Math 1200 section B - Problems, conjectures and proofs - Homework 8

Assigned: February 9, 2008 Due: February 18, 2008, 7:30pm.

The first problem of this assignment is based on chapter 2 of 'Mathematical Proofs.' Please read the chapter (especially sections 2.1 through 2.8) and use the symbols $\wedge$, $\vee$, $\Rightarrow$, $\equiv$, $\tilde{}$ in your answer. For the second and third problem you will use the discussion that we had on February 2 to solve the questions.

(1) The following problem is from the book "What is the name of this book?" by Raymond Smullyan: In an interesting court case, four defendants A, B, C, D were involved and the following four facts were established:
- If both A and B are guilty, then C was an accomplice.
- If A is guilty, then at least one of B, C was an accomplice.
- If C is guilty, then D was an accomplice.
- If A is innocent then D is guilty.

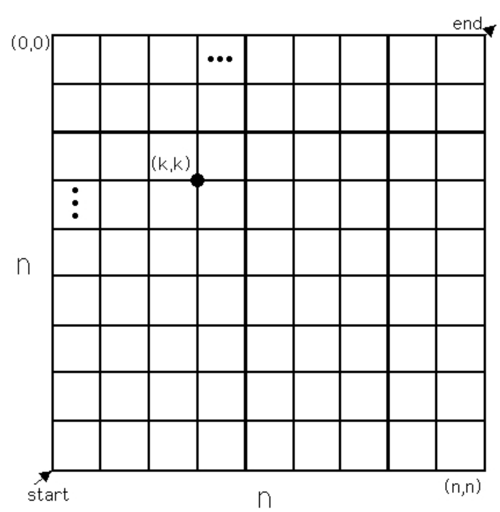
(a) Translate the 4 statements above into compound statements about the guilt of each of the four defendants using the connectives as described in chapter 2 of 'Mathematical Proofs.'

(b) Create a truth table establishing the truth value of each of the four statements in terms of the 16 possible ways of assigning the truth values about the guilt of the individual defendants.

(c) What can you conclude about the guilt or innocence of the defendants? Why?

(2) Explain why the number of paths in a rectangle with n rows and k columns from the bottom left of the rectangle to the top right using only North and East steps is $\binom{n+k}{k}$.

(3) (a) How many paths are there in the $n \times n$ square below that start in the bottom left corner and end in the upper right using only North and East steps that pass through the k^{th} point on the northwest to southeast diagonal (represented by a $\bullet$)?



(b) Establish an identity about binomial coefficients by counting paths in two different ways. First, count the number of paths in an $n \times n$ square from the bottom left corner to the top right with steps in the North and East direction (using the result from the last problem). Second, count the number of paths in the $n \times n$ square using North and East steps which pass through each of the n possible diagonal spots and apply the addition principle.