HOMEWORK ASSIGNMENT NO. 3

DATE: OCTOBER 20, 2009 DUE: NOVEMBER 3, 2009

This homework is a little bit of drill and skill. If you have trouble with the first question, you need to do a bit of algebra review. You should explain what you are doing in the following problems. Clarity is extremely important. Go back and read your answers after you finished and ask yourself if the explanation is as simple and clear as you can make it. Rewrite your solution.

(1) Expand the following and express it as a sum of powers of 2 (assume a and b are positive integers)
$$(1^a + 1^b)(2^a + 2^b)(4^a + 4^b)(8^a + 8^b)$$
(2) In the following expressions, use telescoping sums to explain why the following statements are true or find the resulting sum.
   (a) Show that
$$1^2 \cdot 0 + 2^2 \cdot 1 + 3^2 \cdot 2 + \cdots + n^2 \cdot (n-1) = \frac{n(n-1)(3n+2)(n+1)}{12}$$
   (hint: first show that $\frac{k(k-1)(3k+2)(k+1)}{12} - \frac{(k-1)(k-2)(3k-1)k}{12} = k^2(k-1)$)
   (b) Show that
$$1^4 + 2^4 + 3^4 + \cdots + n^4 = \frac{n(2n+1)(n+1)(3n^2+3n-1)}{30}$$
   (hint: first show that $\frac{k(2k+1)(k+1)(3k^2+3k-1)}{30} - \frac{(k-1)(2k-1)k(3(k-1)^2+3(k-1)-1)}{30} = k^4$)
   (c) Show that
$$1 \cdot 2 \cdot 3 \cdot 4 + 2 \cdot 3 \cdot 4 \cdot 5 + 3 \cdot 4 \cdot 5 \cdot 6 + \cdots + n(n+1)(n+2)(n+3) = \frac{n(n+1)(n+2)(n+3)(n+4)}{5}$$
   (d) Find an expression for
$$1^2 \cdot 2^2 + 2^2 \cdot 3^2 + 3^2 \cdot 4^2 + \cdots + n^2 \cdot (n+1)^2$$
   (hint: first show that $\frac{k(k+2)(k+1)(3k^2+6k+1)}{15} - \frac{(k-1)(k+1)k(3(k-1)^2+6(k-1)+1)}{15} = k^2(k+1)^2$)

(3) Let say that you have an expression $P(k)$ which depends on the integer k and another expression $Q(n)$ which depends on the integer n and say that you are asked to to show the expression
$$P(1) + P(2) + P(3) + \cdots + P(n) = Q(n) \ .$$
Explain how telescoping sums can be used to do this (you have 4 examples in the previous problem, explain how this is done 'in general').