HOMEWORK ASSIGNMENT NO. 4

DATE GIVEN: JANUARY 14, 2013 DUE (FOR SOME): FEBRUARY 4, 2013

For practice for the upcoming quiz, you should do all of the following problems by induction. After the quiz is over I will place you into three groups depending on how well you did.

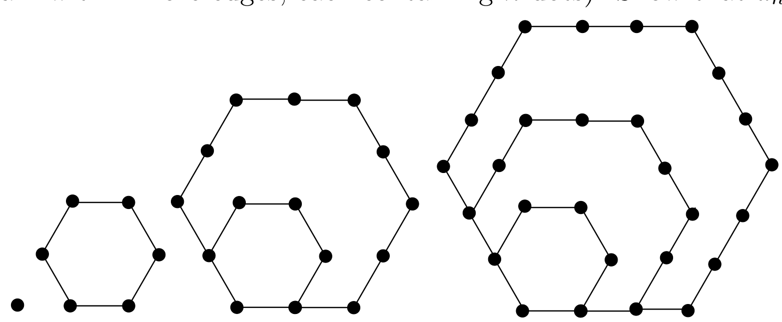
- Those that do well (miss just a few points), will get a free pass on the homework assignment and will not be required to hand in this assignment at all.
- Those that miss more than a few points, but still pass the quiz must do four problems on this homework. Do problem #1 and then follow the instructions for that problem to complete three other questions on this assignment.
- Those that do not pass the quiz will have to complete all of the questions 2 through 12.

Note, when you complete problems #5 through #8, do this (a) by induction and (b) by telescoping sums.

(1) The following computations will determine which problems those that are in the second group do in this assignment.
   (a) Compute your student id number (mod 3) as a number between 0 and 2 and then add 2. Complete this problem.
   (b) Compute your student id number (mod 4) as a number between 0 and 3 and then add 5. Complete this problem.
   (c) Compute your student id number (mod 5) as a number between 0 and 4 and then add 9. Complete this problem.

(2) Show that 3^{n+1} divides $2^{3^n} + 1$ for all $n \geq 0$.

(3) Let $a_n^{(6)}$ be the number of points in the n^{th} diagram of the sequence of drawings of nested hexagons shown below (the n^{th} diagram is defined as a hexagonal diagram that contains the previous diagram with 4 more edges, each containing n dots). Show that $a_n^{(6)} = n(2n-1)$.



(4) Prove that if $a_1, a_2, \ldots, a_n \geq 1$, then

$$2^{n-1}(a_1 a_2 \cdots a_n + 1) \geq (1 + a_1)(1 + a_2) \cdots (1 + a_n) \ .$$

NB: Do problems #5 through #8 by (a) induction and (b) by telescoping sums.

(5) Show that for $n > 0$,
$$1^4 + 3^4 + 5^4 + \cdots + (2n-1)^4 = (48n^5 - 40n^3 + 7n)/15\ .$$

(6) Show that for $n > 0$,
$$1^3 + 3^3 + 5^3 + \cdots + (2n-1)^3 = n^2(2n^2 - 1)\ .$$

(7) Show that for $n > 0$,
$$1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + \cdots + n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}\ .$$

(8) Show that for $n > 0$,
$$\frac{1}{1 \cdot 5} + \frac{1}{5 \cdot 9} + \cdots + \frac{1}{(4n-3)(4n+1)} = \frac{n}{4n+1}\ .$$

(9) Show that if $a_n = 3a_{n-1} - 2a_{n-2} + 2$ and $a_0 = a_1 = 1$ then show that $a_n = 2^{n+1} - (2n+1)$ for all $n > 1$.

(10) Show that if $a_n = a_{n-1} + n(n-1)$ and $a_0 = 1$ then conjecture a closed form formula for a_n and prove that it is correct by induction.

(11) Show that if $a_n = a_{n-1} + a_{n-2} + n$ and $a_0 = 1$ and $a_{-1} = 0$, then $a_n = 2(F_{n+3} - 1) - (n+1)$ for all $n \geq 1$ (where: $F_1 = F_2 = 1$ and $F_n = F_{n-1} + F_{n-2}$ for all $n \geq 3$).

(12) Show that if $a_n = 2a_{n-1} + 2^n$ and $a_0 = 1$ then prove $a_n = (n+1)2^n$.

(13) Show that if $a_n = 2a_{n-1} + (-1)^n$ and $a_0 = 2$ then show that $a_n = (5 \cdot 2^n + (-1)^n)/3$.