

QUIZ 3 : MATH 1200- PROBLEMS, CONJECTURES AND PROOFS

FEBRUARY 18, 2009

You have 45 minutes to complete the following quiz. You may use any notes or books but please do not use any calculators.

- (1) Show by induction that

$$1 \cdot n + 2 \cdot (n - 1) + 3 \cdot (n - 2) + \cdots + (n - 1) \cdot 2 + n \cdot 1 = \binom{n + 2}{3}.$$

Please include all necessary details and structure your induction proof clearly.

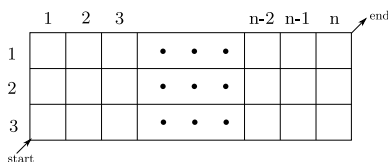
Hint: you may use $1 + 2 + 3 + \cdots + n = \binom{n + 1}{2}$ as something that we have proven before.

- (2) Consider the truth values of the following three statements.

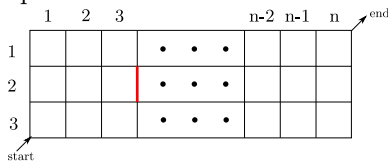
- I love Kathy
- If I love Kathy, then I love Lucy
- the previous two statements are either both true or both false.

Let A represent “I love Kathy” and B represent “I love Lucy.” Write each of the statements above as compound statements in terms of the truth values of these two simple statements. Make a truth table for the three statement in terms of the four possible ways of assigning truth values for A and B . If the third statement is true, can you determine the truth value of the other two statements?

- (3) (a) How many paths are there in the following $3 \times n$ rectangle that start at the bottom left hand corner and end in the upper right using only single steps in the North and East direction? Explain your answer.



- (b) How many paths are there in the following $3 \times n$ rectangle that start at the bottom left hand corner and end in the upper right using only single steps in the North and East direction that pass through middle vertical step in the fourth row from the left? Explain your answer.



- (c) How many paths are there in the following $3 \times n$ rectangle that start at the bottom left hand corner and pass through any of the center vertical lines in the diagram below? Add up each of the numbers of paths which pass through the highlighted vertical lines below. Explain your answer.

