

HOMWORK ASSIGNMENT 9

DATE: MARCH 9, 2009 DUE: MARCH 16, 2009

- (1) Explain whether the following statements are true or false (use at least one English sentence for each of the expressions to say what the expression means and why it is true). Note that this is (essentially) problem 2.48 p.62 of Mathematical Proofs, Chartrand/Polimeni/Zhang. Recall that \mathbb{R} represents the set of real numbers, \mathbb{N} represents the set of natural numbers/positive integers $\{1, 2, 3, \dots\}$, and \mathbb{Q} represents the set of rational numbers.
- (a) $\exists x \in \mathbb{R}, x^2 - x = 0$
 - (b) $\forall n \in \mathbb{N}, n + 1 \geq 2$
 - (c) $\forall x \in \mathbb{R}, \sqrt{x^2} = x$
 - (d) $\forall x \in \mathbb{N}, \sqrt{x^2} = x$
 - (e) $\exists x \in \mathbb{Q}, \frac{1}{x^2} = \frac{1}{2}$
 - (f) $\exists x \in \mathbb{Q}, 3x^2 - 27 = 0$
 - (g) $\exists x \in \mathbb{R}, \exists y \in \mathbb{R}, x + y + 3 = 8$
 - (h) $\forall x, y \in \mathbb{R}, x + y + 3 = 8$
 - (i) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, x + y + 3 = 8$
 - (j) $\exists x \in \mathbb{R}, \forall y \in \mathbb{R}, x + y + 3 = 8$
 - (k) $\exists x, y \in \mathbb{R}, x^2 + y^2 = 9$
 - (l) $\forall x \in \mathbb{R}, \forall y \in \mathbb{R}, x^2 + y^2 = 9$
 - (m) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, x^2 + y^2 = 9$
- (2) Say that you have 8 numbered coins and other than the labels they all look identical but one weighs more than the others. Say also that you have a scale that you can use to weigh the coins against each other in two different pans. When you put an equal number of coins on either side of the scale, if they weigh the same, the scale says = and if they weigh different, then the scale says \neq . Explain a procedure or draw a decision tree which finds which of the 8 coins is heavier in a minimum number of weighings. Since you do not know where the coin is your procedure should be general and explain what to do for every possible location of the heavier coin.