QUIZ 4 : MATH 1200 - PROBLEMS, CONJECTURES AND PROOFS

MARCH 30, 2009

You have 50 minutes to complete the following quiz. You may use any notes or books but please do not use any calculators, cell phones or internet devices.

(1) Let $S = \{1, 2, 3\}$, determine if the following statements are true or false. For each statement, justify in one sentence why your answer is correct.

(a) $\forall x \in S, x^2 - 6x + 11 = 6$

(b) $\forall x \in S, x^3 - 6x^2 + 11x = 6$

(c) $\exists x \in S, x^3 - 2x^2 + 4x = 3$

(d) $\exists x \in S, x^2 + 2x - 4 = 3$

(e) $\forall x \in S, \forall y \in S, x + y = 4$

(f) $\forall x \in S, \exists y \in S, x + y = 4$

(g) $\exists x \in S, \forall y \in S, x + y = 4$

(h) $\exists x \in S, \exists y \in S, x + y = 4$

(2) For each of the following questions, prove them or find a counterexample.
   (a) Let $a, b \in \mathbb{Z}$. If a and b divide n then ab divides n.

   (b) Let $a, b \in \mathbb{Z}$. If $a^2 = b^2$, then $a = b$ or $a = -b$.

   (c) Let $a, b \in \mathbb{Z}$. If $a < b$, then $a^2 < b^2$.

(3) Prove any *one* of the following statements. State clearly which statement you are showing. Use complete sentences:

(a) Let $n \in \mathbb{Z}$. Prove that if $2 - n^2 > 0$, then $n^3 - n = 0$.

(b) Let $x, y \in \mathbb{Z}$. Prove that if $xy + y$ is even, then either x and y are both even or both odd.

(c) Let $n \in \mathbb{Z}$. If 4 divides $n + 3$, then $\binom{n}{2}$ is even.

(4) Say that you have an urn with n labeled balls numbered 1 through n. Let k be a natural number that is less than or equal to n.

(a) How many different ways are there to reach into the urn and pull out k of them and color them red? Explain your answer.

(b) How many ways are there are reaching into the urn and pulling out k of them and coloring them red such that the smallest one colored is 4? Explain your answer

(c) How many ways are there are reaching into the urn and pulling out k of them and coloring them red such that the smallest label on a colored ball is i? Explain your answer.

(d) Prove that for $n \geq k$,

$$\binom{n}{k} = \binom{n-1-k}{0} + \binom{n-k}{1} + \binom{n-k+1}{2} + \cdots + \binom{n-1}{k}$$