HOMEWORK 13 : BETHUNE 1800 : MARCH 25, 2010

DUE: FRIDAY, APRIL 9, 2010

Say that in a game of Texas Hold 'Em that your hole cards are $K\diamondsuit, J\clubsuit$. On the flop you see $3\heartsuit, J\heartsuit, Q\diamondsuit$. On the turn you see $10\spadesuit$.

(1) With just the cards showing, how many hands are better than yours?
(2) Calculate a weighted average

$$\sum_X P(\text{ last card } = X)(\text{ number of hands better than yours })$$

where the sum is over all the possible last cards that could show up on the River.

(3) Say that you have \$30 to play a fair game with and you make 100 \$1 bets. What is the probability of ending the game with exactly \$1 at the end of those 100 games?
(4) Say that you have \$30 to play a fair game with and you make 100 \$1 bets. What is the probability of ending the game with between \$1 and \$5 at the end of those 100 games?
(5) Say that you have \$30 to play a fair game with and you make 100 \$1 bets. What is the probability of still having money left at the end of the 100 games?
(6) Say that you have \$30 to play a fair game with and you make 100 \$3 bets. What is the probability of still having money left at the end of the 100 games?
(7) Let ω_i be a random variable with probability 1/2 that you win \$1 and probability 1/2 that you lose a dollar. For our purposes, you can think of this random variable as a wheel split in half. Draw three wheels representing $S_2 = \omega_1 + \omega_2$, $S_3 = \omega_1 + \omega_2 + \omega_3$ and $S_4 = \omega_1 + \omega_2 + \omega_3 + \omega_4$. Label the arcs of the wheel with the probabilities that each possible value appears and group together arcs where the values that the random variables take on are the same.
(8) What is the expected value of each of the random variables in the previous problem, S_2, S_3, S_4?
(9) Now say that you have a random variable ω_i with probability .4929 of winning \$1 and probability .5071 of losing \$1 (this is equivalent to the pass line bet on craps). Redraw the wheels from the first problem and label the arcs with the probabilities that each of the values takes on.
(10) What is the expected value of each of the random variables in the previous problem, S_2, S_3, S_4?

Say that you have a random variable $S_n = X_1 + X_2 + X_3 + \cdots + X_n$ where X_i are all independent random variables that are 1 with probability p and -1 with probability $1 - p$. Recall that the weak law of large numbers says

$$P\left(|S_n/n - (2p-1)| > \epsilon\right) \leq \frac{4p(1-p)}{n\epsilon^2}$$

(11) In the class experiment we had a game where the p value was 1/2. If someone comes out 32 chips ahead at the end of 130 rounds, what does the weak law of large numbers say about this happening?
(12) We also had players come out 14 chips ahead at the end of 130 rounds. What does the weak law of large numbers say about this event?
(13) Say that someone plays craps (where the probability of winning is $p = .4929$) 130 rounds betting a chip each time. What does the weak law of large numbers say about the probability of coming out at least 33 chips ahead? at least 14 chips ahead?