# QUIZ 5 : MATH 1200 PROBLEMS, CONJECTURES, PROOFS

APRIL 20, 2009

You have 45 minutes to complete the following quiz. You may use any notes or books but please do not use any calculators. Make sure that you write clearly and use complete sentences. If you provide a calculation, explain why that calculation answers the question.

Each of the following expressions below represent a relation on the set of integers. For each of the expressions:

(a) find an example of values of $a$ and $b$ which are related and
(b) find an example of values of $a$ and $b$ which are not related.
(c) Determine if the relation is reflexive and/or symmetric.
(d) Determine if the relation is transitive. If it is, give short explanation why, if not, give an example where it fails to be transitive.

(1) $a$ is related to $b$ if $0 \leq a - b$
   (a) _______ is related to _______
   (b) _______ is not related to _______
   (c) reflexive? symmetric?
   (d) transitive?

(2) $a$ is related to $b$ if $a$ divides $b$ or $b < a$.
   (a) _______ is related to _______
   (b) _______ is not related to _______
   (c) reflexive? symmetric?
   (d) transitive?

(3) $a$ is related to $b$ if $a \equiv -b \ (mod\ 6)$ or $a \equiv b \ (mod\ 6)$.
   (a) _______ is related to _______
   (b) _______ is not related to _______
   (c) reflexive? symmetric?
   (d) transitive?

(4) Give the contrapositive of the statement: if $q$ divides $a$, then there is a $k \in \mathbb{Z}$ such that $a = qk$.

(5) Give the contrapositive of the statement: if $a = k^2$ for some $k \in \mathbb{Z}$, then for all positive integers $n$, $a \equiv 1$ $(mod\ n)$.

(6) Prove or provide a counterexample: If $a, b, c, d \in \mathbb{Z}$ and $n \in \mathbb{N}$, such that $a \equiv b$ $(mod\ n)$ and $c \equiv d$ $(mod\ n)$, then show that $a + c \equiv b + d$ $(mod\ n)$.

(7) Prove or provide a counterexample: If $a, b \in \mathbb{Z}$ and $n, k \in \mathbb{N}$, such that $a^k \equiv b^k$ $(mod\ n)$, then $a \equiv b$ $(mod\ n)$.

(8) Prove or provide a counterexample: If $a, b \in \mathbb{Z}$ and $n, k \in \mathbb{N}$, such that $a \equiv b$ $(mod\ n)$, then $a^k \equiv b^k$ $(mod\ n)$.