

Convener Summary
Assignment # 2

#1. a) $\sum_{i=2}^n (i-1)i = 2 \cdot 6 + 12 + \dots + (n-1)n$

b) $\sum_{i=1}^n [a + (i-1)b] = a + (a+b) + (a+2b) + \dots + [a + (n-1)b]$

c) $\sum_{i=1}^n [1 + 3(n-1)] = 1 + 4 + 7 + 10 + \dots + 100$

d) $\sum_{i=0}^n \binom{n-i}{r} = \binom{n}{r} + \binom{n-1}{r} + \binom{n-2}{r} + \dots + \binom{1}{r}$

#2. a) $\sum_{i=0}^n (a+bi)^2 = a^2 + (a+b)^2 + (a+2b)^2 + \dots + (a+nb)^2$
 $= (n+1)a^2 + ab(n+1)n + b^2 n(n+1)(2n+1)/6$

b) $\sum_{i=0}^n i(i+1)(i+2) = 0 + 6 + 24 + \dots + n(n+1)(n+2)$
 $= n(n+1)(n+2)(n+3)/4$

c) $\sum_{i=2}^n \binom{i}{2} = \binom{2}{2} + \binom{3}{2} + \binom{4}{2} + \dots + \binom{n}{2}$
 $= \binom{n+1}{3}$

d) $\sum_{i=3}^n \binom{i}{3} = \binom{3}{3} + \binom{4}{3} + \binom{5}{3} + \dots + \binom{n}{3}$
 $= \binom{n+1}{4}$

#3. The number of ways of choosing $k+1$ elements from a set of $n+1$ elements is equal to the number of ways of choosing k elements from a set of n elements added to the number of ways of choosing $k+1$ elements from a set of n elements. This can be seen by treating the $+1$ in $n+1$ as an extra element. If this element is in a specified subset of $n+1$, there are $\binom{n}{k}$ subsets in total of this type. The number of subsets that do not contain this extra element is $\binom{n}{k+1}$ (since you are choosing $n+1$ elements in total but not the extra element, or $\binom{n+1}{k+1}$). The subsets contain the extra element added to the subsets that do not contain the extra element gives the whole set of $\binom{n+1}{k+1}$, so $\binom{n}{k} + \binom{n}{k+1} = \binom{n+1}{k+1}$

#4. A trick is to observe the numbers of rectangles that end in each bottom right corner.

1	2	3	4	...
2	4	6	8	...
3	6	9	12	...
...
...
...
...
...
...
...

n	$= n$
$2n$	$= 2n$
$3n$	$= 3n$
\vdots	\vdots
$8n$	$= 8n$

where n is the natural number
 sum up to the numbers of columns (8 in this case)
 n is $\frac{8 \cdot 9}{2} = 36$

total = natural # sum of rows = n

$$\frac{n(n+1)(n+2)}{6} = \frac{36(36+1)(36+2)}{6} = 86 \cdot 36$$

= 1296 rectangles in total

$$\frac{n(n+1)(n+2)}{6} =$$

$$\frac{(1+n)}{2} =$$

$$\frac{(1+n)}{2} =$$