

Math 1200 section B - Problems, conjectures and proofs - Homework 7

Assigned: November 3, 2008 Due: February 9, 2009, 7:30pm.

- (1) Consider the map which sends tuples $(a_1, a_2, a_3, \dots, a_n)$ where $a_i \geq 0$ to the set of polynomials in n variables $\{x_1, x_2, \dots, x_n\}$ where

$$(a_1, a_2, a_3, \dots, a_n) \mapsto x_1^{a_1} x_2^{a_2} x_3^{a_3} \dots x_n^{a_n} .$$

Is this map 1 – 1? Is it onto? Why or why not?

- (2) (see page 187 of *Thinking Mathematically*) On the quiz I asked you prove the formula for the ‘pentagonal numbers’ which were the number of lattice points on a sequence of diagrams consisting of nested pentagons. Find an analogous formula for ‘hexagonal numbers’ which correspond to the number of points in nested six sided figures. Prove your formula using induction.
- (3) Do the same for septagonal numbers for nested 7 sided figures. Find and prove by induction a formula for n -agonal numbers. What does this formula say are the 4-agonal (or square) numbers?
- (4) Find a formula for the number of solutions to

$$x_1 + x_2 + x_3 + \dots + x_k = n$$

where $n \geq 0$ and the the x_i are non-negative integers. For instance, if $n > 0$ then there is exactly one solution to $x_1 = n$. There are $n + 1$ solutions to $x_1 + x_2 = n$. How many are there when $k = 3$?