QUIZ IV - MATH 1021 - NOVEMBER 18, 2004

Note there are several ways of solving these problems and there aren't unique solutions. I am just using properties of vectors here which appear in chapter 4.

(1) Find the equation of the plane which contains the line

$$L_1 = \begin{bmatrix} 3+2t \\ -1+t \\ 5-t \end{bmatrix}$$

and is parallel to the line

$$L_2 = \begin{bmatrix} 1+2t \\ 1-t \\ 2-t \end{bmatrix}$$

Answer: $\vec{u} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$ is a vector parallel to L_1 and $\vec{v} = \begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix}$ is a vector parallel to L_2. A plane which contains these vectors and a point on L_1 is

$$P = \begin{bmatrix} 3 \\ -1 \\ 5 \end{bmatrix} + s \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} + t \begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix}.$$

Also $\vec{u} \times \vec{v} = \begin{bmatrix} -2 \\ 0 \\ -4 \end{bmatrix}$ is a vector normal to the plane, so $-2x + 0y - 4z = r$ for some r and we plug in the point $(3, -1, 5)$ in the equation so that $r = -6 + 0 - 20 = -26$. $-2x - 4z = -26$ or $x + 2z = 13$.

(2) Find the equation of the line which is in the intersection of the plane

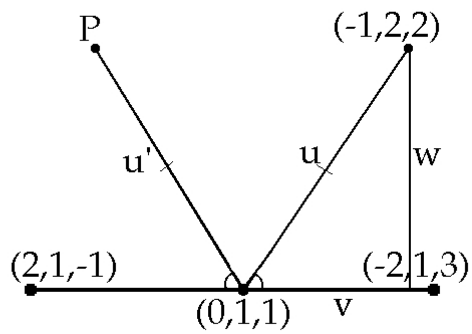
$$P_1 : \begin{bmatrix} 1+2t-s \\ 1-t+s \\ t+2s \end{bmatrix}$$

and the plane $P_2 : 2x + 2y - z = 1$.

Answer: A vector which is perpendicular to P_1 is $\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \times \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -3 \\ -5 \\ 1 \end{bmatrix}$ and P_2 is perpendicular to $\begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$. A line which is in the intersection is perpendicular to both of these vectors so is parallel to $\begin{bmatrix}-3\\-5\\1\end{bmatrix} \times \begin{bmatrix}2\\2\\-1\end{bmatrix} = \begin{bmatrix}3\\-1\\4\end{bmatrix}$. Now a point which lies in the intersection is $(-2,3,1)$. Therefore a line in the intersection is

$$\begin{bmatrix}-2+3t\\3-t\\1+4t\end{bmatrix}$$

(3) Find the coordinates of the point P in the following diagram where the lines marked with a dash have the same length and the marked angles are the same (note: the diagram is not necessarily exactly to scale):



The vector from $(0,1,1)$ to $(-2,1,3)$ is $\vec{v} = \begin{bmatrix}-2\\0\\2\end{bmatrix}$. The vector from $(0,1,1)$ to $(-1,2,2)$ is $\vec{u} = \begin{bmatrix}-1\\1\\1\end{bmatrix}$. The projection from this vector $\vec{u}$ onto $\vec{v}$ is $proj_{\vec{v}}(\vec{u}) = (\vec{u}\cdot\vec{v})\frac{\vec{v}}{|\vec{v}|^2} = \frac{4}{8}\begin{bmatrix}-2\\0\\2\end{bmatrix} = \begin{bmatrix}-1\\0\\1\end{bmatrix}$.

Now $\vec{u} = proj_{\vec{v}}(\vec{u}) + \vec{w}$, so $\begin{bmatrix}-1\\0\\1\end{bmatrix} + \vec{w} = \begin{bmatrix}-1\\1\\1\end{bmatrix}$ this means $\vec{w} = \begin{bmatrix}0\\1\\0\end{bmatrix}$. Now $\vec{v}' = -proj_{\vec{v}}(\vec{u}) + \vec{w} = -\begin{bmatrix}-1\\0\\1\end{bmatrix} + \vec{w} = \begin{bmatrix}1\\1\\-1\end{bmatrix}$ and this is the vector which starts at $(0,1,1)$ and goes to the point P. This means that $P = (0+1, 1+1, 1-1) = (1,2,0)$. You can check that the distance from P to $(2,1,-1)$ is equal to the distance from $(-1,2,2)$ to $(-2,1,3)$.