DISCUSSION FOR ELEVENTH TUTORIAL

DATE: MARCH 17 AND MARCH 24, 2010

(1) Consider the relation on the set of real numbers x is related to y if $|x - y| \leq 1$. Is this relation an equivalence relation? If so, what are the equivalence classes? If not, why does it fail to be an equivalence relation (which of reflexive, symmetric or transitive fails to hold)?

(2) Consider the relation on the set of real numbers x is related to y if $|x - y|$ is an integer. Is this relation an equivalence relation? If so, what are the equivalence classes? If not, why does it fail to be an equivalence relation (which of reflexive, symmetric or transitive fails to hold)?

(3) Prove by induction on n that

$$a^n - b^n = (a - b)(a^{n-1} + a^{n-1}b + a^{n-2}b^2 + \cdots + b^{n-1})$$

Hint: $a^n - b^n = a^n - a^{n-1}b + a^{n-1}b - b^n$.

(4) Show that a is related to b if $a^3 \equiv b^3 \ (mod\ 12)$ is an equivalence relation on the set of integers $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$. What are the equivalence classes of this relation?

(5) (a) Recall the Maclauren series for the following three functions from calculus:

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

$$sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

Write them using the shorthand summation notation as a sum over a variable n with limits $n = 0$ to ∞.

(b) Use the above equations to show that

$$e^{\pi i} = -1$$

where i is a value such that $i^2 = -1$.