Math 1200 section B - Problems, conjectures and proofs - Homework 6

Assigned: October 27, 2008 Due: November 3, 2008, 7:30pm.

Do any 4 of the following 8 problems *by induction.*

(1) Show that for $n \geq 1$,
$$1!1 + 2!2 + 3!3 + \cdots + n!n = (n+1)! - 1$$
(2) Show that for $n \geq 0$,
$$1^3 + 3^3 + 5^3 + \cdots + (2n-1)^3 = n^2(2n^2 - 1)$$
(3) Show that any positive integer n can be uniquely represented in the form
$$c_1 1! + c_2 2! + c_3 3! + \cdots + c_k k!$$
where $0 \leq c_i \leq i$.

(4) Show by induction on m that
$$\sum_{k=0}^{r} \binom{m}{k}\binom{n}{r-k} = \binom{m+n}{r}$$
(5) Show by induction on n that
$$\sum_{k=0}^{r} \binom{m}{k}\binom{n}{r-k} = \binom{m+n}{r}$$
(6) Show by induction on r that
$$\sum_{k=0}^{r} \binom{m}{k}\binom{n}{r-k} = \binom{m+n}{r}$$
(7) Show that for $n \geq 1$,
$$\frac{1}{2n} \leq \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2 \cdot 4 \cdot 6 \cdots (2n)} .$$
(8) Find and prove a formula for the sum
$$\frac{1}{1 \cdot (k+1)} + \frac{1}{(k+1)(2k+1)} + \cdots + \frac{1}{((n-1)k+1)(nk+1)} .$$