

## HOMWORK ASSIGNMENT NO. 2

DATE: ASSIGNED OCT 2, 2012 - DUE OCTOBER 29, 2012

Your assignment should include complete sentences and explanations and not just a few equations or numbers. A solution will not receive full credit unless you explain what your answer represents and where it came from. You may discuss the homework with other students in the class, but please write your own solutions. The first question is adapted from “What is the Name of This Book?” by Raymond Smullyan.

- (1) A bank was robbed and Inspector Craig and Sergeant McPherson were on the case trying to establish the guilt or innocence of four suspects Alice, Bob, Carol and Dave. The nefarious characters are the only people who could be involved in these bank robberies and at least one of them is guilty. In each case the Inspector and Sergeant establish certain facts.

Write an argument in words to establish the guilt or innocence of Alice, Bob and Carol and Dave. Note that the clues provided may not be sufficient to determine the guilt and innocence of all of the suspects, but should be sufficient to establish the guilt of at least one person.

Say that we establish that:

- (1) If Alice was guilty, then she had exactly one other accomplice.
- (2) Bob and Carol were both together at the time of the crime.
- (3) If exactly two are guilty then Alice is one of them.
- (4) Bob and Dave never work together.
- (5) If both Bob and Carol were not involved then Dave is guilty.

Translate each of the clues to a truth valued sentence using the connectives *and*, *or*, *not* and *if ... then* and the propositions:  $A$  representing the statement “Alice is guilty,”  $B$  representing the statement “Bob is guilty,”  $C$  representing “Carol is guilty,” and  $D$  representing “Dave is guilty.” Create a truth table establishing the truth values of the clues in terms of the truth values of  $A$ ,  $B$ ,  $C$  and  $D$ .

- (2) The following questions are about subsets of the integers  $\{1, 2, 3, \dots, 10\}$ . The numbers that are in the subset are sometimes referred to as the elements of the set and the number of elements is the size of the subset.

Give an example of one subset that makes the following statements true and one subset that makes the statement false (if possible) and explain why your subset makes the statement true or false. If the sentence is always true or always false for all subsets, then explain why.

- (a) The subset  $S$  does not contain 1 but does contain 10, or contains both 1 and 10 or the maximum value in  $S$  is less than 7.
- (b) The subset  $S$  contains 1 and it does not contain 9, and for every  $x$  in  $S$ ,  $x + 4$  is also an element in  $S$ .
- (c) If  $S$  has more than 5 elements or 1 is an element of  $S$ , then 10 is not an element of  $S$ .
- (d) For every  $x$  in  $S$ , if  $x$  is prime, then  $x + 1$  is in  $S$ .

- (e) If  $S$  contains 1 and 10 and the size of  $S$  is more than 7, then the largest value in  $S$  less than 10 is at least 5.
  - (f) There is an  $x$  in  $S$  such that either  $x$  is odd and prime or  $3x$  is in  $S$ .
- (3) Let  $A = \{1, 2, 3\}$ ,  $B = \{1, 2, 3, 4\}$ ,  $C = \{1, 2, 3, 4, 5\}$
- (a) Let  $S = \{(b, a) : b \in B, a \in A, a+b \text{ is odd}\}$  and let  $T$  be the non-empty proper subsets of  $A$ . What are  $|S|$  and  $|T|$ ? List the elements of  $S$  and  $T$ .
  - (b) What is  $|A \times B \times C|$ ?
  - (c) Let  $V = \{(a, b) : (a, |b|) \in A \times C\}$ . Let  $U = \{(a, b) \in V : ak = b \text{ for some } k \in \mathbb{Z}\}$ . List the elements of  $U$ .