

QUIZ 2 : MATH 1200- PROBLEMS, CONJECTURES AND PROOFS

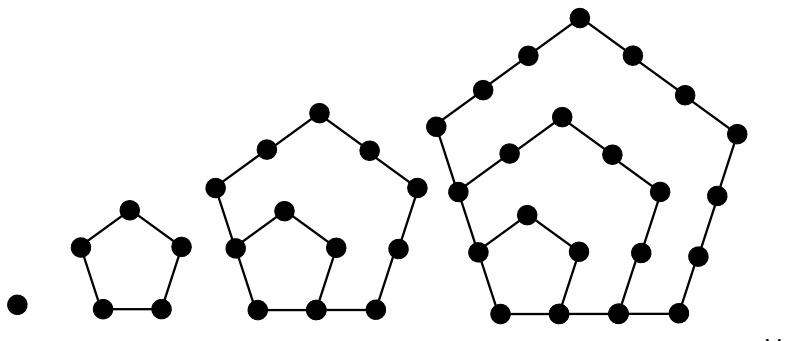
NOVEMBER 3, 2008

You have 1 hour 15 minutes to complete the following quiz. You may use any notes or books but please do not use any calculators.

- (1) (3 points) Use induction to show that for all $n \geq 0$ and all $k \geq 0$ that

$$\sum_{r=0}^n \frac{1}{(rk+1)((r+1)k+1)} = \frac{n+1}{(n+1)k+1}.$$

- (2) (4 points) The n^{th} pentagonal a_n is defined as the number of dots in the n^{th} diagram of the following sequence diagrams.



So we have that $a_1 = 1$, $a_2 = 5$, $a_3 = 12$, $a_4 = 22$ and the next term in the sequence is $a_5 = 35$. Use induction to explain why $a_n = \frac{3n^2-n}{2}$.

- (3) (a) (1 point) Explain why there are 10 solutions to the equation

$$x_1 + x_2 + x_3 = 3$$

with x_1, x_2, x_3 are all integers which are greater than or equal to 0.

- (b) (1 point) Explain why there are 66 solutions to the equation

$$x_1 + x_2 + x_3 = 10$$

with x_1, x_2, x_3 are all integers which are greater than or equal to 0.

- (c) (2 points) Find/guess at a formula for the number of solutions to the equation

$$x_1 + x_2 + x_3 = n$$

with x_1, x_2, x_3 are all integers which are greater than or equal to 0.

- (d) (3 points) Give a proof (explain why) the formula that you found in part (c) for the number of solutions to this equation is correct.