Assignment # 2

1a $\sum_{i=2}^{n} i(i-1)$ $\sum_{i=1}^{n-1} i(i+1)$ $\sum_{i=1}^{n} (i-1)i$

b $\sum_{i=1}^{n} [a+(i-1)b]$ $\sum_{i=0}^{n-1} a+ib$ $\sum_{i=0}^{n} (a+bi)$ X

c $\sum_{i=1}^{34} (3i-2)$ $\sum_{i=0}^{33} (1+3i)$ $\sum_{i=1}^{n} \cancel{(i+3)}$

d $\sum_{i=0}^{n-r} \binom{n-i}{r}$ ✓ $n \geq r$ $\sum_{i=1}^{r} \binom{i+(1-i)}{r}$ X

$= \binom{1}{r} + \binom{1}{r} + \cdots + \binom{1}{r}$

$\sum_{i=0}^{r} \binom{n-i}{r}$ X

$= \binom{n}{r} + \binom{n-1}{r} + \cdots + \binom{n-r}{r}$

2a. $a^2 + (a+b)^2 + (a+2b)^2 + \ldots + [a+b(n)]^2$

$= a^2 + (a^2 + 2ab + b^2) + (a^2 + 4ab + 4b^2) + \ldots + (a^2 + 2abn + n^2b^2)$

$= (n+1)a^2 + 2ab \cdot \frac{n(n+1)}{2} + b^2 \frac{n(n+1)(2n+1)}{6}$

b. $0+6+24+60+\ldots+n(n+1)(n+2)$

$= 0+(1\cdot2\cdot3)+(2\cdot3\cdot4)+(3\cdot4\cdot5)+\ldots+n(n+1)(n+2)$

c $\binom{0}{2}+\binom{1}{2}+\binom{2}{2}+\binom{3}{2}+\ldots\ n$

$$=\frac{(i)!}{(i-2)!\,2!} = \frac{i(i-1)\cancel{(i-2)}!}{\cancel{(i-2)}!\ 2!} = \frac{i^2-i}{2!}$$

$$=\frac{\frac{n(n+1)(2n+1)}{6}-\frac{n(n+1)}{2}}{2!}$$

$$i^2=\frac{n(n+1)(2n+1)}{6}$$

$$i=\frac{n(n+1)}{2}$$

$$=\frac{2n^3+3n^2+n-3n^2-3n}{2!}$$

$$=\frac{2n^3-2n}{2!}=\frac{2n(n-1)}{2!}$$

$\binom{0}{2}+\binom{1}{2}+\binom{2}{2}=1$

hmm... for $n=2$

but $\frac{2\cdot2(2-1)}{2!}=2$

d $$\frac{i!}{(i-3)!\,3!} = \frac{i(i-1)(i-2)(i-3)!}{(i-3)!\,3!} = \frac{i^3-3i^2+2i}{3!}$$

$$=\frac{\left(\frac{n(n+1)^2}{4}\right)-3\left(\frac{n(n+1)(2n+1)}{6}\right)+2\left(\frac{n(n+1)}{2}\right)}{3!}$$

Expend and simplify

$$=\frac{3n^4-6n^3-3n^2+6n}{3!}$$

$$=\frac{3n(n^3-2n^2-n+6)}{3!}=\frac{3n(n+1)(n-1)(n-2)}{3!}$$

for $n=3$ this isn't right

$\binom{0}{3}+\binom{1}{3}+\binom{2}{3}+\binom{3}{3}=1$

$\frac{3\cdot3\cdot4\cdot2\cdot1}{3!}=12$

3 sub in $n=4$ $k=2$

$$\binom{4}{2} + \binom{4}{3} = \binom{5}{3}$$

$LS = 6 + 4$ $RS = 10$

$\therefore LS = RS$

If $\binom{n}{k}$ is the number of ways of choosing k elements from an n element set then when this value is added to the number of ways of choosing $k+1$ elements (i.e, one more element) from the same sized set, what is being added is $n(n-k-1)$ and the sum will always be equivalent to choosing $k+1$ elements from an $n+1$ element set Yes, but looking for an explanation "Why?"

4 Length or width cannot be greater than 8 units

$T = (9-L)(9-W)$ T = total # of rectangles
L = length
W = Width

Plug in the dimensions to find the total # of rectangles that fit in 8x8 chessboard

dimension	# rectangles	d	#r	d	#r	d	#r
1x1	64	1x4	40	3x5	24	4x6	15
1x2	56	2x4	35	4x5	20	5x6	12
2x2	49	3x4	30	5x5	16	6x6	9
1x3	48	4x4	25	1x6	24	1x7	16
2x3	42	1x5	32	2x6	21	2x7	14
3x3	36	2x5	28	3x6	18	3x7	12

d	# r
4 x 7	10
5 x 7	8
6 x 7	6
7 x 7	4
1 x 8	8
2 x 8	7
3 x 8	6
4 x 8	5

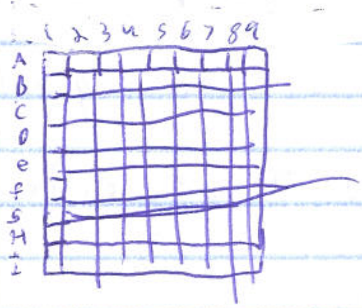
d	# r.
5 x 8	4
6 x 8	3
7 x 8	2
8 x 8	1

The sum of the squares is 204, take the sum of the rectangles $(L \neq W)$ multiply them by 2 and add them to the sum of square.

$$546(2) + 204 = 1296$$

$\therefore$ 1296 rectangles in a 8x8 chessboard

Solution 2



to choose two sides from a to i we have $\binom{9}{2}$ different ways,

to choose two sides from 1 to 9

we have $\binom{9}{2}$ different ways

According to rule of product, to get a rectangle we have two steps, so we have $\binom{9}{2} \cdot \binom{9}{2}$ different ways:

$$\binom{9}{2} \cdot \binom{9}{2} = \frac{9 \times 8}{2} \times \frac{9 \times 8}{2} = 36 \times 36 = 1296$$

$\therefore$ There is 1296 rectangles