Math 1200 section B - Problems, conjectures and proofs

Assigned: September 15, 2008 Due: September 22, 2008, 7pm.

You should read chapters 1 and 2 from the textbook 'Thinking Mathematically.' The first three problems should be done following the discussion from in class.

(1) Write the following sums using summation notation. Make clear about what assumptions are implied about the variables a, b, r and n in the expressions.
- $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + (n-1)n$
- $a + (a+b) + (a+2b) + \cdots + (a+(n-1)b)$
- $1 + 4 + 7 + 10 + \cdots + 100$
- $\binom{n}{r} + \binom{n-1}{r} + \binom{n-2}{r} + \cdots + \binom{r}{r}$

(2) Find the values of the sums. Explain with a few sentences how you determined them and what conditions you are assuming about any variables or parameters.
- $\sum_{i=0}^{n} (a+bi)^2$
- $\sum_{i=0}^{n} i(i+1)(i+2)$
- $\sum_{i=0}^{n} \binom{i}{2}$
- $\sum_{i=0}^{n} \binom{i}{3}$

(3) Recall that we defined $\binom{n}{k}$ to be the number of ways of choosing k elements from a set of n elements. Using this definition (and not the algebraic one of $\binom{n}{k} = \frac{n!}{k!(n-k)!}$), explain why

$$\binom{n}{k} + \binom{n}{k+1} = \binom{n+1}{k+1} .$$

(4) Find the number of rectangles in an 8×8 chessboard.