Assignment Summary   #3.

①   Firstly, on the course web page it is noted that "words" are not English words, but just a sequence of letters. Only one person agreed with the given solution. Everyone else realized that it was less than one and therefore not correct because there can be atleast 3 ways; all As, all bs and all Cs. Also too, few people noted that dividing by 10! would only be correct if there were 10 different letters but there are only 3 - A, B & C.

Assignment Summary.

④ How many ways are there of rolling a die 10 times such that the results are all 4s, 5s and 6s and in increasing order?

If the die was rolled once, there will be 3 ways - 4, 5 or 6. If the die was rolled twice, we can get the following combination of numbers:

4 5   6 6
4 4   5 6
5 5   4 6

- 6 ways.

If the die is rolled 3 times, there will be 10 ways:

4 4 4   5 5 5   6 6 6   4 4 6
4 4 5   4 5 6   5 6 6
4 5 5   5 5 6   4 6 6

If the die is rolled 4 times, there will be 15 ways:

4 4 4 4   5 5 5 6   5 6 6 6
5 5 5 5   4 4 5 6   4 5 5 5
6 6 6 6   4 4 5 5   4 6 6 6
4 4 4 5   5 5 6 6   5 5 6 6
4 4 4 6   4 5 5 6   4 4 6 6

From these 4 cases, we can make a table and find a pattern.

Rolls	Possibilities
1	3 2+3
2	6 2+4
3	10 2+5
4	15

Yes, but should explain the "pattern"

Hence, for 10 rolls, there will be $(3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11)$ ways
= 66 ways.

② There are 26 letters in the alphabet and hence 26 options for the first letter. Since repetition is allowed, there will be 26 options for all four letters. This means that there are 26^4 ways of obtaining the first 4 letters.

There are 10 numbers to choose from for all 3 numbers since repetition is allowed. This means that there are 10^3 ways of obtaining the 3 numbers. Since the letters and numbers go together [ordered pairs], the total no. of ways;

$$= 26^4 \times 10^3$$
$$= 456976000$$

③ There are only 3 possibilities and the die is rolled 10 times. This means that for each roll, there can be 3 different outcomes.

$$\Rightarrow 3^{10} \text{ ways}$$
$$= 59049.$$