Math 1200 section B - Problems, conjectures and proofs

Assigned: October 6, 2008 Due: October 20, 2008, 7:30pm.

(1) Show by induction on n that for all $n \geq 0$ and $r \geq 0$,

$$\sum_{i=0}^{r} \binom{n+i}{i} = \binom{n+r+1}{r}$$

(2) Show by induction on r that for all $n \geq 0$ and $r \geq 0$,

$$\sum_{i=0}^{r} \binom{n+i}{i} = \binom{n+r+1}{r}$$

(3) Consider the following statement and proof: For $k \geq 1$ and $n \geq 0$,

$$\sum_{r=0}^{n} (r+1)(r+2)(r+3)\cdots(r+k) = \frac{(n+1)(n+2)(n+3)\cdots(n+k+1)}{k+1} \tag{1}$$

Proof: For $k = 1$, let $S_1 = \sum_{r=0}^{n}(r+1) = 1 + 2 + 3 + \cdots + (n+1)$. Then $S_1 = \frac{1}{2}(S_1 + S_1) = \frac{1}{2}((1+2+3+\cdots+(n+1)) + (1+2+3+\cdots+(n+1))) = \frac{1}{2}((1+(n+1)) + (2+n) + (3+(n-1)) + \cdots + ((n+1)+1)) = \frac{1}{2}(n+1)(n+2)$. Therefore equation (1) is true for $k = 1$ and all $n \geq 0$.

Next we assume that equation (1) holds for some fixed k. Then we have

$$\begin{aligned}
&\sum_{r=0}^{n+1}(r+1)(r+2)(r+3)\cdots(r+k) \\
&\quad = (n+2)(n+3)(n+4)\cdots(n+k+1) + \sum_{r=0}^{n}(r+1)(r+2)(r+3)\cdots(r+k) \\
&\quad = (n+2)(n+3)\cdots(n+k+1) + \frac{(n+1)(n+2)(n+3)\cdots(n+k+1)}{k+1} \\
&\quad = \frac{(n+2)(n+3)\cdots(n+k+2)}{k+1}
\end{aligned}$$

Therefore equation (1) holds for the next value of k. By the principle of mathematical induction this formula must hold for all $k \geq 0$.

There is something wrong with this proof and even if the equation above holds true the justification is not correct. Clearly state what it wrong with the explanation and give a correct version of it.

(4) Answer the question "Triangular Count" on page 198 of Thinking Mathematically. Compare your answer to what you found for "Chessboard Rectangles" on page 43 (this was on the 2nd homework assignment). Are these questions/answers related?