HOMEWORK ASSIGNMENT NO. 4

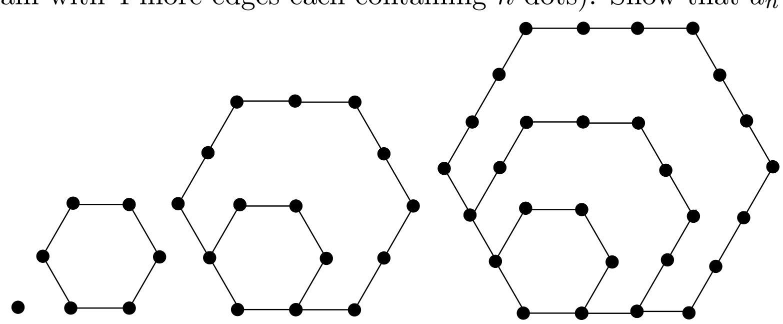
DATE GIVEN: JANUARY 23, 2012 DUE: FEBRUARY 27, 2011

You are not expected to write up of the problems on this homework assignment, however it is a good idea to do all of them. You should do all of the following problems by induction.

NOTE: Change of plans. If you got 9, 10, 11 or 12 on the fourth quiz, I don't care if you hand in this assignment and I give you a free pass on it. If you got a 6,7,8 on the quiz I would like you to do the homework as planned. If you got less than 6 on the quiz, then you must do ALL THE PROBLEMS ON THIS PAGE.

For those of you that got 6,7 or 8 on the quiz: First do problem number 1. Do problem number (your answer to (1)(a)) + 2. Do problem number (your answer to (1)(b)) + 5 in two ways: first, by induction; then by telescoping sums. Do (your answer to (1)(c)) + 9.

(1) The following computations will determine which problems you do in this assignment.
  (a) Compute your student id number (mod 3) as a number between 0 and 2.
  (b) Compute your student id number (mod 4) as a number between 0 and 3.
  (c) Compute your student id number (mod 5) as a number between 0 and 4.
(2) Show that 3^{n+1} divides $2^{3^n} + 1$ for all $n \geq 0$.
(3) Let $a_n^{(6)}$ be the number of points in the n^{th} diagram of the sequence of drawings of nested hexagons shown below (the n^{th} diagram is defined as a hexagonal diagram that contains the previous diagram with 4 more edges each containing n dots). Show that $a_n^{(6)} = n(2n-1)$.



(4) Prove that if $a_1, a_2, \ldots, a_n \geq 1$, then
$$2^{n-1}(a_1 a_2 \cdots a_n + 1) \geq (1 + a_1)(1 + a_2) \cdots (1 + a_n) \ .$$
(5) Show that for $n > 0$,
$$1^4 + 3^4 + 5^4 + \cdots + (2n-1)^4 = (48n^5 - 40n^3 + 7n)/15 \ .$$
(6) Show that for $n > 0$,
$$1^3 + 3^3 + 5^3 + \cdots + (2n-1)^3 = n^2(2n^2 - 1) \ .$$

(7) Show that for $n > 0$,
$$1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + \cdots + n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4} .$$
(8) Show that for $n > 0$,
$$\frac{1}{1 \cdot 5} + \frac{1}{5 \cdot 9} + \cdots + \frac{1}{(4n-3)(4n+1)} = \frac{n}{4n+1} .$$
(9) Show that if $a_n = 3a_{n-1} - 2a_{n-2} + 2$ and $a_0 = a_1 = 1$ then show that $a_n = 2^{n+1} - (2n+1)$ for all $n > 1$.
(10) Show that if $a_n = a_{n-1} + n(n-1)$ and $a_0 = 1$ then conjecture a closed form formula for a_n and prove that it is correct by induction.
(11) Show that if $a_n = a_{n-1} + a_{n-2} + n$ and $a_0 = 1$ and $a_{-1} = 0$, then $a_n = 2(F_{n+3} - 1) - (n+1)$ for all $n \geq 1$ (where: $F_1 = F_2 = 1$ and $F_n = F_{n-1} + F_{n-2}$ for all $n \geq 3$).
(12) Show that if $a_n = 2a_{n-1} + 2^n$ and $a_0 = 1$ then prove $a_n = (n+1)2^n$.
(13) Show that if $a_n = 2a_{n-1} + (-1)^n$ and $a_0 = 2$ then show that $a_n = (5 \cdot 2^n + (-1)^n)/3$.