Math 1200 section B - Problems, conjectures and proofs - Homework 3

Assigned: September 22, 2008 Due: October 6, 2008, 7:30pm.

Please follow these instructions carefully:

- Give your name and tutorial name (either Cat Ladies or Apple Pi). If you don't know, check the website.
- Rewrite the question so it is clear what question you are answering.
- Explain your answers clearly and completely. You should use complete sentences. Although an numerical value may answer the question, I am looking for a justification of why your answer is correct.
- These questions can be solved by applying the addition and multiplication principle so when you write your explanations you should state clearly why you add and multiply numbers together using these two fundamental rules.
- If you can't see the answer clearly to begin with, you might try to solve an easier problem by reducing the question (what if the license plate only has one letter and one number? what if you roll the die 2 times?)

(1) How many 10 letter words can be made with the letters A, B, C where the order of the letters does not matter (e.g. AAB is the same as ABA is the same as BAA)? Critique the following solution:
Answer: Each letter of an ordered word can be chosen with 3 different possibilities and since there are 10 letters chosen in each ordered word, there are 3^{10} possible ordered words by the multiplication principle. Now we find the number of unordered words by dividing by the number of ways of arranging 10 letters. This is $P(10, 10) = 10 \cdot 9 \cdot 8 \cdots 2 \cdot 1 = 10!$ so there are $3^{10}/10!$ words where the order of the letters does not matter.

(2) An Ontario license plate for a car consists of 4 letters followed by a three digit number. How many different license plates are possible? Answer: 456976000

(3) How many ways are there of rolling a die 10 times such that the results are all 4s, 5s, 6s? Answer: 59049.

(4) How many ways are there of rolling a die 10 times such that the results are all 4s, 5s and 6s and in increasing order? Answer: 66.