HOMEWORK #5

DATE GIVEN: OCTOBER 28, 2019 DUE: NOVEMBER 13, 2019

(1) Let n be an integer, and k be a positive integer. Justify the statement:

"The last k digits of n are divisible by 2^k if and only if n is divisible by 2^k."

(2) Similarly, let n be an integer, justify the statement

"The sum of the digits of n is divisible by 9, if and only if n is divisible by 9."

(3) Show that 3^{n+1} divides $2^{3^n} + 1$ for all $n \geq 0$.

(4) The following is a justification for the statement

"if 3 divides n^2, then 3 divides n."

Let n be a positive integer such that n^2 is a multiple of 3. Then $n = 3m$ where m is some positive integer. So $n^2 = (3m)^2 = 9m^2 = 3(3m^2)$. This breaks down into $3m$ times $3m$ which shows that m is a multiple of 3.

Read this argument critically. Is the argument correct, i.e., does it make sense logically? What is the reason each sentence is true? If the argument is not correct, is there a minor (notational or other) change that would yield a correct argument? Does the argument (or your minor revision) prove the result? If not, does it prove anything else? Justify your answers.