

### Homework Assignment no. 3

Date: Due week of Nov 29 (Nov 29 for lecture B and Dec 2 for lecture C)

Your assignment should include complete sentences and explanations and not just a few equations, tables or numbers. A solution will not receive full credit unless you explain what your answer represents and where it came from. You may discuss the homework with other students in the class, but please write your own solutions.

For some of the problems below you may need to know the division algorithm (which we will discuss in class):

For every  $m > 0$ , and every integer  $a$ , there exists integers  $q, r$  such that  $0 \leq r < m$  and  $a = qm + r$ .

- (1) Translate the following statements into logical shorthand using the symbols  $\forall, \exists, \in, \mathbb{Z}, \mathbb{R}, \mathbb{Q}$  and  $O(n) = n$  is odd,
  - (a) If  $n$  is an integer, then  $n^2/2 - 3n/2$  is an integer.
  - (b) If  $x$  is an integer, then  $x^2/2 + 3x/2 + 5$  is an integer.
  - (c) If  $y$  is an integer, then  $y^3/3 + y^2 - 10y/3 + 2$  is an integer.
  - (d) For every rational number  $x$ , there is an integer  $y$  such that  $xy$  is a integer.
  - (e) The product of a rational number and an integer is a rational number.
- (2) The statements above are all true, but I would like you to explain why. For each part (1a)-(1e) provide a justification.
- (3) Explain in English what the following mathematical statements mean. Which of the statements are true, which are false? Why?
  - (a)  $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, x < y$
  - (b)  $\exists x \in \mathbb{R}, \forall y \in \mathbb{R}, x < y$
  - (c)  $\exists x \in \mathbb{R}, \exists y \in \mathbb{R}, x < y$
  - (d)  $\forall x \in \mathbb{R}, \forall y \in \mathbb{R}, x < y$
  - (e)  $\forall x \in \mathbb{Z}, \exists y, z \in \mathbb{Z}, x^2 + y^2 = z^2$
  - (f)  $\exists x, y, z \in \mathbb{Z}, x^2 + y^2 = z^2$
  - (g)  $\exists x, y, z \in \mathbb{Z}, x^3 + y^3 = z^3$
  - (h)  $\exists x, y, z \in \mathbb{Z}, x^3 + y^3 = z$