

## HOMEWORK #6

DATE GIVEN: NOVEMBER 13, 2019 DUE: NOVEMBER 27, 2019

Follow the instructions on the webpage for preparing this homework.

- (1) Bob was born in 2000 and has a birthday on March 15. Find the day of the week his birthday occurred on in 2000 through 2019. Explain this pattern and find a way of determining the day of the week in any year.  
Note: In the Gregorian calendar, years that are divisible by 100, but not by 400, do not contain a leap day. That means you need to figure out how the formula is correct until 2100 and then you should have a different formula.
- (2) Every day of the week I bought the same lunch and always found that I received 3 quarters in change. At the end of some number of weeks, I rolled my quarters and there were 13 left over. What is the minimum number of weeks that I had been buying the same lunch? Note: Rolls of quarters come in 40's (total of \$10).
- (3) Prove or disprove: Let  $a, b, c$  be integers. If  $\gcd(a, b) = 1$  and  $c|(a + b)$ , then  $\gcd(a, c) = 1$ .
- (4) Only one of the following two statements is true. Find a proof for one and a counterexample for the other. Let  $r$  and  $s$  be integers and  $a, b$  be positive integers.
  - (a) if  $r \equiv s \pmod{a}$ , then  $r \equiv s \pmod{ab}$ .
  - (b) if  $r \equiv s \pmod{ab}$ , then  $r \equiv s \pmod{a}$ .
- (5) Let  $a$  and  $b$  be positive integers. Prove or disprove: If  $b \equiv 0 \pmod{a}$  and  $a \equiv 0 \pmod{b}$ , then  $a = b$ .