

HOMEWORK ASSIGNMENT NO. 5

DATE GIVEN: FEBRUARY 4, 2013 DUE: MARCH , 2013

- (1) Let n be an integer. Justify the following statements.
- (a) The last digit of n is even if and only if n is divisible by 2.
 - (b) The last two digits of n are divisible by 4 if and only if n is divisible by 4.
 - (c) The last three digits of n are divisible by 8 if and only if n is divisible by 8.
 - (d) The last k digits of n are divisible by 2^k if and only if n is divisible by 2^k .

Recall that we call a function $f : A \rightarrow B$ 'injective' or '1-1' if for all $x, y \in A$, if $f(x) = f(y)$, then $x = y$. And we call a function 'surjective' or 'onto' if for every $y \in B$, there is an $x \in A$ such that $f(x) = y$.

- (2) Consider the function $f : \mathbb{Z} \rightarrow \mathbb{Z}$ where $f(x) = x^3 - 4x$
- (a) Is f injective? Why or why not? If it is demonstrate or explain why. If not, give an example of where it fails to be injective.
 - (b) Is f surjective? Why or why not? If it is demonstrate or explain why. If not, give an example of where it fails to be surjective.
- (3) Consider the function $g : \mathbb{R} \rightarrow \mathbb{R}$ where $g(x) = x^3 - 1$.
- (a) Is g injective? Why or why not? If it is demonstrate or explain why. If not, give an example of where it fails to be injective.
 - (b) Is g surjective? Why or why not? If it is demonstrate or explain why. If not, give an example of where it fails to be surjective.
- (4) Let \mathfrak{S} be the set of finite subsets of \mathbb{Z} . Consider the following three functions $f, g, h : \mathfrak{S} \rightarrow \mathfrak{S}$ defined below.
- (a) Let $f(S) = S \cap \{2n : n \in \mathbb{Z}\}$. For example $f(\{1, 4, 5, 8, 10\}) = \{4, 8, 10\}$.
 - (i) Is f onto? Why or why not?
 - (ii) Is f 1-to-1? Why or why not?
 - (b) Let $g(S) = \{n + 1 : n \in S\}$. Also let $g(\{\}) = \{\}$. For example $g(\{-3, 5, 6\}) = \{-2, 6, 7\}$. Another example is $g(\{-1, 2, 3, 4, 5, 10\}) = \{0, 3, 4, 5, 6, 11\}$.
 - (i) Is g onto? Why or why not?
 - (ii) Is g 1-to-1? Why or why not?
 - (c) Let $h(S) = S \cap \{n + 1 : n \in S\}$. For example $h(\{-3, 5, 6\}) = \{-3, 5, 6\} \cap \{-2, 6, 7\} = \{6\}$. Another example is $h(\{-1, 2, 3, 4, 5, 10\}) = \{-1, 2, 3, 4, 5, 10\} \cap \{0, 3, 4, 5, 6, 11\} = \{3, 4, 5\}$.
 - (i) Is h onto? Why or why not?
 - (ii) Is h 1-to-1? Why or why not?