when point is 5 or 9	the point is rolled before 7 win 7 before the point loses (on existing pass bet only)	3:2			
take odds when point is 4 or 10	the point is rolled before 7 win 7 before the point loses (on existing pass bet only)	2:1			
lay odds when point is 6 or 8	the point is rolled before 7 lose 7 before the point wins (on existing pass bet only)	5:6			
lay odds when point is 5 or 9	the point is rolled before 7 lose 7 before the point wins (on existing pass bet only)	2:3			
lay odds when point is 4 or 10	the point is rolled before 7 lose 7 before the point wins (on existing pass bet only)	1:2			
place 6	if 6 is rolled before an 7, then win; if 7 is before 6, then lose.	7:6			
place 5	if 5 is rolled before an 7, then win; if 7 is before 5, then lose.	7:5			
place 4	if 4 is rolled before an 7, then win; if 7 is before 4, then lose.	9:5			
place 6 to lose	if 6 is rolled before an 7, then win; if 7 is before 6, then lose.	4:5			
place 5 to lose	if 5 is rolled before an 7, then win; if 7 is before 5, then lose.	5:8			
place 4 to lose	if 4 is rolled before an 7, then win; if 7 is before 4, then lose.	5:11			