

Assignment # 2

1a $\sum_{i=2}^n i(i-1)$ $\sum_{i=1}^{n-1} i(i+1)$ $\sum_{i=1}^n (i-1)i$

b $\sum_{i=1}^n [a+(i-1)b]$ $\sum_{i=0}^{n-1} a+ib$ ~~$\sum_{i=0}^n (a+bi)$~~

c $\sum_{i=1}^{34} (3i-2)$ $\sum_{i=0}^{33} (1+3i)$ ~~$\sum_{i=1}^n (i+3)$~~

d $\sum_{i=0}^{n-r} \binom{n-i}{r}$ ✓ $n \geq r$ $\sum_{i=1}^r \binom{i+(1-i)}{r}$ ✗
 $\sum_{i=0}^r \binom{n-i}{r}$ ✗ $\binom{1}{r} + \binom{1}{r} + \dots + \binom{1}{r}$
 \parallel $\binom{n}{r} + \binom{n-1}{r} + \dots + \binom{n-r}{r}$

2a $a^2 + (a+b)^2 + (a+2b)^2 + \dots + [a+bn]^2$
 $\circ a^2 + (a^2 + 2ab + b^2) + (a^2 + 4ab + 4b^2) + \dots + (a^2 + 2abn + n^2b^2)$
 $\circ (n+1)a^2 + 2ab \cdot \frac{n(n+1)}{2} + b^2 \cdot \frac{n(n+1)(2n+1)}{6}$

hmm... for $n=2$ $\binom{0}{2} + \binom{1}{2} + \binom{2}{2} = 1$
 $\frac{2 \cdot 2(2-1)}{2!} = 2$

b. $0 + 6 + 24 + 60 + \dots + n(n+1)(n+2)$
 $0 + (1 \cdot 2 \cdot 3) + (2 \cdot 3 \cdot 4) + (3 \cdot 4 \cdot 5) + \dots + n(n+1)(n+2)$

c. $\binom{0}{2} + \binom{1}{2} + \binom{2}{2} + \binom{3}{2} + \dots + n$

$$= \frac{(i)!}{(i-2)! \cdot 2!} = \frac{i(i-1) \cancel{(i-2)!}}{\cancel{(i-2)!} \cdot 2!} = \frac{i^2 - i}{2!}$$

$$\frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2}$$

$$i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$i = \frac{n(n+1)}{2}$$

but $\frac{2n^3 + 3n^2 + n - 3n^2 - 3n}{2!}$

$$= \frac{2n^3 - 2n}{2!} = \frac{2n(n-1)}{2!}$$

d. $\frac{i!}{(i-3)! \cdot 3!} = \frac{i(i-1)(i-2)(i-3)!}{(i-3)! \cdot 3!} = \frac{i^3 - 3i^2 + 2i}{3!}$

$$= \frac{\left(\frac{n(n+1)}{2}\right)^2}{4} - 3 \left(\frac{n(n+1)(2n+1)}{6}\right) + 2 \left(\frac{n(n+1)}{2}\right)$$

Expand and simplify

$$= \frac{3n^4 - 6n^3 - 3n^2 + 6n}{3!}$$

for $n=3$ this isn't right
 $\binom{0}{3} + \binom{1}{3} + \binom{2}{3} + \binom{3}{3} = 1$
 $\frac{3 \cdot 3 \cdot 4 \cdot 2 \cdot 1}{3!} = 12$

$$= \frac{3n(n^3 - 2n^2 - n + 6)}{3!} = \frac{3n(n+1)(n-1)(n-2)}{3!}$$

3 sub in $n=4$ $k=2$

$$\binom{4}{2} + \binom{4}{3} = \binom{5}{3}$$

$$LS = 6 + 4 \quad RS = 10$$

$$\therefore LS = RS$$

If $\binom{n}{k}$ is the number of ways of choosing k elements from an n element set, then when this value is added to the number of ways of choosing $k+1$ elements (i.e., one more element) from the same sized set, what is being added is $n(n-k-1)$ and the sum will always be equivalent to choosing $k+1$ elements from an $n+1$ element set

Yes, but looking for an explanation "Why?"

4 Length or width cannot be greater than 8 units

$$T = (9-L)(9-W)$$

$T =$ total # of rectangles
 $L =$ length
 $W =$ width

Plug in the dimensions to find the total # of rectangles that fit in 8x8 chessboard

dimension	# rectangles	d	#r	d	#r	d	#r
1x1	64	1x4	40	3x5	24	4x6	15
1x2	56	2x4	35	4x5	20	5x6	12
2x2	49	3x4	30	5x5	16	6x6	9
1x3	48	4x4	25	1x6	24	1x7	16
2x3	42	1x5	32	2x6	21	2x7	14
3x3	36	2x5	28	3x6	18	3x7	12

d	#
4 x 7	10
5 x 7	8
6 x 7	6
7 x 7	4
1 x 8	8
2 x 8	7
3 x 8	6
4 x 8	5

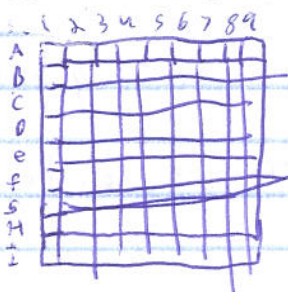
d	#
5 x 8	4
6 x 8	3
7 x 8	2
8 x 8	1

The sum of ^{the} squares is 204, take the sum of the rectangles (L ≠ w) multiply them by 2 and add them to the sum of square.

$$546(2) + 204 = 1296$$

∴ 1296 rectangles in a 8x8 chessboard

Solution 2



to choose two sides from 9 to
 we have $\binom{9}{2}$ different ways,

to choose two sides from 1 to 9

we have $\binom{9}{2}$ different ways

According to rule of product, to get a rectangle we have two steps, so we have $\binom{9}{2} \cdot \binom{9}{2}$ different ways:

$$\binom{9}{2} \cdot \binom{9}{2} = \frac{9 \times 8}{2} \times \frac{9 \times 8}{2} = 36 \times 36 = 1296$$

∴ There is 1296 rectangles