HOMEWORK ASSIGNMENT NO. 4

DATE ASSIGNED: OCOBER 9, 2019; DUE: WEDNESDAY, OCTOBER 23, 2019

In a proof by induction, be sure to clearly indicate your base case, inductive assumption and your conclusion (you must have a statement which says that you conclude that some statement is true for all values greater than the base case).

(1) Let $L_1 = 1, L_2 = 3$ and $L_{n+1} = L_n + L_{n-1}$ for $n \geq 3$. Prove by induction that
$$L_n = \left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{2}\right)^n$$
for $n \geq 1$.

(2) Prove (by induction) that for all $n \geq 1$,
$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}^n = \begin{bmatrix} 1 & n & \frac{n(n-1)}{2} \\ 0 & 1 & n \\ 0 & 0 & 1 \end{bmatrix}$$
Prove the same result holds true for all $n < 1$ as well.

(3) I've said in class that the complex solutions to $z^n = 1$ are all equally spaced about a circle of radius 1 about the origin. To show this properly, you would need to know that the sum of the x coordinates of all these points are all 0 and the sum of the y coordinates are all 0. Note: showing that the sum of the y coordinates is all 0, (what I am asking in part (a) of this problem) does not require induction and will probably handle n even and n odd differently because if n is even there are two points on the x-axis and if n is odd there is only one point on the x-axis.

(a) Show that for all $n \geq 1$,
$$\sin(0) + \sin(2\pi/n) + \sin(4\pi/n) + \cdots + \sin((2n-2)\pi/n) = 0\ .$$

(b) Show that for all $n \geq 0$,
$$1 + \cos x + \cos(2x) + \cdots + \cos(nx) = 1 + \frac{\cos\left(\frac{(n+1)x}{2}\right)\sin\left(\frac{nx}{2}\right)}{\sin\left(\frac{x}{2}\right)}\ .$$
Explain clearly what trigonometric identities that you used in your proof and then use this identity to show that
$$\cos(0) + \cos(2\pi/n) + \cos(4\pi/n) + \cdots + \cos((2n-2)\pi/n) = 0$$