Math 1200 section B - Problems, conjectures and proofs - Homework 10

Assigned: March 16, 2008 Due: March 23, 2009, 7:30pm.

(1) The following is a justification for the statement "if 3 divides n^2, then 3 divides n."

Let n be a positive integer such that n^2 is a multiple of 3. Then $n = 3m$ where m is some positive integer. So $n^2 = (3m)^2 = 9m^2 = 3(3m^2)$. This breaks down into $3m$ times $3m$ which shows that m is a multiple of 3.

Read this argument critically. Is the argument correct, i.e., does it make sense logically? What is the reason each sentence is true? If the argument is not correct, is there a minor (notational or other) change that would yield a correct argument? Does the argument (or your minor revision) prove the result? If not, does it prove anything else? Justify your answers.

(2) Read pages 67-73 from Mathematical Proofs by Chartrand/Polimeni/Zhang. Do problems 3.6 through 3.11 on page 83.

3.6 Prove that if x is an odd integer, then $9x + 5$ is even.

3.7 Prove that if x is an even integer, then $5x - 3$ is an odd integer.

3.8 Prove that if a and c are odd integers, then $ab + bc$ is even.

3.9 Let $n \in \mathbb{Z}$. Prove that if $1 - n^2 > 0$, then $3n - 2$ is an even integer.

3.10 Let $x \in \mathbb{Z}$. Prove that if 2^{2x} is an odd integer, then 4^x is an odd integer.

3.11 Let $S = \{0, 1, 2\}$ and let $n \in S$. Prove that if $(n+1)^2(n+2)^2/4$ is even, then $(n+2)^2(n+3)^2/4$ is even.