QUIZ III - MATH 1021 - NOVEMBER 18, 2004

Note there are several ways of solving these problems and there aren't unique solutions. I am just using properties of vectors here which appear in chapter 4.

(1) Find the equation of the plane which contains both of the lines

$$L_1 = \begin{bmatrix} 3+2t \\ -1+t \\ 5-t \end{bmatrix}$$

and

$$L_2 = \begin{bmatrix} -4-2t \\ 1-t \\ 3+t \end{bmatrix}$$

Answer: Normally we can take the cross product of the two vectors to find a normal vector. In this case L_1 is in the direction of $\begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$ and L_2 is in the direction of $\begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}$ and the cross product of these vectors is 0. This means that these two lines are parallel. Take any vector which goes from a point on L_1 to a point on L_2, this will be a vector in the plane and it will not be parallel to either L_1 or L_2. Since $(3,-1,5)$ is a point on L_1 and $(-4,1,3)$ is a point on L_2, take $\begin{bmatrix} 3-(-4) \\ -1-1 \\ 5-3 \end{bmatrix} = \begin{bmatrix} 7 \\ -2 \\ 2 \end{bmatrix}$ as a second vector. Then (for example, there are many solutions here):

$$\begin{bmatrix} -4 \\ 1 \\ 3 \end{bmatrix} + s\begin{bmatrix} 7 \\ -2 \\ 2 \end{bmatrix} + t\begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$$

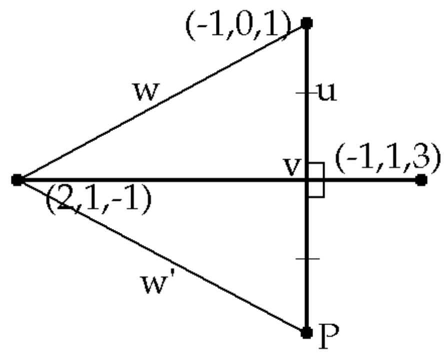
is the plane which contains them. You can also give this in non-parametric form since $\begin{bmatrix} 7 \\ -2 \\ 2 \end{bmatrix} \times \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 11 \\ 11 \end{bmatrix}$, so $y+z=4$ is the the non-parametric form.

(2) Find the equation of the line which lies in the plane $P_1 : 3x - y - z = 4$ and which is perpendicular to the line of intersection of P_1 and $P_2 : 2x + 2y - z = 1$ at the point $(0,-1,-3)$.

Answer: The line of intersection is parallel to the vector $\begin{bmatrix}3\\-1\\-1\end{bmatrix} \times \begin{bmatrix}2\\2\\-1\end{bmatrix} = \begin{bmatrix}3\\1\\8\end{bmatrix}$. If a line lies in the plane P_1 then the direction will be perpendicular to $\begin{bmatrix}3\\-1\\-1\end{bmatrix}$. Therefore the line must be parallel to $\begin{bmatrix}3\\-1\\-1\end{bmatrix} \times \begin{bmatrix}3\\1\\8\end{bmatrix} = \begin{bmatrix}-7\\-27\\6\end{bmatrix}$. If it also passes through the point $(0,-1,-3)$, then the equation is

$$\begin{bmatrix}0\\-1\\-3\end{bmatrix} + t\begin{bmatrix}-7\\-27\\6\end{bmatrix}$$

(3) Find the coordinates of the point P in the following diagram where the lines marked with a dash have the same length (note: the diagram is not necessarily exactly to scale):



Answer: $\vec{v}$ is the vector from $(2,1,-1)$ to $(-1,1,3)$ and so $\vec{v} = \begin{bmatrix}-3\\0\\4\end{bmatrix}$ and $\vec{w}$ is the vector from $(2,1,-1)$ to $(-1,0,1)$ or $\vec{w} = \begin{bmatrix}-3\\-1\\2\end{bmatrix}$. Now $proj_{\vec{v}}(w) = \frac{(\vec{w}\cdot\vec{v})\vec{v}}{|\vec{v}|^2} = \frac{17}{25}\vec{v}$ and because we have $proj_{\vec{v}}(w) + \vec{u} = \vec{w}$ we know that $\vec{u} = \vec{w} - proj_{\vec{v}}(w)$ and this means that $\vec{w}' = proj_{\vec{v}}(w) - \vec{u} = proj_{\vec{v}}(w) - (\vec{w} - proj_{\vec{v}}(w)) = 2proj_{\vec{v}}(w) - \vec{w}$. If you work this all out then you have $\vec{w}' = \begin{bmatrix}-27/25\\1\\86/25\end{bmatrix}$ and if you add $\vec{w}'$ to the point $(2,1,-1)$, then you get that $P = (23/25, 2, 61/25)$.